London Academy of Mathematics

AQA - GCSE Maths
Aiming for a 9
Paper 2&3 (calculator) - Set 4
Worked Solutions

Compiled by Chris Nunes, created by AQA Education

Answer **all** questions in the spaces provided.

1 (a) The nth term of a sequence is $\frac{3-5n}{2}$

Work out the difference between the 20th term and the 8th term.

[2 marks]

$\frac{3-5(8)}{2} - \frac{3-5(20)}{2}$

$= \frac{-37}{2} - \frac{-97}{2} = \frac{60}{2} = 30$

Answer 30

3

3 (a) p, q and r are all integers greater than 1

$pqr = 1365$

Work out one possible set of values for p, q and r.

[2 marks]

1365

5 | 273

3 | 91

1

$p =$ 5 $q =$ 3 $r =$ 91

3 (b) a and b are both **square** numbers greater than 1

$ab - 11b$ is also a **square** number.

By factorising $ab - 11b$, work out one possible pair of values for a and b.
You **must** show your working.

[2 marks]

$b(a-11)$

if $b = 25$, $a = 36$ then

$b(a-11) = 25 \times 25 = 25^2$

Many other examples work as well, e.g.

$b = 16$, $a = 36$ $b(a-11) = 16 \times 25 = (4 \times 5)^2 = 20^2$

$a =$ 36 $b =$ 25

4 Solve $\frac{56}{\sqrt[3]{x}} = 4$

[2 marks]

$56 = 4\sqrt[3]{x}$

$14 = \sqrt[3]{x}$

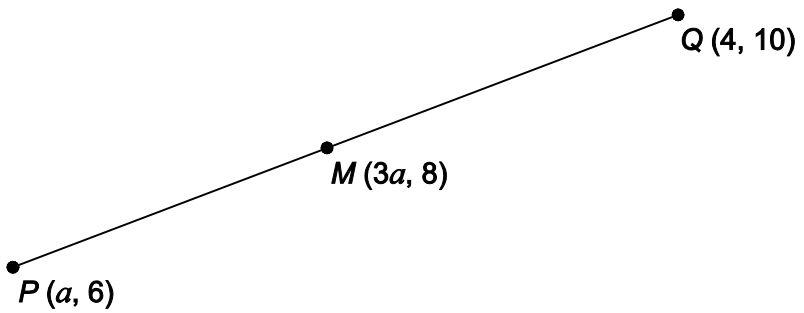
$x = 2744$

$x =$ 2744

Turn over for the next question

6

5 *M* is the midpoint of *PQ*.



Not drawn accurately

Work out the value of a.

[3 marks]

$\frac{a+4}{2} = 3a$

$a + 4 = 6a$

$5a = 4$

$a = 4/5$

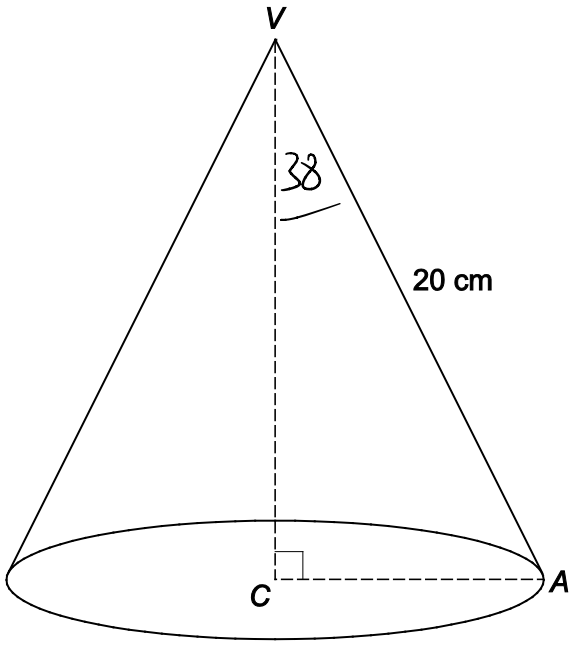
Answer _____

6 A cone has vertex *V*.

C is the centre of the base.

The slant height, *VA*, is 20 cm

The angle between *VA* and *VC* is 38°



Work out the radius of the base.

[3 marks]

$\sin(38) = \frac{r}{20}$

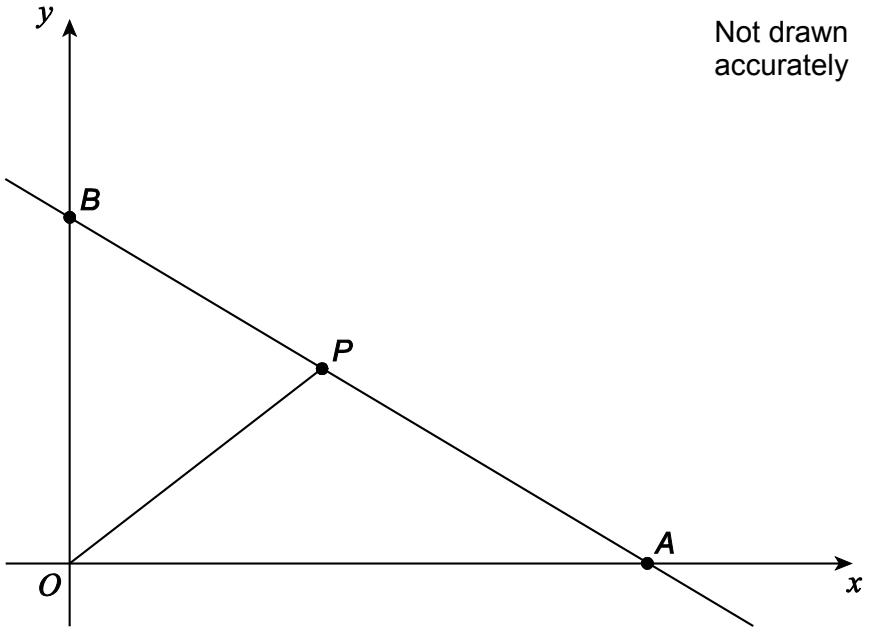
$r = 20\sin(38) = 12.31$ cm to 2dp

Answer 12.31 cm

6

7 The equation of the line through B, P and A is $4x + 5y = 40$

$BP : PA = 2 : 3$



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Work out the area of triangle OBP.

