London Academy of Mathematics

Edexcel - A-level Maths
Pure
Domain and Range
Worked Solutions

Compiled by Chris Nunes, Created by Pearson Edexcel

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4. The function f is defined by

$$\mathrm{f} : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{x+1}, \quad x > 3.$ **(4)**

(b) Find the range of f. **(2)**

(c) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function. **(3)**

The function g is defined by

$$\mathrm{g} : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $\mathrm{fg}(x) = \dfrac{1}{8}$. **(3)**

a) $f(x) = \dfrac{2(x-1)}{(x-3)(x+1)} - \dfrac{1}{x-3}$

$= \dfrac{2(x-1) - (x+1)}{(x-3)(x+1)}$

$= \dfrac{2x-2-x-1}{(x-3)(x+1)}$

$= \dfrac{x-3}{(x-3)(x+1)} = \dfrac{1}{x+1}$

b) $0 < f(x) < \dfrac{1}{4}$

c) $x = f(y)$

$= \dfrac{1}{y+1}$

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Question 4 continued

$x(y+1) = 1$

$xy + x = 1$

$xy = 1 - x$

$y = \frac{1-x}{x}$

$f^{-1}(x) = \frac{1-x}{x}$ $\quad 0 < x < 1/4$

since domain of $f^{-1}(x) \approx$ range of $f(x)$

d) $f(g(x)) = \frac{1}{8}$

$\frac{1}{2x^2 - 3 + 1} = \frac{1}{8}$

$8 = 2x^2 - 2$

$5 = x^2$

$x = \pm\sqrt{5}$

both since domain of $g(x)$ is $x \in \mathbb{R}$.

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5. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{fg}(x)\right] = x(x\mathrm{e}^{x^2} + 2)$.

(6)

a) $x^2 \geqslant 0 \Rightarrow e^{x^2} \geqslant e^0 = 1$

So $g(x) \geqslant 1$

b) $f(g(x)) = f(e^{x^2}) = 3e^{x^2} + \ln(e^{x^2})$

$= 3e^{x^2} + x^2 \cancel{\ln(e)}^{1}$

$= 3e^{x^2} + x^2$

c) $f(g(0)) = 3e^0 + 0 = 3$

$\therefore f(g(x)) \geqslant 3$ similar reasons to (a)

d) $\frac{d}{dx}\left[f(g(x))\right] = 6xe^{x^2} + 2x$

$6xe^{x^2} + 2x = x(xe^{x^2} + 2)$

$6xe^{x^2} + \cancel{2x} = x^2e^{x^2} + \cancel{2x}$

$x^2e^{x^2} - 6xe^{x^2} = 0$

H 3 1 1 2 3 A 0 1 2 2 8

$e^{x^2} > 0$ for all $x \in \mathbb{R}$, so $e^{x^2} \neq 0$.

Question 5 continued

$$x e^{x^2} (x-6) = 0$$

$x = 0$ or $x = 6$

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5.

