London Academy of Mathematics

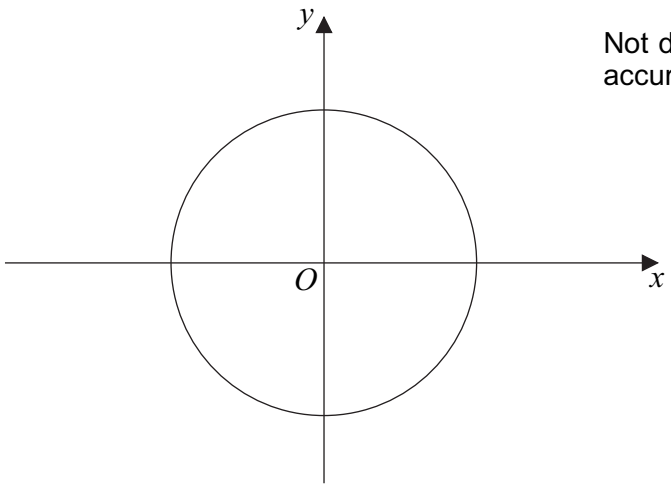
AQA - GCSE Maths
Aiming for a 9
Paper 2&3 (calculator) - Set 1
Worked solutions

Compiled by Chris Nunes, created by AQA Education

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Here is a sketch of the circle $x^2 + y^2 = 36$



Not drawn accurately

Work out the circumference of the circle.

$r^2 = 36$

$r = 6$

$d = 12$

Answer 12 *(3 marks)*

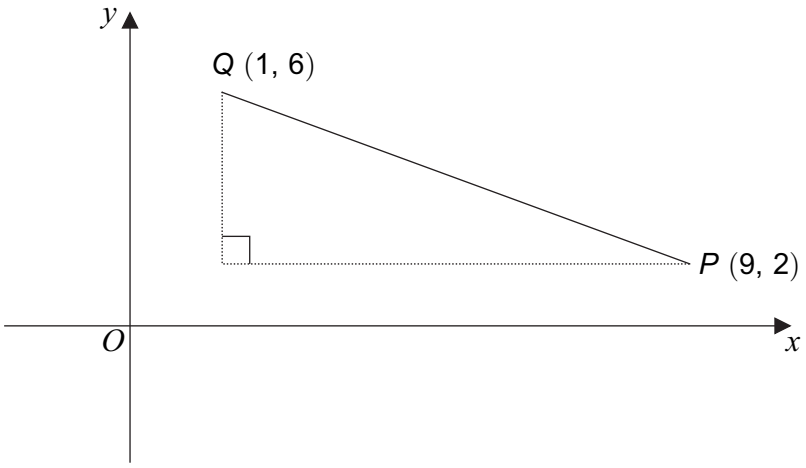
Turn over for the next question

3

Turn over ▶

Do not write outside the box

3



Not drawn accurately

Work out the length of *PQ*.
Give your answer to 3 significant figures.

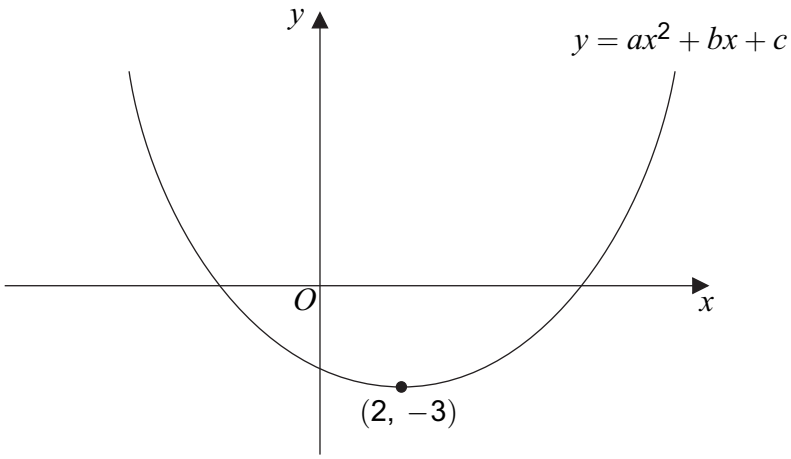
$|PQ| = \sqrt{(9-1)^2 + (2-6)^2}$

$= \sqrt{8^2 + (-4)^2} = \sqrt{80} = 8.94$ (to 3sf)

PQ = 8.94 *(4 marks)*

Do not write outside the box

4 A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is $(2, -3)$.



For the sketch shown, circle the correct answer in each of the following.

