London Academy of Mathematics

AQA - GCSE Maths
Aiming for a 9
Paper 1 (non calculator) - Set 1
Worked Solutions

Compiled by Chris Nunes, created by AQA Education

3 Work out the greatest integer value of x that satisfies the inequality $3x + 10 < 1$

$3x + 10 < 1$

$3x < -9$

$x < -3$

Answer -4 *(2 marks)*

4 (a) Factorise fully $2x^2 - 2x - 40$

$= 2(x^2 - x - 20)$

$= 2(x-5)(x+4)$

Answer $2(x-5)(x+4)$ *(3 marks)*

4 (b) Factorise fully $(x + y)^2 + (x + y)(2x + 5y)$

$(x+y)(x+y + 2x+5y)$

$(x+y)(3x+6y)$

$(x+y)3(x+2y)$

$3(x+y)(x+2y)$

Answer $3(x+y)(x+2y)$ *(3 marks)*

N.B. for 4(a) $(2x+8)(x-5)$ would have scored $2/3$

Do not write outside the box

5 Simplify $(2cd^4)^3$

$2^3 c^3 d^{12} = 8c^3d^{12}$

Answer $8c^3d^{12}$ *(2 marks)*

6 Solve the simultaneous equations

$$2y = 3x + 4$$

$$2x = -3y - 7$$

Do **not** use trial and improvement.

A $-3x + 2y = 4$

B $2x + 3y = -7$

A $\times 2 =$ C $-6x + 4y = 8$

B $\times 3$: D $6x + 9y = -21$

C + D $13y = -13$

$y = -1$

Sub into B: $2x - 3 = -7$

$2x = -4 \Rightarrow x = -2$

Answer $x = -2$, $y = -1$ *(4 marks)*

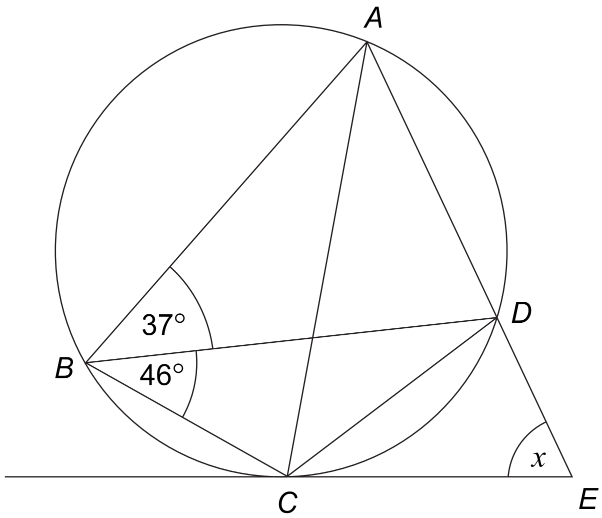
14

Turn over ▶

Do not write outside the box

7 The diagram shows a cyclic quadrilateral *ABCD*.

ADE is a straight line.
CE is a tangent to the circle.



Not drawn accurately

Work out the size of angle x.

$\angle ADC = 180 - (37 + 46) = 97°$

opposite angles in a cyclic quadrilateral add up to 180°

$\angle EDC = 180 - 97° = 83°$

angles on a straight line add up to 180°

$\angle DCE = 46°$

alternate Segment theorem

$x =$ 51 degrees *(3 marks)*

$x = 180 - (\angle EDC + \angle DCE)$

$= 180 - (46 + 83)$

$= 51°$

angles in a triangle add up to 180°

N.B. in this question reasons weren't required.

Do not write outside the box

9 Write this ratio in its simplest form

$$\sqrt{12} : \sqrt{48} : \sqrt{300}$$

$\sqrt{4}\sqrt{3} : \sqrt{16}\sqrt{3} : \sqrt{100}\sqrt{3}$

$= 2\sqrt{3} : 4\sqrt{3} : 10\sqrt{3}$

$= 2 : 4 : 10$

$= 1 : 2 : 5$

Answer 1 : 2 : 5 *(3 marks)*

10 The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1 .

Prove algebraically that **all** the terms in the new sequence are multiples of 5 .