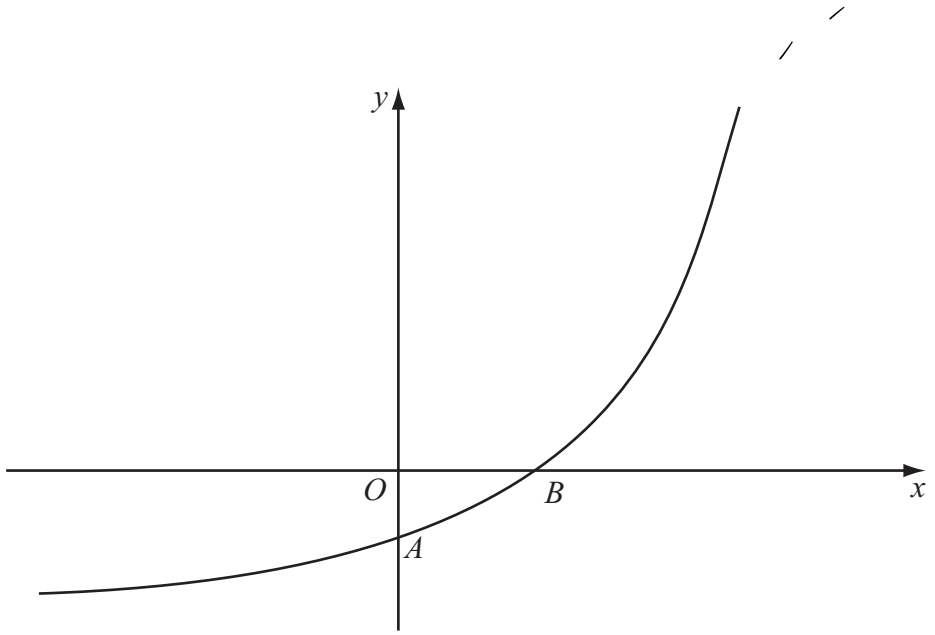

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.
The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$,
where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f ,

(1)

(d) find $\mathrm{f}^{-1}(x)$,

(3)

(e) write down the domain of f^{-1}.

(1)

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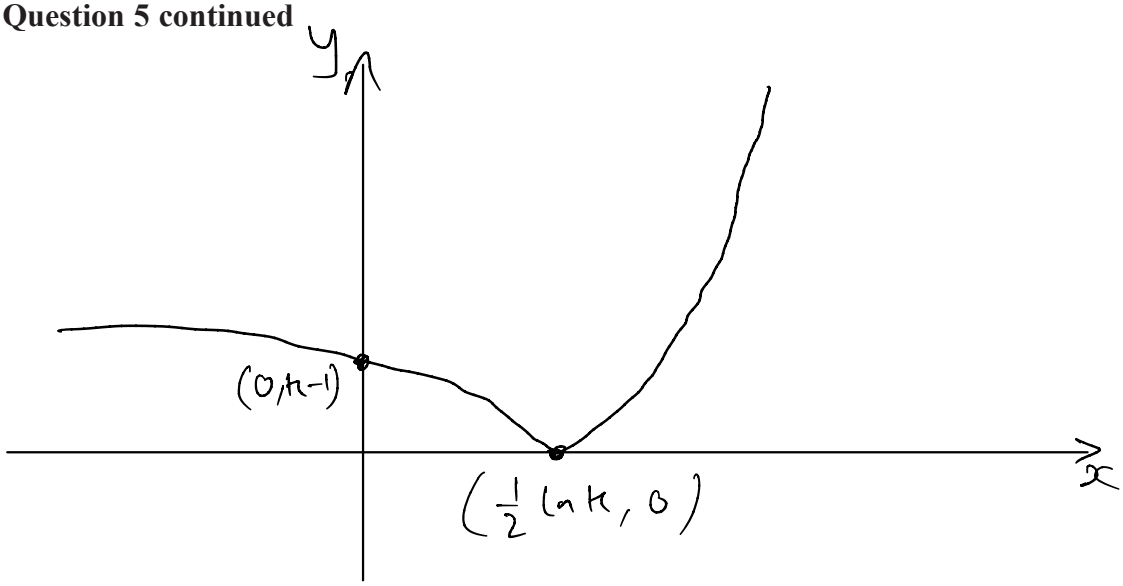
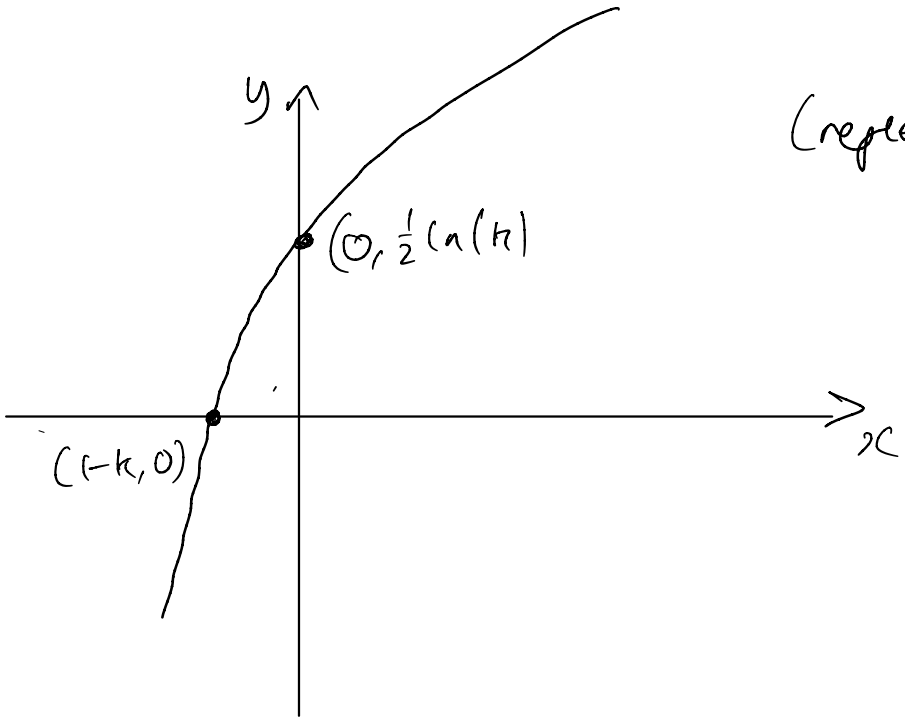
Question 5 continued