[4 marks]

$B: \quad x = 0 \Rightarrow 5y = 40 \Rightarrow y = 8$

So $B = (0, 8)$

similarly $A = (10, 0)$ since $y = 0$

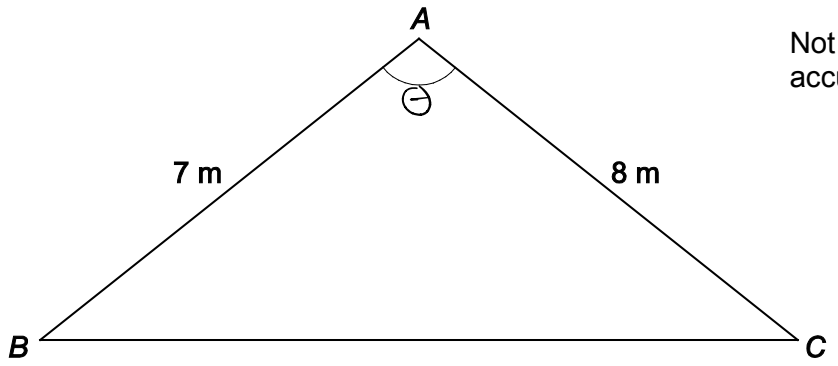
$P = (4, 4.8)$

$\therefore$ Area $OBP = \frac{1}{2} \times x_P \times y_B$

$= \frac{1}{2} \times 4 \times 8 = 16$

Answer 16 square units

8 The perimeter of a triangular flower bed, *ABC*, is marked out using 27 metres of rope.



Not drawn accurately

Work out the size of angle *BAC*.

[4 marks]

$P = 27 \Rightarrow BC = 27 - 7 - 8 = 12$

$$\cos(\theta) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{8^2 + 7^2 - 12^2}{2(8)(7)} = \frac{-31}{112}$$

$$\theta = \cos^{-1}\left(\frac{-31}{112}\right) = 106.1^\circ \quad \text{to 1dp}$$

Answer 106.1 degrees

Turn over for the next question

8

9 $-11 < 5x \leqslant 5$ and $6x + 7 \leqslant 4x + 4$

Show that there is **exactly** one integer that x can be.

[5 marks]

÷5($-11 < 5x \leq 5$)÷5

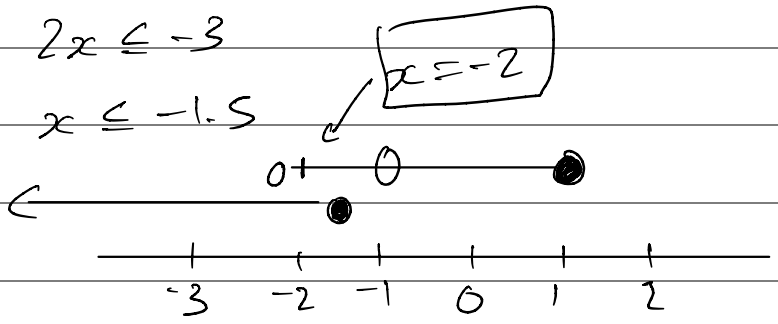
$-2.2 < x \leq 1$

$6x + 7 \leq 4x + 4$

$2x + 7 \leq 4$

$2x \leq -3$

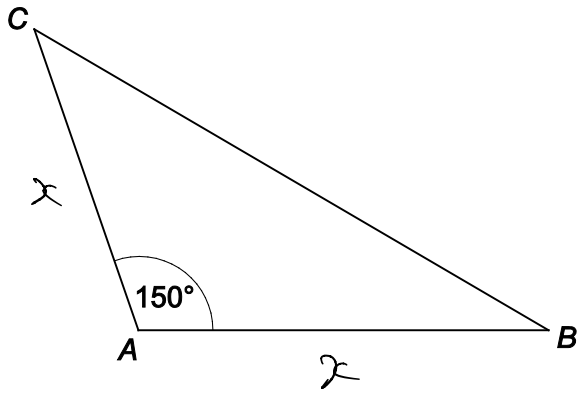
$x \leq -1.5$



$x = -2$

10 *ABC* is an isosceles triangle with $AB = AC$

The area of *ABC* is 57.76 cm^2



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Work out the length of *AB*.