$(5n-3)^2 + 1 = (5n-3)(5n-3) + 1$

$= 25n^2 - 15n - 15n + 9 + 1$

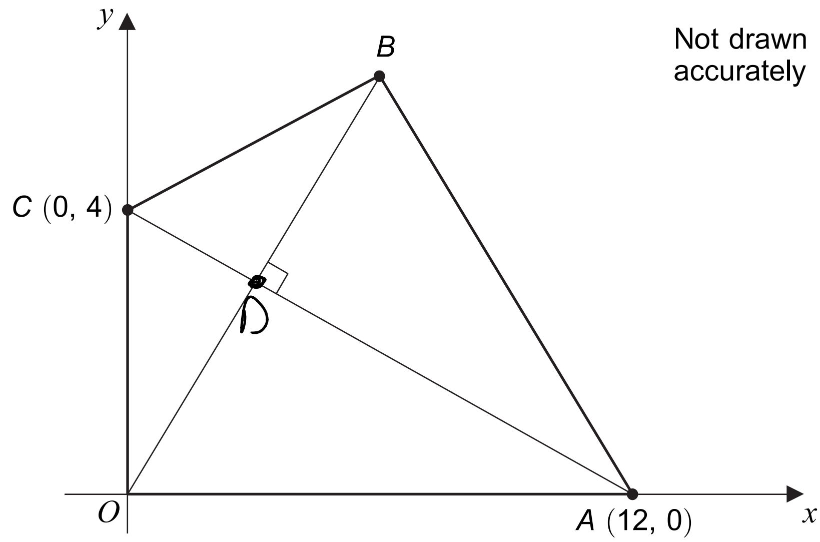
$= 25n^2 - 30n + 10$

$= 5(5n^2 - 6n + 2)$

which is a multiple of 5

(4 marks)

11 *OABC* is a kite.



11 (a) Work out the equation of *AC*.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{12-0} = \frac{-4}{12} = \frac{-1}{3}$

$y = -\frac{1}{3}x + 4$

Answer $y = -\frac{1}{3}x + 4$ *(2 marks)*

11 (b) Work out the coordinates of *B*.

We know that $m_{OB} = 3$

because AC and OB are perpendicular

line OB has equation : $y = 3x$.

To find D :

$3x = -\frac{1}{3}x + 4$ $\qquad \vec{OB} = 2 \times \vec{OD}$

$9x = -x + 12$ $\qquad = 2 \times \begin{pmatrix} 6/5 \\ 18/5 \end{pmatrix} = \begin{pmatrix} 12/5 \\ 36/5 \end{pmatrix}$

$10x = 12$

$x = 6/5$ $\qquad \therefore B = (12/5, 36/5)$

$\Rightarrow y = 18/5$

Answer ($12/5$, $36/5$) *(6 marks)*

15

Turn over ▶

Do not write outside the box

13 Simplify $\dfrac{x^2+4x-12}{x^2-25} \div \dfrac{x+6}{x^2-5x}$

Factorise first !

$\dfrac{(x+6)(x-2)}{(x-5)(x+5)} \div \dfrac{x+6}{x(x-5)}$

$= \dfrac{(x+6)(x-2)}{(x-5)(x+5)} \times \dfrac{x(x-5)}{x+6}$

$= \dfrac{\cancel{(x+6)}(x-2)\,x\,\cancel{(x-5)}}{\cancel{(x-5)}(x+5)\cancel{(x+6)}}$ Don't expand

$= \dfrac{x(x-2)}{x+5}$

Answer $\dfrac{x(x-2)}{x+5}$ *(5 marks)*

14 $x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \dfrac{25}{4}$ where $y > 0$

Work out the value of $\dfrac{x}{y}$.