4 (a) The value of a is

zero positive negative *(1 mark)*

4 (b) The value of c is

zero positive negative *(1 mark)*

4 (c) The solutions of $ax^2 + bx + c = 0$ are

both zero both positive both negative one positive and one negative *(1 mark)*

4 (d) The **number** of solutions of $ax^2 + bx + c = -6$ is

0 1 2 3 *(1 mark)*

4 (e) The equation of the tangent to $y = ax^2 + bx + c$ at $(2, -3)$ is

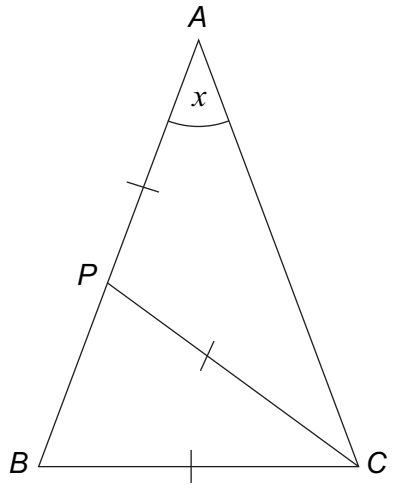
$x = 2$ $y = 2$ $x = -3$ $y = -3$ *(1 mark)*

11

Turn over ▶

Do not write outside the box

5 *ABC* is a triangle.
P is a point on *AB* such that $AP = PC = BC$
Angle $BAC = x$



Not drawn accurately

5 (a) Prove that angle $ABC = 2x$

$\angle ACP = x$ APC is isosceles

$\angle APC = 180 - 2x$ angles in a triangle add up to 180°

$\angle CPB = 180 - (180 - 2x) = 2x$ angles on a straight line add up to 180°

$\angle ABC = \angle CPB = 2x$ BPC is isosceles. *(3 marks)*

5 (b) You are also given that $AB = AC$

Work out the value of x.

$\angle ACB = \angle ABC = 2x$ Since ABC is now isosceles

$\therefore 2x + 2x + x = 180^\circ$

$5x = 180^\circ$

$x = 36^\circ$

$x =$ 36 degrees *(3 marks)*

Do not write outside the box

6 (a) Expand $3x(2x - 5y)$

Answer $6x^2 - 15xy$ *(2 marks)*

6 (b) Expand and simplify $(3x + 2y)(3x - 4y)$

$9x^2 - 12xy + 6xy - 8y^2$

$= 9x^2 - 6xy - 8y^2$

Answer $9x^2 - 6xy - 8y^2$ *(3 marks)*

6 (c) Work out the ratio $(3x + 2y)(3x - 4y) : 3x(2x - 5y)$ when $y = 0$

Give your answer as simply as possible.

$(3x)(3x) : 3x(2x)$

$= 9x^2 : 6x^2$

$= 9 : 6$

$= 3 : 2$

Answer 3 : 2 *(2 marks)*

7 $1 \leqslant m \leqslant 5$ and $-9 \leqslant n \leqslant 2$

7 (a) Work out an inequality for $m + n$.

Answer $-8 \leqslant m + n \leqslant 7$ *(2 marks)*

7 (b) Work out an inequality for $(m + n)^2$.

Answer $0 \leqslant (m + n)^2 \leqslant 64$ *(2 marks)*

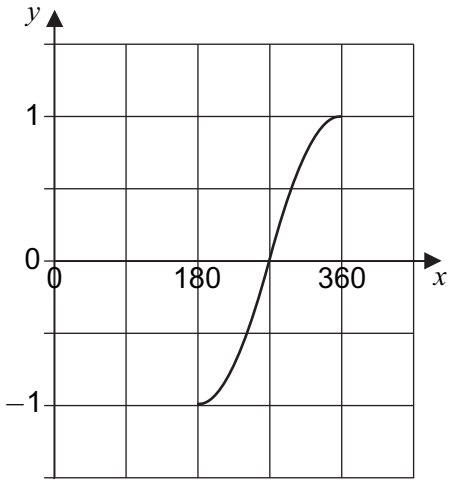
17

Turn over ▶

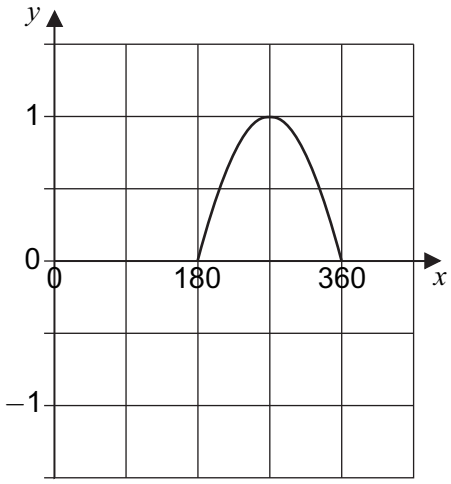
Do not write outside the box

8 Four graphs are shown for $180° \leqslant x \leqslant 360°$

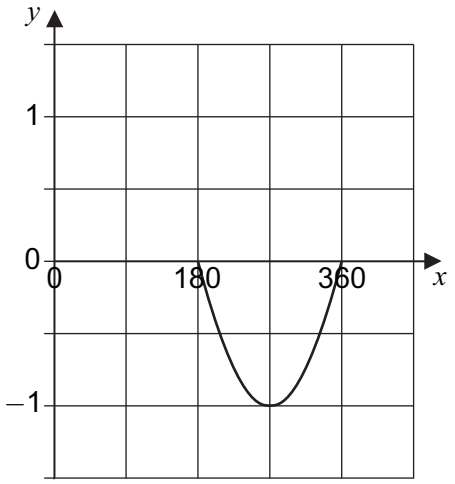
Graph A



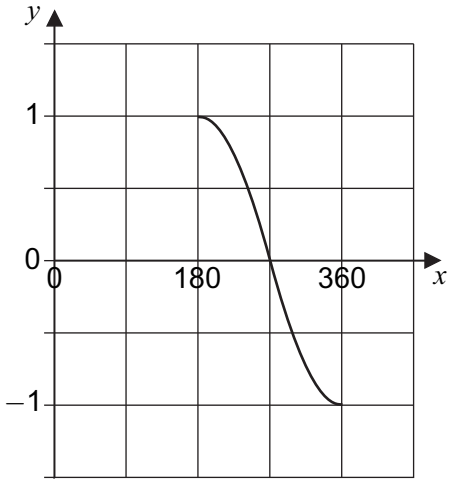
Graph B



Graph C



Graph D



8 (a) Which graph is $y = \sin x$?