[3 marks]

$\frac{1}{2} x^2 \sin(150) = 57.76$

$x^2 = \frac{2 \times 57.76}{\sin(150)} = 231.04$

$x = 15.2$ cm

Answer 15.2 cm

Turn over for the next question

8

12 (a) Factorise fully $75 - 3x^2$

[2 marks]

$3(25 - x^2) = 3(5 - x)(5 + x)$

Answer $3(5 - x)(5 + x)$

12 (b) Simplify fully $(3n + 1)^2 - (3n - 1)^2$

[2 marks]

$((3n+1) - (3n-1))((3n+1) + (3n-1))$

$= 2(6n) = 12n$

Answer $12n$

Or

$9n^2 + 6n + 1 - (9n^2 - 6n + 1)$

$= 9n^2 + 6n + 1 - 9n^2 + 6n - 1$

$= 12n$

10

13 Simplify fully $\frac{8a}{3a+6} \times \frac{5a+10}{3a^2} \div \frac{4}{15a^3}$

[3 marks]

$$\frac{2\cancel{8a}}{3(a+2)} \times \frac{5(a+2)}{\cancel{3}a^2} \times \frac{5\cancel{15}a^3}{\cancel{4}^1}$$

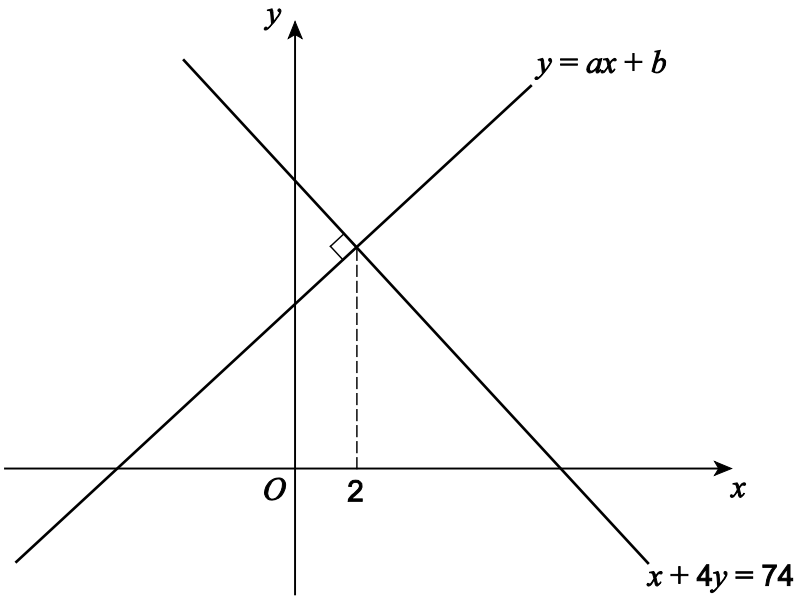
$$= \frac{50\cancel{(a+2)}\,a^3}{3\cancel{(a+2)}a^2}$$

$$= \frac{50a}{3}$$

Answer _____

14 The line $y = ax + b$ is perpendicular to the line $x + 4y = 74$

The lines intersect at the point where $x = 2$



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Work out the values of a and b.

[5 marks]

$4y = -x + 74$

$y = -\frac{1}{4}x + 74$

$\therefore a = 4$ Since lines are perpendicular when $x = 2$, lines intersect so,

$-\frac{1}{4}(2) + 74 = 4(2) + b$

$\frac{147}{2} = 8 + b$

$b = \frac{131}{2}$

$a =$ 4 $b =$ $\frac{131}{2}$

8

15 Rearrange $w = \frac{8x - y}{y}$ to make y the subject.

[3 marks]

$wy = 8x - y$

$wy + y = 8x$

$y(w+1) = 8x$

$y = \frac{8x}{w+1}$

Answer $y = \frac{8x}{w+1}$

16 (a) $a = 3^{2b}$

Circle the correct expression for $\frac{1}{a}$

[1 mark]

3^{2b-1} 3^{-2b} -3^{2b} $\left(\frac{1}{3}\right)^{-2b}$

16 (b) $y = 5^{x}$

Circle the correct expression for $25y$

[1 mark]

5^{x+2} 25^{x} 5^{2x} 125^{x}

16 (c) $w = 2^{m}$

Circle the correct expression for w^3

[1 mark]

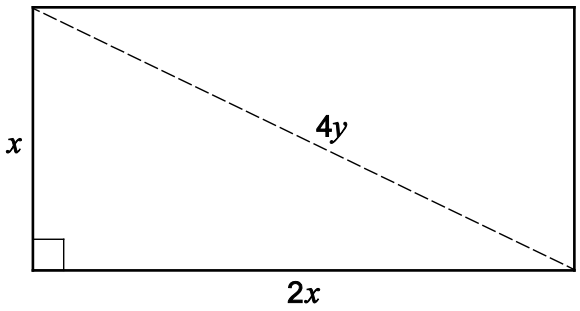
8^{3m} 6^{m} 2^{m+3} 2^{3m}

Turn over for the next question

6

18 The diagram shows a rectangle with a diagonal drawn.

The given expressions for the measurements are in centimetres.



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Work out an expression for the area of the rectangle, in cm^2
Give your answer in its simplest form, in terms of y.