a)



b)



(reflection in line $y = x$)

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Question 5 continued

c) $f(x) = e^{2x} - k$

we know range of $y = e^x$ is $y > 0$

$e^x \rightarrow e^{2x}$ horizontal stretch s.f $\frac{1}{2}$ (range unchanged)

$e^{2x} \rightarrow e^{2x} - k$ translation $\begin{pmatrix} 0 \\ -k \end{pmatrix}$

range is now: $\underline{\underline{f(x) > -k}}$ *

d) $x = f(y)$

$= e^{2y} - k$

$x + k = e^{2y}$

$y = \frac{\ln(x+k)}{2} \Rightarrow f^{-1}(x) = \frac{1}{2}\ln(x+k)$

Domain $x > -k$

Since either Domain of $f^{-1}(x) \simeq$ Range of $f(x)$

* This level of detail is not required but is shown to help the reader with understanding.

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Question 5 continued

Q5

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9. (i) Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$

(3)

(b) $3^x e^{7x+2} = 15$

(5)

(ii) The functions f and g are defined by

$$f(x) = e^{2x} + 3, \qquad x \in \mathbb{R}$$
$$g(x) = \ln(x - 1), \qquad x \in \mathbb{R},\ x > 1$$

(a) Find f^{-1} and state its domain.

(4)

(b) Find fg and state its range.

(3)

i)a) $3x - 7 = e^5$

$x = \frac{e^5 + 7}{3}$

i) b) $\ln(3^x e^{7x+2}) = \ln(15)$

$\ln(3^x) + \ln(e^{7x+2}) = \ln(15)$

$x\ln(3) + 7x + 2 = \ln(15)$

$x\ln(3) + 7x = \ln(15) - 2$

$x(\ln(3) + 7) = \ln(15) - 2$

$x = \frac{\ln(15) - 2}{\ln(3) + 7}$

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Question 9 continued

ii) a) $x = e^{2y} + 3$

$x - 3 = e^{2y}$

$\ln(x-3) = 2y$

$y = \frac{1}{2}\ln(x-3)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\ln(x-3)$

Domain: $x > 3$

ii) b) $f(g(x)) = e^{2\ln(x-1)} + 3$

$= e^{\ln((x-1)^2)} + 3$

$= \underbrace{(x-1)^2}_{>0 \text{ since } x>1} + 3$

Common mistake

$\boxed{\neq 2(x-1)+3}$

$\therefore$ range is $f(g(x)) > 3$

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4. The function f is defined by

$$\mathrm{f} : x \mapsto |2x-5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = \mathrm{f}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $\mathrm{f}(x) = 15 + x$.