Graph ....C.... *(1 mark)*

8 (b) Which graph is $y = \cos x$?

Graph ....B.... *(1 mark)*

Do not write outside the box

9 Here is a formula.

$$5t + 3 = 4w(t + 2)$$

9 (a) Rearrange the formula to make t the subject.

$5t + 3 = 4wt + 8w$

$5t - 4wt = 8w - 3$

$t(5 - 4w) = 8w - 3$

$t = \frac{8w - 3}{5 - 4w}$

Answer $t = \frac{8w - 3}{5 - 4w}$ *(4 marks)*

9 (b) Work out the exact value of t when $w = -\frac{1}{8}$

Give your answer in its simplest form.

$t = \frac{8(-1/8) - 3}{5 - 4(-1/8)} = \frac{-1 - 3}{5 + 1/2} = \frac{-4}{11/2}$

$= \frac{-8}{11}$

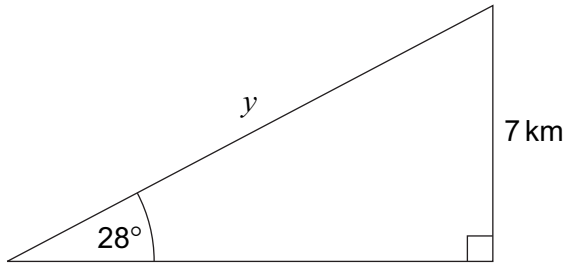
$t =$ $\frac{-8}{11}$ *(3 marks)*

9

Turn over ▶

Do not write outside the box

10 An aircraft flies y kilometres in a straight line at an angle of elevation of 28°.
The gain in height is 7 kilometres.



Not drawn accurately

Work out the value of y.

$\sin(28) = \frac{7}{y}$

$y \sin(28) = 7$

$y = \frac{7}{\sin(28)} = 14.9$ km to 3sf

$y =$ 14.9 km *(3 marks)*

11 A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\frac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

$V_{sphere} = V_{hemisphere}$

$\Rightarrow \frac{4}{3}\pi x^3 = \frac{2}{3}\pi y^3$

$2x^3 = y^3$

$\sqrt[3]{2}\,x = y$

$\therefore \frac{y}{x} = \frac{\sqrt[3]{2}\,x}{x} = \sqrt[3]{2}$

$= 2^{1/3}$

$\frac{y}{x} =$ $2^{1/3}$ *(3 marks)*

Do not write outside the box

12 Expand and simplify $(t+4)^3$

$(t+4)(t+4)(t+4)$

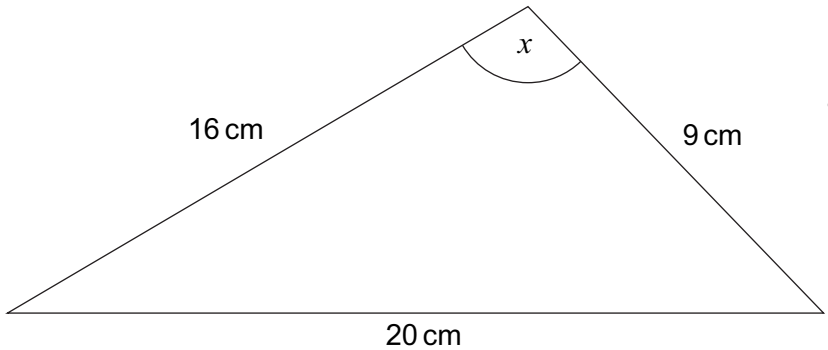
$= (t^2+8t+16)(t+4)$

$= t^3+8t^2+16t+4t^2+32t+64$

$= t^3+12t^2+48t+64$

Answer $t^3+12t^2+48t+64$ *(3 marks)*

13



Not drawn accurately

Work out angle x.

using cosine rule:

$\cos(x) = \frac{b^2+c^2-a^2}{2bc} = \frac{16^2+9^2-20^2}{2(16)(9)}$

$\cos(x) = \frac{-7}{32}$ $x = 102.6°$ to 1dp

$x =$ 102.6 degrees *(3 marks)*

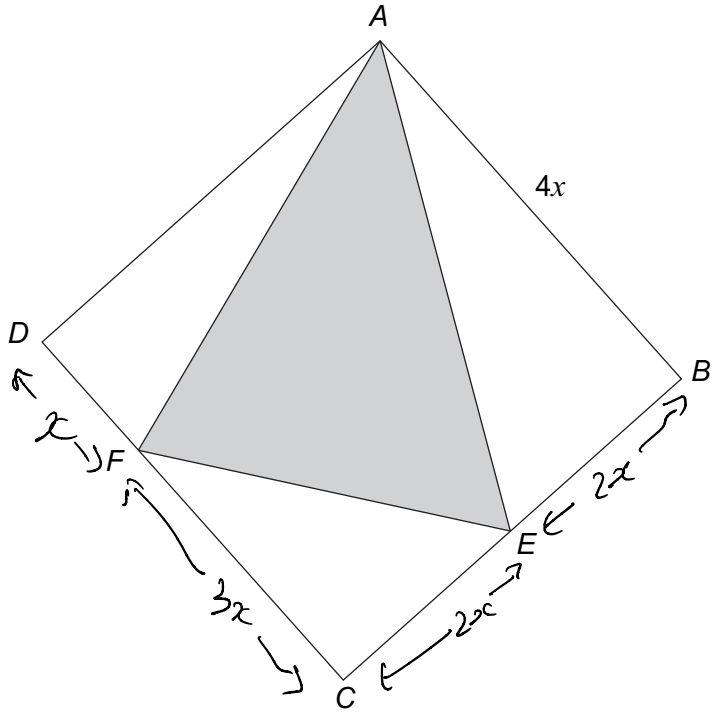
12

Turn over ▶

Do not write outside the box

17 *ABCD* is a square of side length $4x$.

E is the midpoint of *BC*.
$DF : FC = 1 : 3$



Not drawn accurately

You are given that

area of triangle $AEF = kx^2$

Work out the value of k.

Area of Square $= (4x)^2 = 16x^2$

Area of ADF $= 4x \times x \times \frac{1}{2} = 2x^2$

Area of EFC $= 3x \times 2x \times \frac{1}{2} = 3x^2$

Area of ABE $= 4x \times 2x \times \frac{1}{2} = 4x^2$

$\therefore$ AEF $= 16x^2 - (2x^2 + 3x^2 + 4x^2)$

$= 7x^2$

$k =$ 7 *(5 marks)*

12

Turn over ▶

Do not write outside the box

18 $(x-5)^2 + a \equiv x^2 + bx + 28$

Work out the values of a and b.

$x^2 - 10x + 25 + a \equiv x^2 + bx + 28$

$[x^2]$: $1 = 1$

$[x]$: $-10 = b$

$[k]$: $25 + a = 28$

$a = 3$

Comparing coefficients.

$a =$ 3 $b =$ −10 *(3 marks)*

19 Solve the simultaneous equations

$$x + y = 4$$
$$y^2 = 4x + 5$$

Do **not** use trial and improvement.

Make x or y subject of linear equation:

$x = 4 - y$

$y^2 = 4(4-y) + 5$

$y^2 = 16 - 4y + 5$

$y^2 = -4y + 21$

$y^2 + 4y - 21 = 0$

$(y+7)(y-3) = 0$

$y = -7$ or $y = 3$

when $y = -7$, $x = 11$

when $y = 3$, $x = 1$

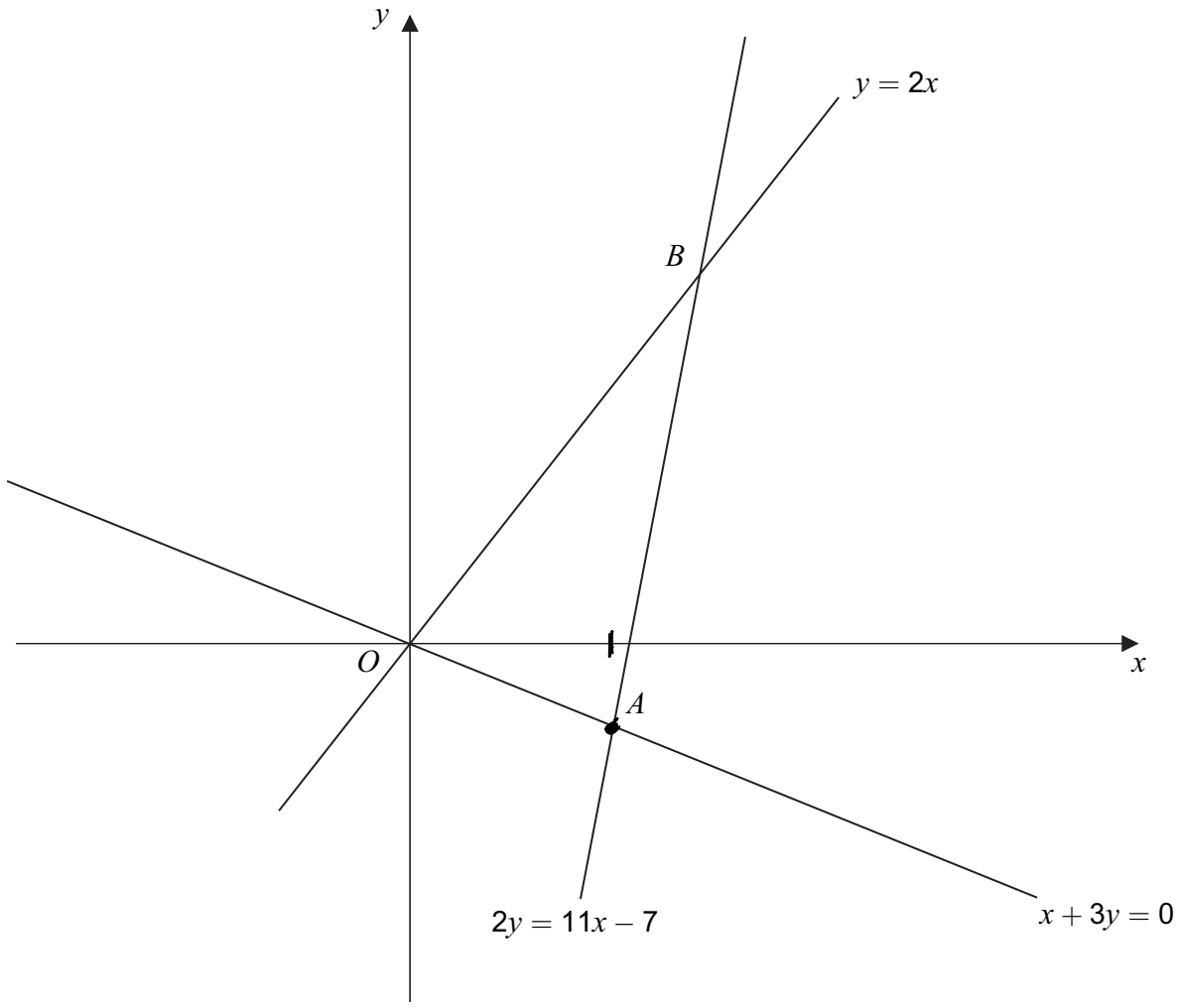
Answer $x = 11$, $y = -7$ or $x = 1$, $y = 3$ *(6 marks)*