[4 marks]

Area $= x \times 2x = 2x^2$

By Pythagoras Theorem,

$x^2 + (2x)^2 = (4y)^2$

$5x^2 = 16y^2$

$x^2 = \frac{16}{5}y^2$

$\therefore$ Area $= 2\left(\frac{16}{5}y^2\right) = \frac{32}{5}y^2$

Answer $\frac{32}{5}y^2$ cm^2

20 Two circles overlap.

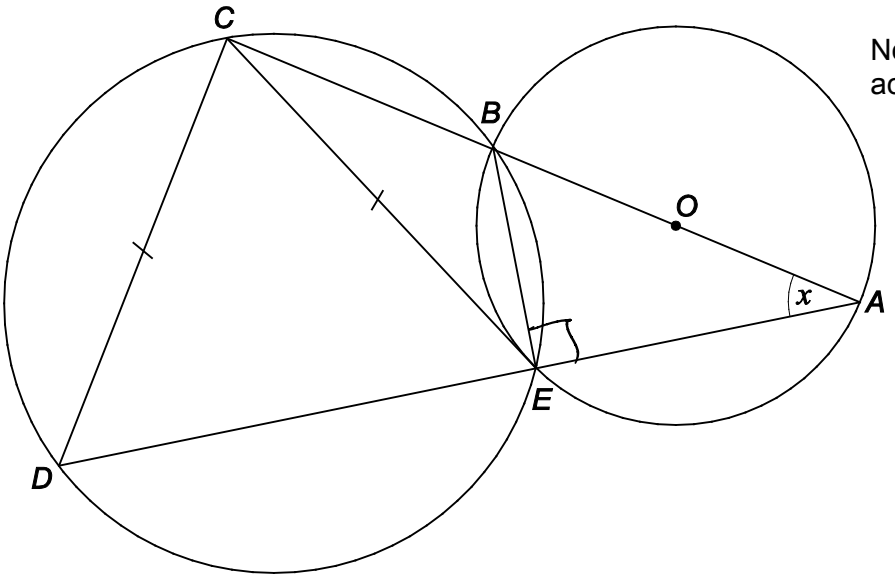
A, *B* and *E* lie on the circle, centre *O*.

B, *C*, *D* and *E* lie on the other circle.

AOBC and *AED* are straight lines.

CD = *CE*

angle *BAE* = x



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20 (a) Give a reason why angle *BEA* = 90°

[1 mark]

Angle in a semicircle is a right angle

(BA is a diameter)

20 (b) Prove that angle $DCE = 2x$

[4 marks]

$\angle ABE = 90 - x$ angles in a triangle add up to 180°

$\angle EBC = 180 - (90 - x)$

$= 90 + x$ angles on a straigh line add up to 180°

$\angle CDE = 180 - (90 + x)$

$= 90 - x$ opposite angles in a cyclic quadrilateral add up to 180°

$\angle CED = 90 - x$ $\triangle CDE$ is isosceles

$\angle DCE = 180 - 2(90 - x)$

$= 180 - 180 + 2x$ angles in a triangle add up to 180°

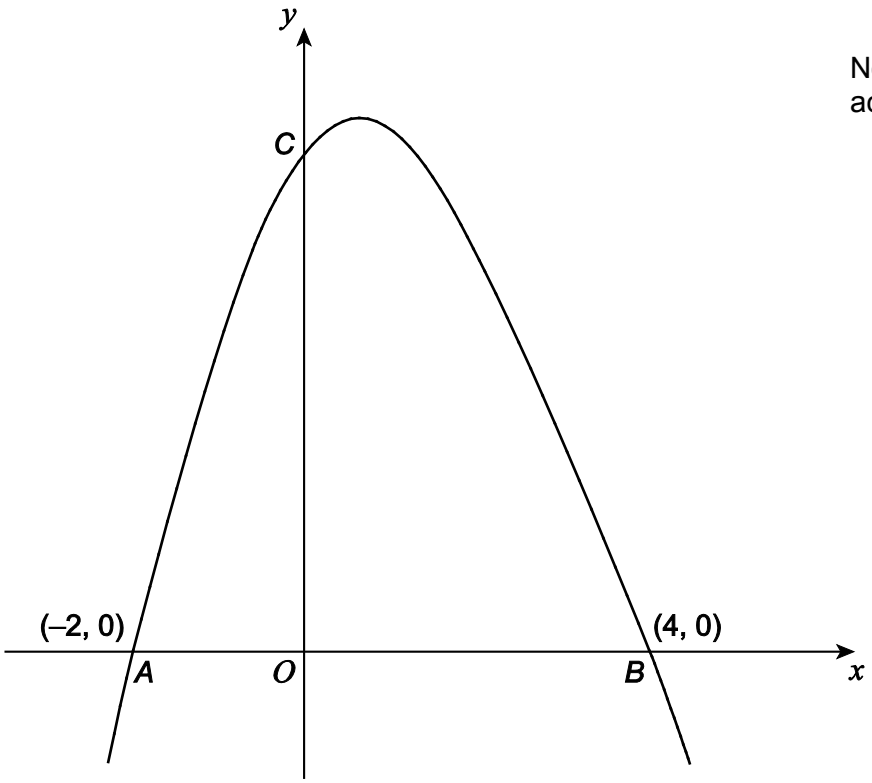
$= 2x$

Turn over for the next question

5

21 Here is a sketch of $y = (x + 2)(4 - x)$

The graph intersects the axes at A (–2, 0), B (4, 0) and C.



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21 (a) Work out the coordinates of C.

[1 mark]

$x = 0 \Rightarrow y = 8$

Answer (0 , 8)

22 The equation of a circle is $(x-2)^2 + (y-1)^2 = 16$

The equation of a line is $y = 2x + 1$

The circle and the line intersect at two points.

Work out the coordinates of the two points.
You **must** show your working.
Do **not** use trial and improvement.