(3)

The function g is defined by

$$\mathrm{g} : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

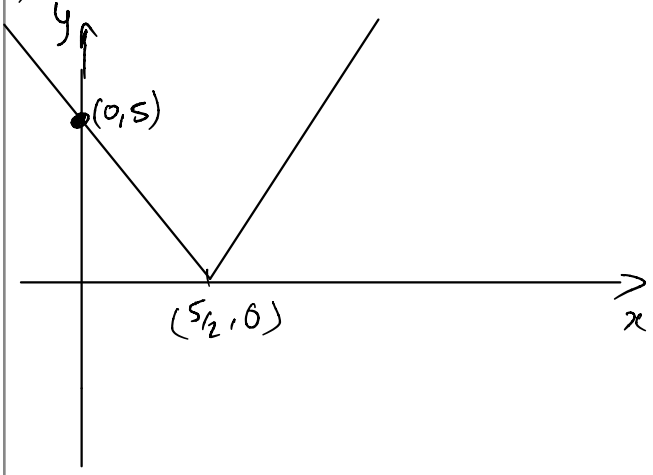
(c) Find fg(2).

(2)

(d) Find the range of g.

(3)

a)



b) $|2x-5| = 15+x$

$2x-5 = 15+x$ or $-2x+5 = 15+x$

$x = 20$ or $-3x = 10$

$x = \frac{-10}{3}$

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Question 4 continued

c) $f(g(2)) = f(4-8-1)$

$= f(-5)$

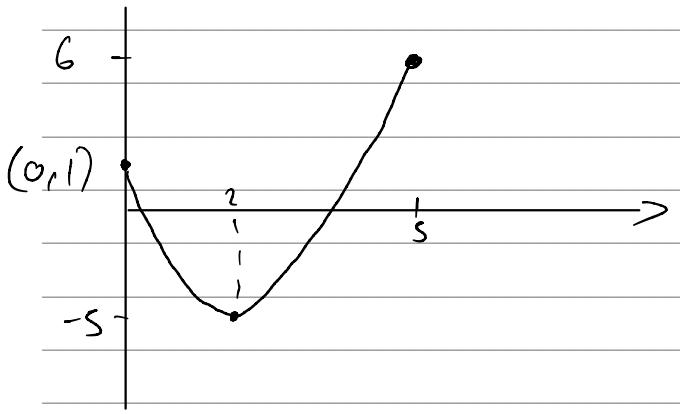
$= |-10-5| = 15$

d) $g(x) = x^2 - 4x + 1$

$g'(x) = 2x - 4 = 0$

$\Rightarrow x = 2$, $g(2) = -5$ from c)

$g(0) = 1$, $g(5) = 25 - 20 + 1 = 6$



$-5 \leq g(x) \leq 6$

Note that completing the square to find the minimum is also a correct way to do this.

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6. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \ x \neq 5$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