Do not write outside the box

21 The equations of three straight lines are

$y = 2x$ $\quad$ $x + 3y = 0$ $\quad$ $2y = 11x - 7$

The lines intersect at the points O, A and B as shown on this sketch.



Do not write outside the box

Show that length OB = length AB

First let's find coordinates of B:

By substitution:

$y = 2x$

$2y = 11x - 7$

$2(2x) = 11x - 7$

$4x = 11x - 7$

$-7x = -7$

$x = 1$

$\Rightarrow y = 2$

so $B = (1, 2)$

By elimination

① $2y = 4x$

② $2y = 11x - 7$

① − ② $0 = -7x + 7$

$7x = 7$

$x = 1$

$\Rightarrow y = 2$

so $B = (1, 2)$

To find coordinates of A:

① $x + 3y = 0 \Rightarrow x = -3y$

② $2y = 11x - 7$

sub $x = -3y$ into ②:

$2y = -33y - 7$

$35y = -7 \quad y = -\frac{1}{5} \Rightarrow x = \frac{3}{5}$

(6 marks)

Turn over for the next question

$|OB| = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}$

$|AB| = \sqrt{(\frac{3}{5} - 1)^2 + (-\frac{1}{5} - 2)^2}$

$= \sqrt{\frac{4}{25} + \frac{121}{25}} = \sqrt{\frac{125}{25}} = \sqrt{5}$

$\therefore |OB| = |AB|$

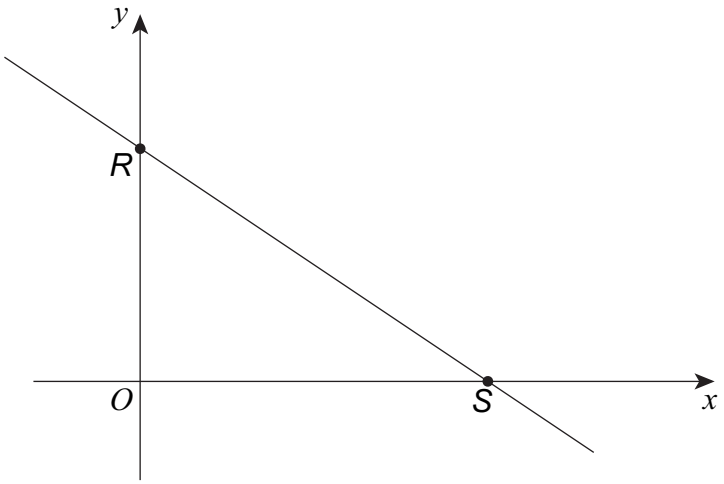
6

Turn over ▶

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A sketch of $2x + 3y = 12$ is shown.



1 (a) Work out the coordinates of R.

$x = 0$

Answer (0 , 4) *(1 mark)*

1 (b) Work out the coordinates of the midpoint of RS.