[5 marks]

$(x-2)^2 + (2x+1-1)^2 = 16$

$x^2 - 4x + 4 + (2x)^2 = 16$

$x^2 - 4x + 4 + 4x^2 = 16$

$5x^2 - 4x - 12 = 0$

$5x^2 - 10x + 6x - 12 = 0$

$5x(x-2) + 6(x-2) = 0$

$(5x+6)(x-2) = 0$

$x = -6/5 \qquad x = 2$

$\downarrow \qquad \downarrow$

$y = 2(-6/5) + 1 \qquad y = 2(2) + 1$

$= -7/5 \qquad = 5$

Answer ($-6/5$, $-7/5$) and (2 , 5)

24 Write $12x^2 - 60x + 5$ in the form $a(bx + c)^2 + d$ where a, b, c and d are integers.

[5 marks]

$3(4x^2 - 20x) + 5$ Not fully factorising because of this

$3((2x-5)^2 - 25) + 5$

$3(2x-5)^2 - 75 + 5$

$= 3(2x-5)^2 - 70$

or $12(x^2 - 5x) + 5$

$= 12((x - 5/2)^2 - \frac{25}{4}) + 5$

$= 12(x - 5/2)^2 - 75 + 5$

$= 3 \times 2^2 (x - 5/2)^2 - 70$

$= 3(2(x - 5/2))^2 - 70$

Answer _____

$= 3(2x-5)^2 - 70$

END OF QUESTIONS

5

Answer **all** questions in the spaces provided.

1 The nth term of a sequence is $\dfrac{1420-5n}{1420+5n}$

1 (a) Work out the **position** of the term that has the value zero.

[2 marks]

$\frac{1420-5n}{1420+5n} = 0$

$1420 - 5n = 0$

$5n = 1420 \qquad n = 284$

Answer 284

Turn over for the next question

3

2 P (–3, –10) and Q (a, b) are points on a straight line with gradient 12

Work out one possible pair of integer values for a and b.

[2 marks]

$$\frac{b+10}{a+3} = 12$$

if $a = 0$, $\frac{b+10}{3} = 12$

$b + 10 = 36$

$b = 26$

other simple pairs:

$a = -2$ $b = 2$

$a = -1$ $b = 14$

$a = 0$, $b = 26$

$a = 1$, $b = 38$

$a = 2$, $b = 50$

$a = 3$, $b = 62$

$a =$ 0 $b =$ 26

3 $p = \dfrac{m+2}{m^2+1}$

3 (a) Work out the value of p when $m = -5.5$

[1 mark]

$$\frac{-5.5+2}{(-5.5)^2+1} = \frac{-14}{125}$$

Answer $\dfrac{-14}{125}$

3 (b) Work out the values of m when $p = 2$

[3 marks]

$$2 = \frac{m+2}{m^2+1}$$

$$2m^2 + 2 = m + 2$$

$$2m^2 - m = 0$$

$$m(2m-1) = 0$$

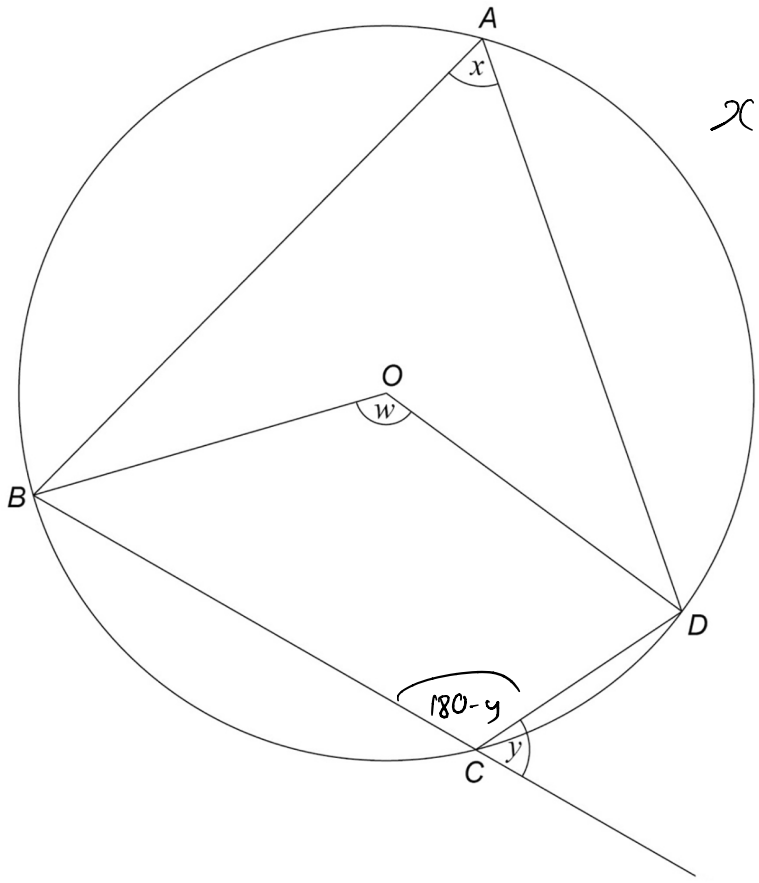
$m = 0$ or $m = 1/2$

Answer _____

Turn over for the next question

6

4 *A*, *B*, *C* and *D* are points on a circle, centre *O*.



$x + 180 - y = 180$

$x - y = 0$

$x = y$

Which statement is correct?

Tick **one** box.