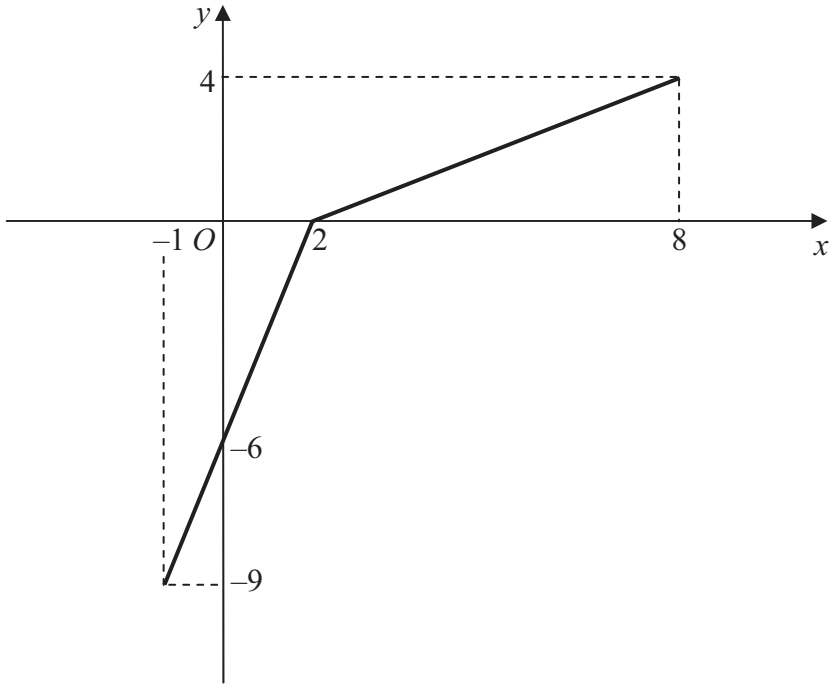

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = \mathrm{g}(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |\mathrm{g}(x)|$,

(ii) $y = \mathrm{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

H 3 5 4 0 4 R A 0 1 8 2 8

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Question 6 continued

a) $x = f(y)$

$= \dfrac{3-2y}{y-5}$

$xy - 5x = 3 - 2y$

$xy + 2y = 3 + 5x$

$y(x+2) = 3 + 5x$

$y = \dfrac{3+5x}{x+2} \Rightarrow f^{-1}(x) = \dfrac{3+5x}{x+2}$

b) $-9 \leq g(x) \leq 4$

c) $g(g(2)) = g(0) = -6$

d) $f(g(8)) = f(4) = \dfrac{3-2(4)}{4-5} = \dfrac{-5}{-1} = 5$

e) P.T.O

f) $-9 \leq x \leq 4$

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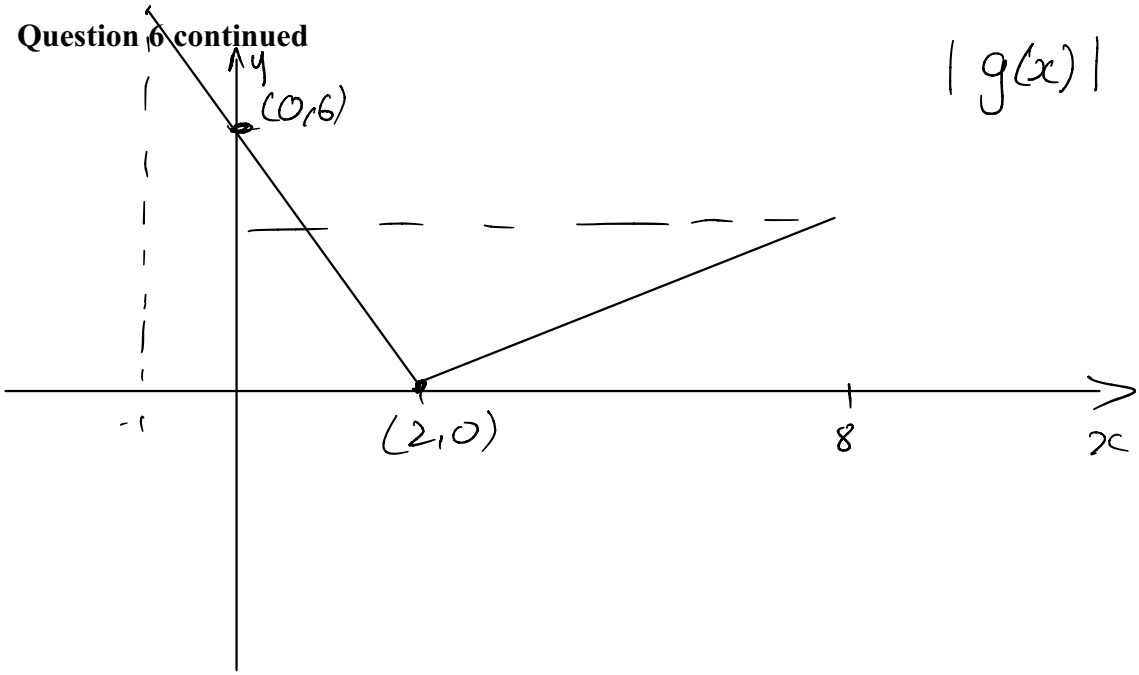
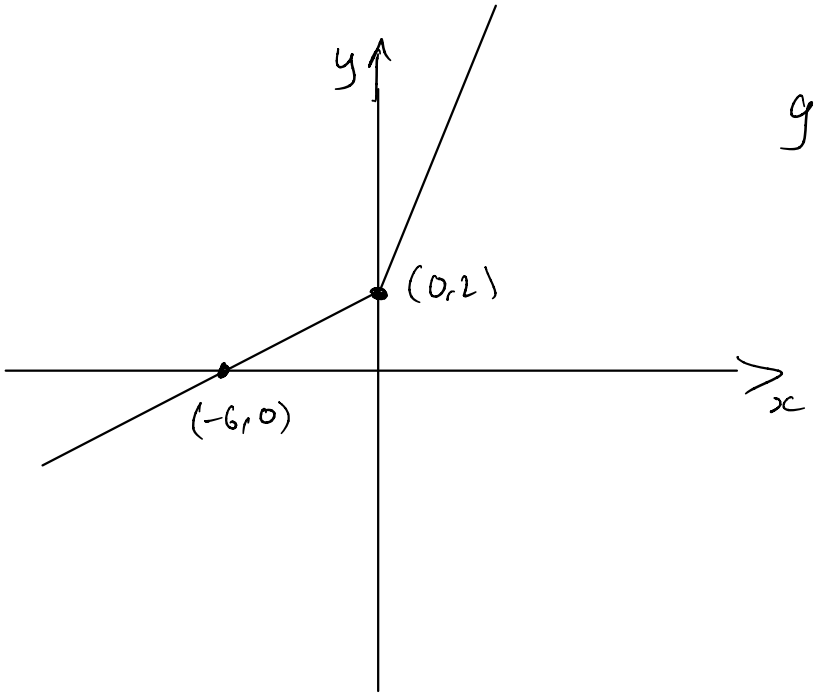
Question 6 continued