S has coordinates (6, 0)

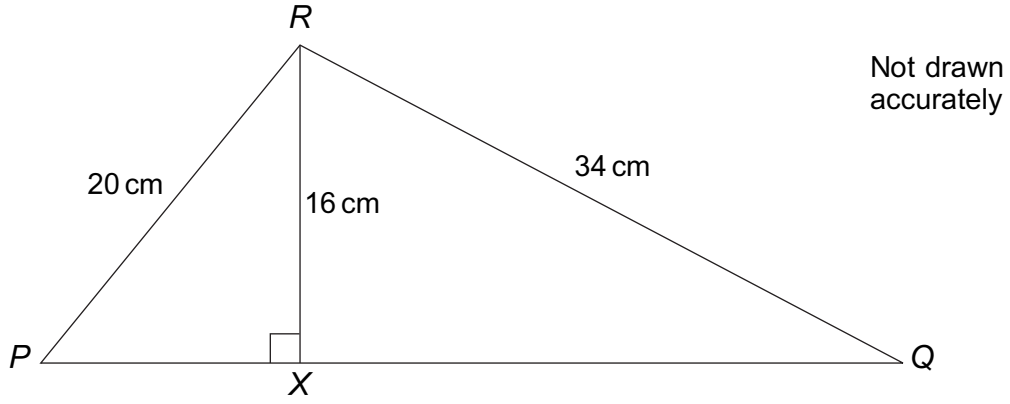
Answer (3 , 2) *(2 marks)*

3

Turn over ▶

Do not write outside the box

2 In triangle *PQR*, *X* is a point on *PQ*.
RX is perpendicular to *PQ*.



Work out the ratio *PX* : *XQ*
Give your answer in its simplest form.

$(PX)^2 = 20^2 - 16^2 = 144$

$PX = \sqrt{144} = 12$

$(XQ)^2 = 34^2 - 16^2 = 900$

$XQ = \sqrt{900} = 30$

$PX : XQ = 12 : 30$
$= 2 : 5$

Answer 2 : 5 *(4 marks)*

Do not write outside the box

3 Solve $5d - 3 > d + 17$

$4d > 20$

$d > 5$

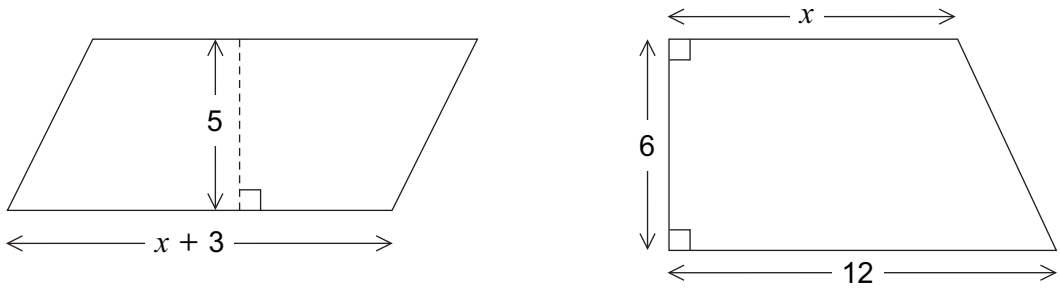
Answer ..... *(2 marks)*

9

Turn over ▶

Do not write outside the box

5 A parallelogram and a trapezium are shown.
All lengths are in centimetres.



Not drawn accurately

The area of the parallelogram is equal to the area of the trapezium.

Work out the value of x.

$A_{parallelogram} = A_{trapezium}$

$5(x+3) = \frac{(12+x) \times 6}{2}$

$5x + 15 = (12+x)3$

$5x + 15 = 36 + 3x$

$2x = 21$

$x = 10.5$

$x =$ 10.5 cm *(4 marks)*

Do not write outside the box

7 Here are the first four terms of a sequence.

$4a \qquad 9a \qquad 14a \qquad 19a$

The nth term of the sequence is $\dfrac{10n-2}{3}$

Work out the value of a.

Sub $n=1$ in : First term $= \frac{8}{3}$

$\therefore 4a = \frac{8}{3} \Rightarrow a = \frac{8}{3} \times \frac{1}{4} = \frac{2}{3}$

$a =$ $\frac{2}{3}$ *(2 marks)*

8 (a) Factorise fully $5m^2 - 20p^2$

$5(m^2 - 4p^2) = 5(m-2p)(m+2p)$

Answer $5(m-2p)(m+2p)$ *(3 marks)*

8 (b) You are given that $p = 15$ and $5m^2 - 20p^2 = 0$

Using your answer to part (a), or otherwise, work out the values of m.

$5m^2 - 20(15)^2 = 0$

$5m^2 - 4500 = 0$

$m^2 - 900 = 0$

$m^2 = 900$ OR $(m-30)(m+30) = 0$

$m = \pm\sqrt{900} = \pm 30$ $\quad m = 30$ or $m = -30$

Answer 30 or -30 *(2 marks)*

Do not write outside the box

9 (a) Expand $(x+m)(x+n)$

.....

.....

Answer $x^2 + mx + nx + mn$ *(1 mark)*

9 (b) $x^2 + qx + r \equiv (x+m)(x+n)$

Use your answer to part (a) to write q and r in terms of m and n.

$q =$ $m+n$

$r =$ mn *(2 marks)*

9 (c) r is an odd integer.