$\left(x^{3/2}\right)^{2/3} = (8)^{2/3}$ $\left(y^{-2}\right)^{-1/2} = \left(\frac{25}{4}\right)^{-1/2}$

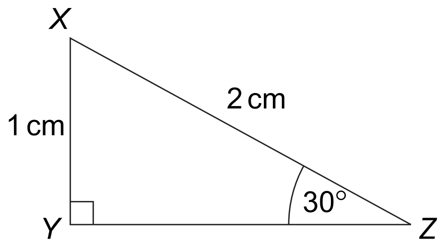
$x = 4$

$y = \frac{2}{5}$

$\therefore \dfrac{x}{y} = \dfrac{4}{2/5} = 4 \times \dfrac{5}{2} = 10$

$\dfrac{x}{y} =$ 10 *(5 marks)*

15 (a) *XYZ* is a right-angled triangle.



Not drawn accurately

Use triangle *XYZ* to show that $\sin 60° = \frac{\sqrt{3}}{2}$

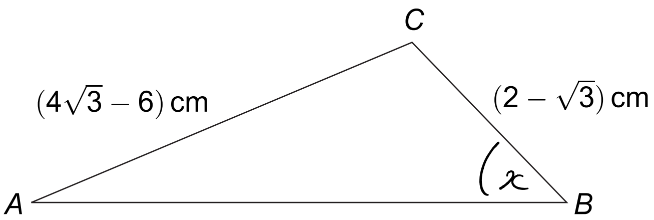
$\angle YXZ = 60°$ & $(YZ)^2 = 2^2 - 1^2 = 3$

$\Rightarrow YZ = \sqrt{3}$

$\therefore \sin(60) = \frac{YZ}{XZ} = \frac{\sqrt{3}}{2}$

(2 marks)

15 (b) Triangle *ABC* has an obtuse angle at *C*.



Not drawn accurately

Given that $\sin A = \frac{1}{4}$, use triangle *ABC* to show that angle $B = 60°$

using sine rule:

$\frac{1/4}{2-\sqrt{3}} = \frac{\sin(x)}{4\sqrt{3}-6}$

$\frac{4}{4} \times \quad \frac{\frac{1}{4}(4\sqrt{3}-6)}{2-\sqrt{3}} = \sin(x)$

$\frac{4\sqrt{3}-6}{8-4\sqrt{3}} = \sin(x)$

$\sin(x) = \frac{(4\sqrt{3}-6)(8+4\sqrt{3})}{(8-4\sqrt{3})(8+4\sqrt{3})}$

$\sin(x) = \frac{32\sqrt{3}+48-48-24\sqrt{3}}{64+\cancel{32\sqrt{3}}-\cancel{32\sqrt{3}}-48}$

$= \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$

$\sin(x) = \frac{\sqrt{3}}{2}$

$\Rightarrow x = 60°$

(6 marks)

18

Answer **all** questions in the spaces provided.

1 The line $y = mx + c$ passes through the point $(4, 3)$.
It is parallel to the line $y = 5x + 6$

Work out the values of m and c.

$m = 5$ since lines are parallel

$y = 5x + C$

$3 = 5(4) + C$

$3 - 20 = C$

$c = -17$

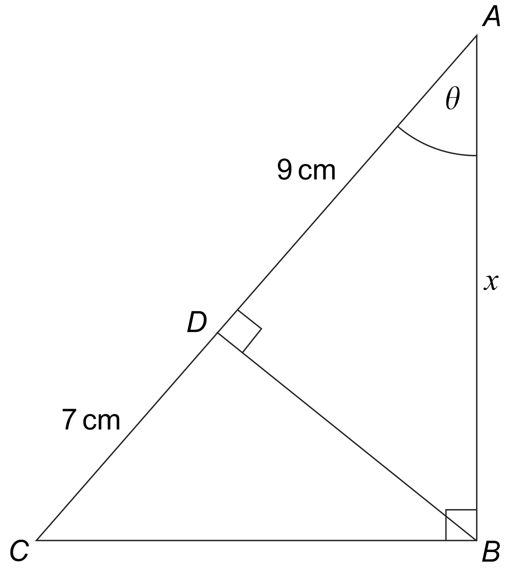
$m =$ 5, $c =$ -17 *(3 marks)*

s)

7

Turn over ▶

3 *ABC* is a right-angled triangle.
D is a point on *AC*.
BD is perpendicular to *AC*.