[1 mark]

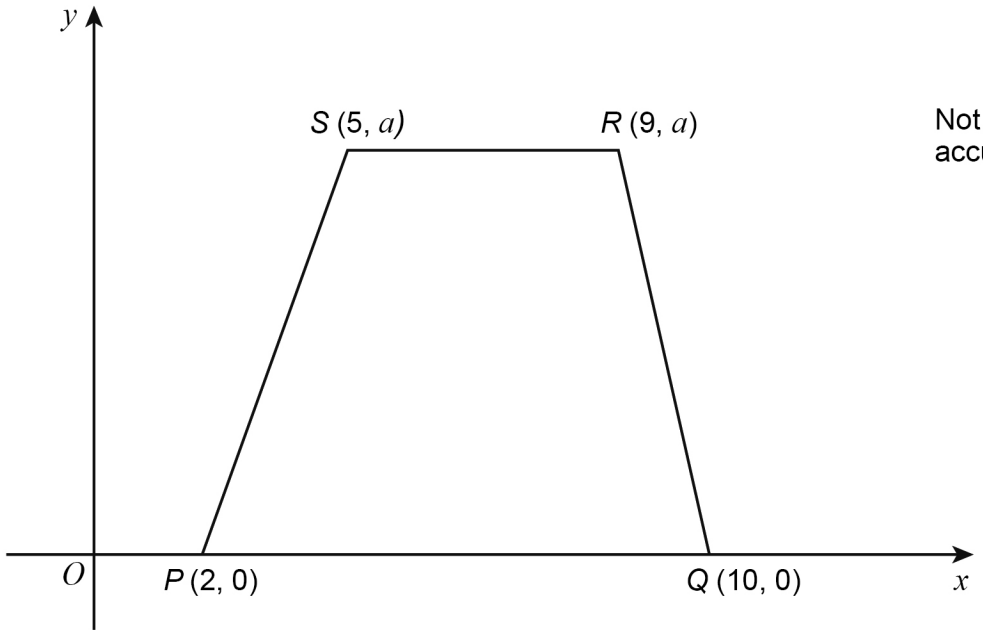
$x + y = 180°$ and $w = 2x$ ☐

$x + y = 180°$ and $x = 2w$ ☐

$x = y$ and $w = 2x$ ☑

$x = y$ and $x = 2w$ ☐

7 *PQRS* is a trapezium.



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The area of the trapezium is 63 square units.

Work out the value of a.

[2 marks]

$(10-2+9-5) \times a \times \frac{1}{2} = 63$

$12 \times a \times \frac{1}{2} = 63$

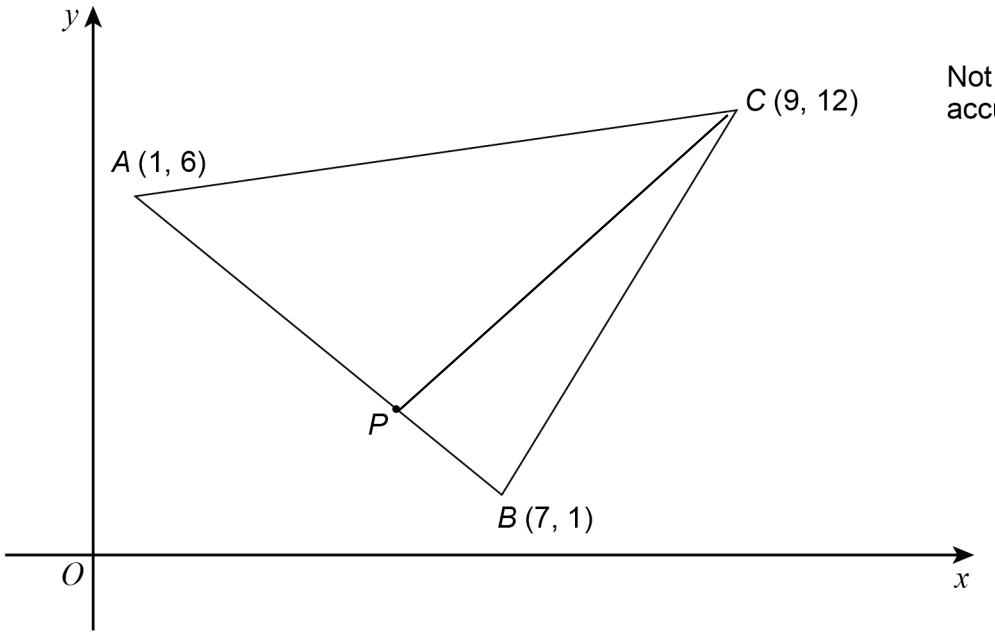
$6a = 63$

$a = 10.5$

Answer 10.5

8 Here is a sketch of triangle *ABC*.

P is a point on *AB*.



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AP : *PB* is 3 : 1

Work out the length *PC*.

Give your answer to 4 significant figures.

[4 marks]

$\vec{OP} = \vec{OA} + \vec{AP} = \vec{OA} + \frac{3}{4}\vec{AB}$

$= \begin{pmatrix} 1 \\ 6 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} 6 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 6 \end{pmatrix} + \begin{pmatrix} 9/2 \\ -15/4 \end{pmatrix} = \begin{pmatrix} 11/2 \\ 9/4 \end{pmatrix}$

$|\vec{OP}| = \sqrt{(9 - 11/2)^2 + (12 - 9/4)^2}$

$= 10.36$

Answer 10.36 units

6

Turn over ►

11 Expand and simplify fully $(x + 2)(x + 3)(x + 4)$

[3 marks]

$(x^2 + 5x + 6)(x+4)$

$= x^3 + 5x^2 + 6x + 4x^2 + 20x + 24$

$= x^3 + 9x^2 + 26x + 24$

Answer $x^3 + 9x^2 + 26x + 24$

12 (a) Write $\frac{7}{9x} + \frac{2}{3x^2}$ as a single fraction in its simplest form.