Use your answer to part (b) to explain why q is an even integer.

if r is odd then both neither of m and n can be even since even × odd and even × even = even.
∴ m & n are both odd.
odd + odd = even
so q is even

(2 marks)

12

Turn over ▶

Do not write outside the box

10 $S = \dfrac{a}{1 - r}$

10 (a) Show that $r = \dfrac{S - a}{S}$

$S(1 - r) = a$

$S - rS = a$

$\times -1$ $-rS = a - S$ $\times -1$

$rS = S - a$

$r = \dfrac{S - a}{S}$

(3 marks)

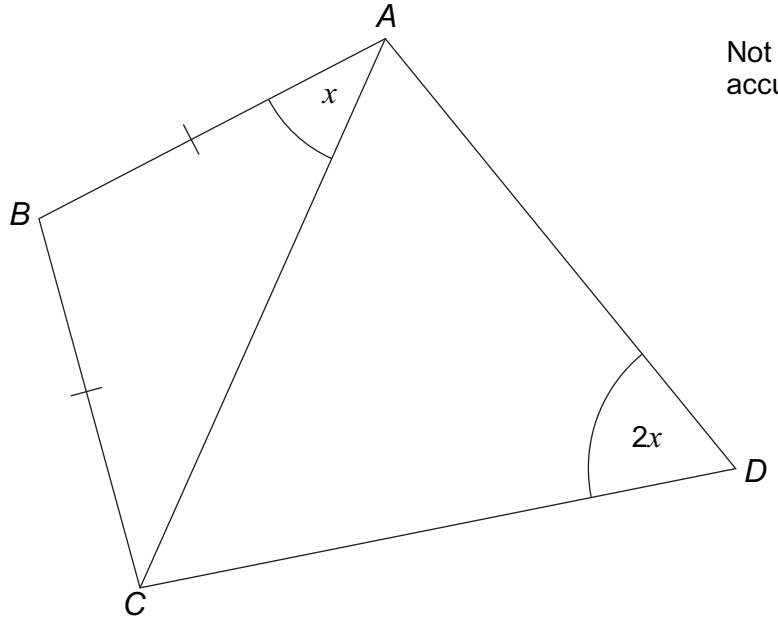
10 (b) Work out the value of r when $S = 10a$

$r = \dfrac{10a - a}{10a} = \dfrac{9a}{10a} = \dfrac{9}{10}$

$r = \dfrac{9}{10}$ *(2 marks)*

Do not write outside the box

11 In the diagram, $AB = BC$



Not drawn accurately

Prove that $ABCD$ is a cyclic quadrilateral.
Give reasons for any statements you make.

$\angle ACB = x$ ABC is isosceles

$\angle ABC = 180 - 2x$ angles in triangle add up to 180°

$\angle ABC + \angle ADC = 180 - 2x + 2x$

$= 180°$ as required.

$\therefore$ ABCD is a cyclic quadrilateral since opposite angles add up to 180°

(3 marks)

8

Turn over ▶

Do not write outside the box

13 (a) Show that $\frac{4}{x} + \frac{2}{x-1}$ simplifies to $\frac{6x-4}{x(x-1)}$

$$\frac{4(x-1)+2x}{x(x-1)} = \frac{4x-4+2x}{x(x-1)}$$

$$= \frac{6x-4}{x(x-1)}$$

(2 marks)

13 (b) Hence, or otherwise, solve $\frac{4}{x} + \frac{2}{x-1} = 3$

Give your solutions to 3 significant figures.

$$\frac{6x-4}{x(x-1)} = 3$$

$$6x-4 = 3x(x-1)$$

$$6x-4 = 3x^2-3x$$

$$0 = 3x^2-9x+4$$

Can't factorise, use quadratic formula:

$$x = \frac{9 \pm \sqrt{(-9)^2-4(3)(4)}}{2(3)}$$

$$= \frac{9 \pm \sqrt{33}}{6}$$

$$= 2.46 \text{ or } 0.543$$

Answer 2.46, 0.543 *(5 marks)*

11

Turn over ▶

Do not write outside the box

14 The value of x is 50% **more** than the value of t.
The value of y is 10% **less** than the value of w.

$x = y$

Work out $\frac{t}{w}$

Give your answer as a decimal.

$x = 1.5t \Rightarrow t = \frac{x}{1.5} \times \frac{2}{2} = \frac{2x}{3}$

$y = 0.9w \Rightarrow w = \frac{y}{0.9} \times \frac{10}{10} = \frac{10y}{9}$

$x = y$

$\frac{t}{w} = \frac{2x/3}{10y/9} = \frac{2x}{3} \times \frac{9}{10y} = \frac{18x}{30y} = \frac{3x}{5y} = \frac{3x}{5x}$

$= \frac{3}{5}$

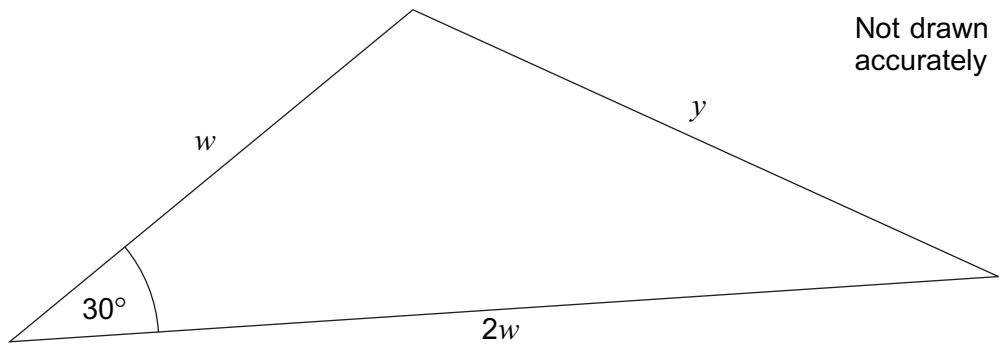
Since $x = y$

$\frac{t}{w} =$ 0.6 *(4 marks)*

(3 marks)

Do not write outside the box

20 The area of this triangle is 18 cm^2.



Work out y.