Not drawn accurately

3 (a) Use triangle *ABC* to write $\cos\theta$ in terms of x.

$\cos(\theta) = \frac{adj}{hyp} = \frac{x}{9+7} = \frac{x}{16}$

$\cos\theta = \frac{x}{16}$ *(1 mark)*

3 (b) By writing another expression for $\cos\theta$ in terms of x, or otherwise, work out the value of x.

triange ABD :

$\cos(\theta) = \frac{9}{x}$

$\frac{x}{16} = \frac{9}{x} \Rightarrow x^2 = 9 \times 16$

$x = 3 \times 4 = 12$

$x =$ 12 cm *(2 marks)*

5 (a) n is a positive integer.

Write down the **next** odd number after $2n - 1$

Answer $2n+1$ *(1 mark)*

5 (b) Prove that the product of two consecutive odd numbers is **always** one less than a multiple of 4 .

$(2n-1)(2n+1)+1 = 4n^2 - 2n + 2n - 1 + 1$

$= 4n^2$

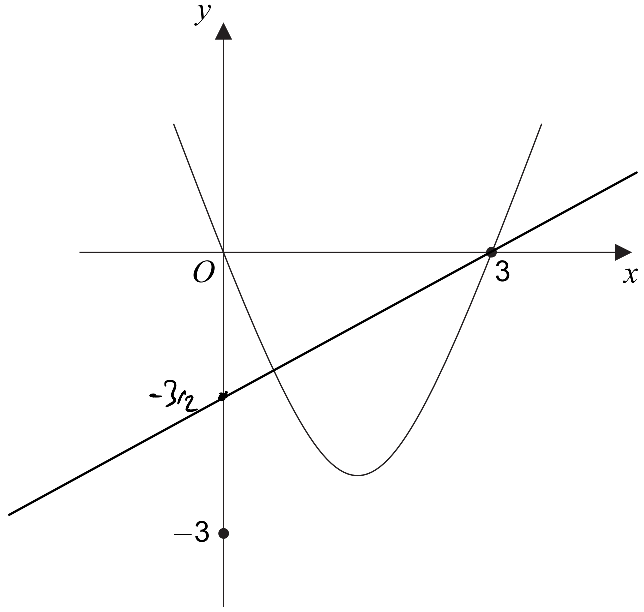
$= 4(n^2)$

which is a multiple of 4

(3 marks)

Do not write outside the box

6 The diagram shows a sketch of $y = x^2 - 3x$



6 (a) Sketch the line $y = \frac{1}{2}(x - 3)$ on the diagram.

Mark the value where this line crosses the y-axis. *(2 marks)*

6 (b) By factorising $x^2 - 3x$, or otherwise, work out the smaller solution of

$$x^2 - 3x = \frac{1}{2}(x - 3)$$

$x(x-3) = \frac{1}{2}(x-3)$

$\div x - 3$ $\div x - 3$

$(x = 3)$ $x = \frac{1}{2}$ $(x = 3)$

Note that you need to be careful when dividing by variables as you can't divide by 0.

$x =$ $\frac{1}{2}$ *(2 marks)*

8

Turn over ▶

Do not write outside the box

9 Bag A contains $7x$ counters.

Bag B contains $2x$ counters.

Five counters are taken from bag A and put in bag B.

9 (a) Write an expression, in terms of x, for the number of counters now in bag B.

Answer $2x + 5$ *(1 mark)*

9 (b) The ratio of counters in bag A to bag B is now 8 : 3

Use algebra to work out the **total** number of counters in the bags.

$7x - 5 + 2x + 5 = 11k$ where k is some positive integer

$9x = 11k$ $\therefore k = 9$

Since x must also be an integer

$11 \times 9 = 99$

Answer 99 *(4 marks)*

Do not write outside the box

10 Solve the simultaneous equations

$$\frac{x-1}{y-2} = 3 \qquad \frac{x+6}{y-1} = 4$$

Do **not** use trial and improvement.
You **must** show your working.