[3 marks]

$\frac{7}{9x} \times \frac{x}{x} + \frac{2}{3x^2} \times \frac{3}{3} = \frac{7x}{9x^2} + \frac{6}{9x^2} = \frac{7x+6}{9x^2}$

Answer $\frac{7x+6}{9x^2}$

12 (b) Show that $\frac{x^4}{x+4} \times \frac{x+2}{x} \div \frac{x^2}{3x+12}$

simplifies to the form $ax^2 + bx$ where a and b are integers.

[4 marks]

$\frac{x^4}{x+4} \times \frac{x+2}{x} \times \frac{3(x+4)}{x^2}$

$= \frac{3x^4(x+2)(x+4)}{(x+4)\,x \times x^2}$

$= \frac{3x^4(x+2)}{x^3} = 3x(x+2)$

$= 3x^2 + 6x$

Turn over for the next question

10

14 (a) Factorise fully $12pq^3r - 18pq^2r^2 + 24pq^2r$

[2 marks]

Answer $6pq^2r(2q - 3r + 4)$

14 (b) Factorise fully $6(y + 3)^5 + 4(y + 3)^4$

Give your answer in its simplest form.

Do **not** attempt to expand $(y + 3)^5$ or $(y + 3)^4$

[3 marks]

$2(y+3)^4(3(y+3) + 2)$

$= 2(y+3)^4(3y + 9 + 2)$

$= 2(y+3)^4(3y + 11)$

Answer $2(y+3)^4(3y+11)$

14 (c) Factorise fully $48 - 75x^2$

[2 marks]

$3(16 - 25x^2) = 3(4 - 5x)(4 + 5x)$

Answer

16 $A = 2 - 5x \qquad B = 3x - 1 \qquad C = x^2$

Show that $(2A + 3B)^2 \equiv A + B + C$

[4 marks]

$LHS = \left(2(2-5x) + 3(3x-1)\right)^2$

$= \left(4 - 10x + 9x - 3\right)^2$

$= (1 - x)^2 = 1 - 2x + x^2$

$RHS = 2 - 5x + 3x - 1 + x^2$

$= 1 - 2x + x^2$

$\therefore LHS = RHS$

17 A circle has equation $x^2 + y^2 = 29$

P is the point (–5, 2)

17 (a) Show that P is on the circle.

[1 mark]

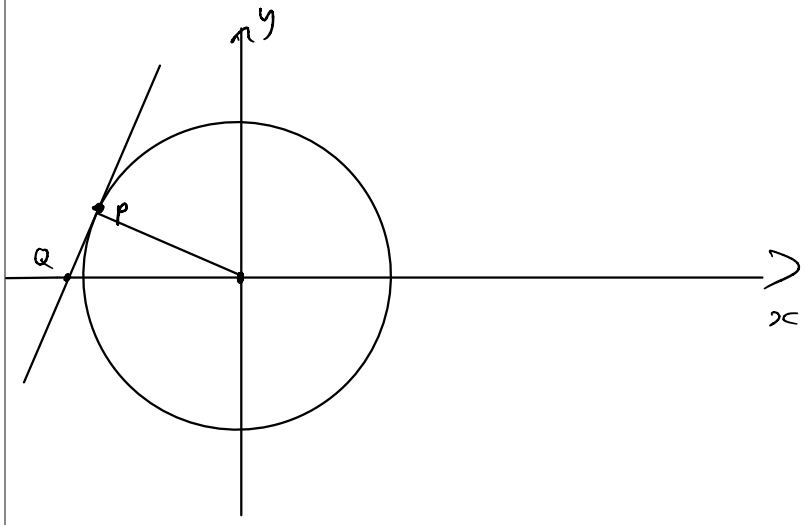
$(-5)^2 + 2^2 = 25 + 4 = 29$

17 (b) The tangent to the circle at P intersects the x-axis at point Q.

Work out the x-coordinate of Q.

You **must** show your working.

[4 marks]



$m_{OP} = \frac{2-0}{-5-0} = \frac{-2}{5}$

$m_{PQ} = \frac{5}{2}$

$y = \frac{5}{2}x + c$

$2 = \frac{5}{2}(-5) + c$

$2 = -\frac{25}{2} + c$

$c = 2 + \frac{25}{2} = \frac{29}{2}$

$y = \frac{5}{2}x + \frac{29}{2}$

$0 = \frac{5}{2}x + \frac{29}{2}$

$\frac{5}{2}x = -\frac{29}{2}$

$x = -\frac{29}{5}$

Answer $-\frac{29}{5}$

9

18 (a) Work out all the **integer** values of x for which

$$-5 < 4x + 3 \leqslant 13$$

[3 marks]

$-8 < 4x \leq 10$

$-2 < x \leq 2.5$

Answer $-1, 0, 1, 2$

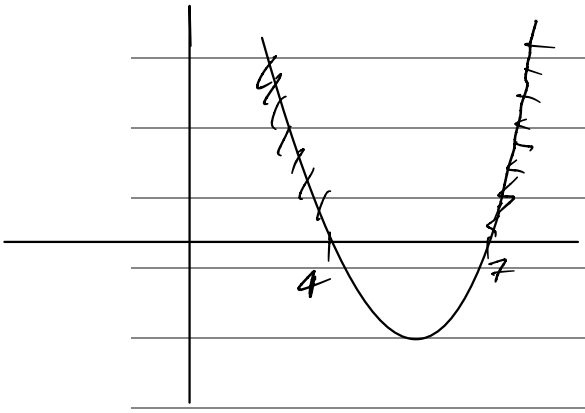
18 (b) Work out the range of values of x for which

$$x^2 - 11x + 28 > 0$$

You **must** show your working.