$\frac{1}{2} \times w \times 2w \times \sin(30) = 18$

$\frac{1}{2} w^2 = 18$

$w^2 = 36$

$w = 6$ since $w > 0$.

cosine rule: $y^2 = 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos(30)$

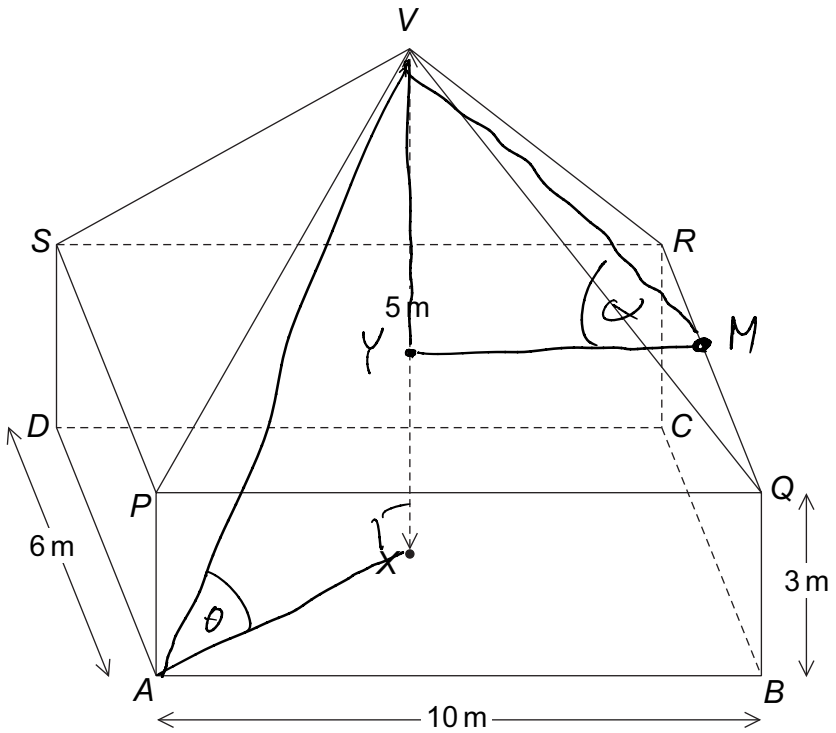
$= 55.29\ldots$

$y = \sqrt{55.29\ldots} = 7.44$ to 3sf

$y =$ 7.44 cm *(5 marks)*

Do not write outside the box

23 The diagram shows a cuboid *ABCDPQRS* and a pyramid *PQRSV*.
V is directly above the centre, *X*, of *ABCD*.



The total height, *VX*, is 5 metres.

Do not write outside the box

23 (a) Work out the angle between the line *VA* and the plane *ABCD*.

$(AX)^2 = 5^2 + 3^2$

$AX = \sqrt{34}$

$\tan(\theta) = \frac{5}{\sqrt{34}}$

$\theta = \tan^{-1}\left(\frac{5}{\sqrt{34}}\right) = 40.6^\circ$ to 3sf.

Answer 40.6 degrees *(4 marks)*

23 (b) Work out the angle between the planes *VQR* and *PQRS*.

$VY = 5 - 3 = 2$

$MY = 5$

$\therefore \tan(\alpha) = \frac{2}{5}$

$\alpha = \tan^{-1}\left(\frac{2}{5}\right) = 21.8^\circ$ to 3sf

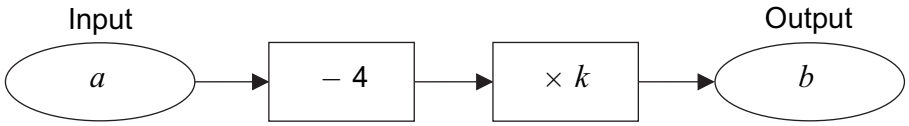
Answer 21.8 degrees *(2 marks)*

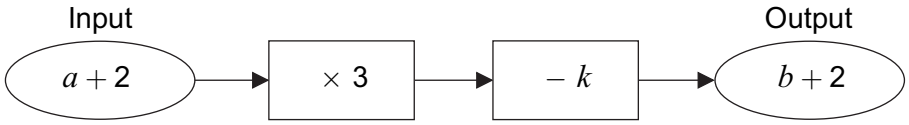
6

Turn over ▶

Do not write outside the box

25 Here are two number machines.





Work out a in terms of k.

$k(a-4) = b$

$3(a+2) - k = b+2$

(1) $ak - 4k = b$

$3a + 6 - k = b + 2 \Rightarrow 3a + 4 - k = b$ (2)

We want to eliminate b so

$ak - 4k = 3a + 4 - k$

$ak - 3a = 4 + 3k$

$a(k-3) = 4 + 3k$

$a = \frac{4+3k}{k-3}$

$a = \frac{4+3k}{k-3}$ *(6 marks)*

END OF QUESTIONS

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