$x - 1 = 3y - 6$ $\qquad$ $x + 6 = 4y - 4$

(1) $x - 3y = -5$ $\qquad$ (2) $x - 4y = -10$

(1) − (2) $\quad y = 5$

sub $y = 5$ into (1) :

$x - 15 = -5$

$x = 10$

$x =$ 10 , $y =$ 5 *(5 marks)*

10

Turn over ▶

Do not write outside the box

11 Write $\sqrt{500} - 2\sqrt{45}$ in the form $a\sqrt{5}$ where a is an integer.

$\sqrt{100}\sqrt{5} - 2\sqrt{9}\sqrt{5}$

$= 10\sqrt{5} - 6\sqrt{5}$

$= 4\sqrt{5}$

Answer $4\sqrt{5}$ *(2 marks)*

12 Simplify fully $\dfrac{4x^2 + 19x - 5}{9x^2 - 16} \div \dfrac{x+5}{3x-4}$

$4x^2 + 19x - 5$

$= 4x^2 + 20x - x - 5$

$= 4x(x+5) - (x-5)$

$= (x+5)(4x-1)$

$\dfrac{(x+5)(4x-1)}{(3x-4)(3x+4)} \times \dfrac{3x-4}{x+5}$

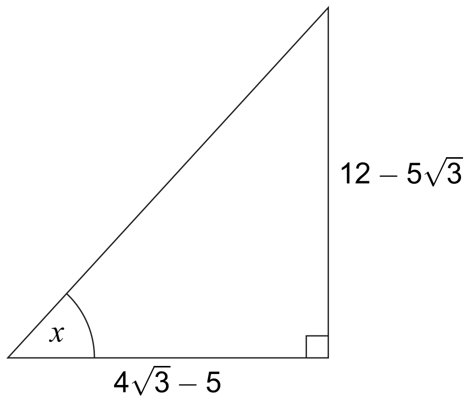
$= \dfrac{\cancel{(x+5)}(4x-1)\cancel{(3x-4)}}{\cancel{(3x-4)}(3x+4)\cancel{(x+5)}}$

$= \dfrac{4x-1}{3x+4}$

Answer .... *(5 marks)*

Do not write outside the box

15 Show that angle $x = 60°$



Not drawn accurately

You **must** show your working.

$$\tan(x) = \frac{12-5\sqrt{3}}{4\sqrt{3}-5}$$

$$= \frac{(12-5\sqrt{3})(4\sqrt{3}+5)}{(4\sqrt{3}-5)(4\sqrt{3}+5)}$$

$$= \frac{48\sqrt{3}+60-60-25\sqrt{3}}{48+20\sqrt{3}-20\sqrt{3}-25}$$

$$= \frac{23\sqrt{3}}{23}$$

$$= \sqrt{3}$$

$$x = \tan^{-1}(\sqrt{3}) = 60°$$

(4 marks)