[3 marks]

$(x-7)(x-4) > 0$



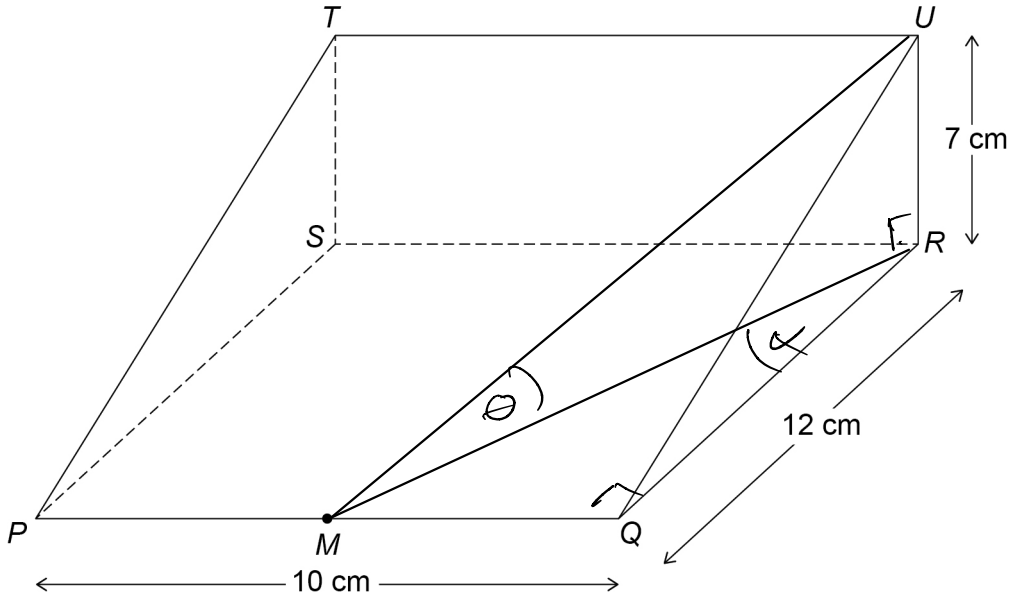
Answer $x < 4$ or $x > 7$

20 *PQRSTU* is a triangular prism.

PQRS is a rectangle and angle *QRU* = 90°

PQ = 10 cm *QR* = 12 cm *UR* = 7 cm

M is the midpoint of *PQ*.



20 (a) Calculate the size of the angle between the line *UM* and the plane *PQRS*.

[4 marks]

$(RM)^2 = 12^2 + 5^2$

$RM = \sqrt{12^2+5^2} = 13$

$\tan(\theta) = \frac{7}{13}$

$\theta = \tan^{-1}\left(\frac{7}{13}\right) = 28.3^\circ$ to 1dp

Answer 28.3° degrees

20 (b) Calculate the size of the angle between the planes *UMR* and *UQR*.

[2 marks]

$\tan(\alpha) = \frac{5}{12}$

$\alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$ to 1dp

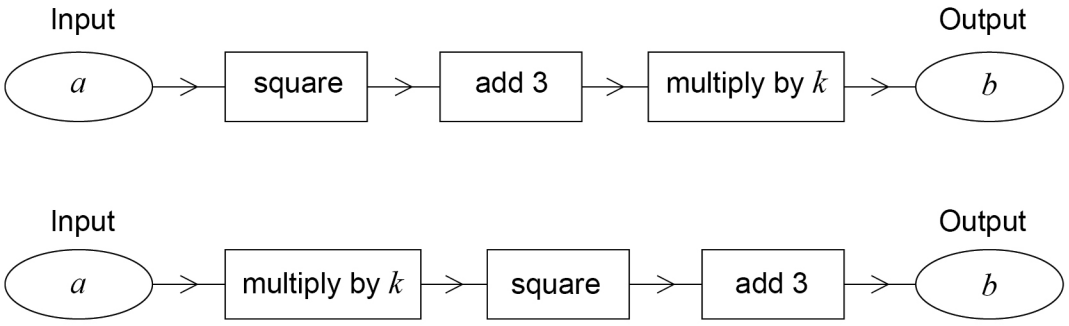
Answer 22.6 degrees

Turn over for the next question

6

23 For each of these two function machines, when the input is a the output is b.

$k > 0$ and $k \neq 1$ and $a > 0$



Work out an expression for a in terms of k.

Give your answer in its simplest form.

[6 marks]

$k(a^2+3) = b$

$(ak)^2 + 3 = b$

$\Rightarrow \quad k(a^2+3) = (ak)^2 + 3$

$ka^2 + 3k = a^2k^2 + 3$

$ka^2 - a^2k^2 = 3 - 3k$

$a^2(k - k^2) = 3 - 3k$

$a^2 = \frac{3-3k}{k-k^2} = \frac{3(1-k)}{k(1-k)}$

$a^2 = \frac{3}{k} \qquad k \neq 1$

$a = \pm\sqrt{\frac{3}{k}} \qquad a > 0$

So $a = \sqrt{\frac{3}{k}}$

Answer $a = \sqrt{\frac{3}{k}}$

24 Work out the value of p when

$$9^{0.5p} \times 81 = 27^{2p-1}$$

[4 marks]

$(3^2)^{0.5p} \times 3^4 = (3^3)^{2p-1}$

$3^p \times 3^4 = 3^{6p-3}$

$3^{p+4} = 3^{6p-3}$

$p + 4 = 6p - 3$

$5p = 7 \qquad p = \frac{7}{5}$

Answer $\frac{7}{5}$

END OF QUESTIONS

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