9

Turn over ▶

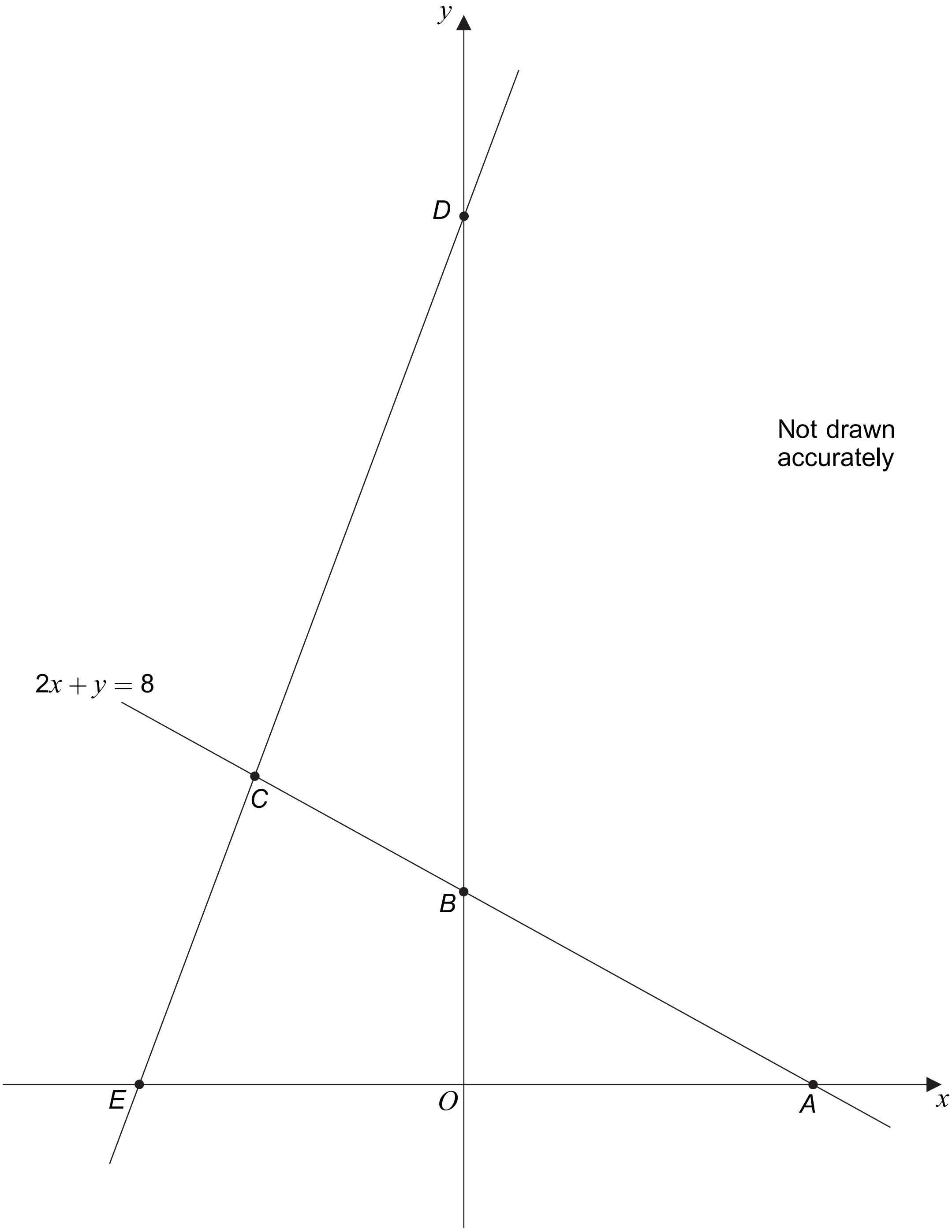
Do not write outside the box

16 A, B and C are points on the line $2x + y = 8$

DCE is a straight line.

$AB : BC = 2 : 1$

$EC : CD = 1 : 2$



Do not write outside the box

Work out the ratio Area of triangle *AEC* : Area of triangle *BCD*

Give your answer in its simplest form.

At point A, $y=0$ $\Rightarrow$ $A=(4,0)$
At point B, $x=0$ $\Rightarrow$ $B=(0,8)$

point $C=(-2,12)$ using either vectors or by inspection. (since $AB:BC=2:1$)

E has coordinates $(e,0)$
and D has coordinates $(0,d)$

$\vec{ED} = 3\vec{EC}$

$\begin{pmatrix}-e\\ d\end{pmatrix} = 3\begin{pmatrix}-2-e\\ 12-0\end{pmatrix}$

$\begin{pmatrix}-e\\ d\end{pmatrix} = \begin{pmatrix}-6-3e\\ 36\end{pmatrix}$

$e=-3$
$d=36$

$\therefore$ Area AEC $=$

$\dfrac{(3+4)\times 8}{2} = 28$

Area BCD $= \dfrac{(36-12)\times 2}{2}$

$= 24$

$28:24 = 7:6$

Answer 7 : 6 *(6 marks)*

END OF QUESTIONS

6