$|g(x)|$



$g^{-1}(x)$

(reflection in the $y=x$ axis)

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Question 6 continued

(Total 13 marks)

Q6

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4. The function f is defined by

$$\mathrm{f} : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R},\ x \geqslant -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

(b) Find the domain of f^{-1}.

(1)

The function g is defined by

$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

a) $x = 4 - \ln(y+2)$

$\ln(y+2) = 4 - x$

$y + 2 = e^{4-x}$

$y = e^{4-x} - 2$

$f^{-1}(x) = e^{4-x} - 2$

b) Range of $f(x)$:

$f(-1) = 4 \quad \therefore \quad f(x) \leq 4$

So domain of $f^{-1}(x)$: $\quad x \leq 4$

c) $f(g(x)) = f(e^{x^2} - 2) = 4 - \ln(e^{x^2} - 2 + 2)$

$= 4 - \ln(e^{x^2})$

$= 4 - x^2 \ln(e) = 4 - x^2$

P 3 8 1 5 9 A 0 8 2 4

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Question 4 continued

d) $f(g(x)) \leq 4$

Since domain of g is $x \in \mathbb{R}$ therefore maximum of $f(g(x))$ is $f(g(0)) = 4$.

Q4

(Total 8 marks)

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7. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{2x-1}$ **(4)**

(b) Find $\mathrm{f}^{-1}(x)$ **(3)**

(c) Find the domain of f^{-1} **(1)**

$$\mathrm{g}(x) = \ln(x+1)$$

(d) Find the solution of $\mathrm{fg}(x) = \dfrac{1}{7}$, giving your answer in terms of e. **(4)**

a) $f(x) = \dfrac{3(x+1)}{(2x-1)(x+4)} - \dfrac{1}{x+4}$

$= \dfrac{3(x+1) - (2x-1)}{(2x-1)(x+4)}$

$= \dfrac{3x+3-2x+1}{(2x-1)(x+4)}$

$= \dfrac{\cancel{x+4}}{(2x-1)\cancel{(x+4)}} = \dfrac{1}{2x-1}$ since $x > 1/2$

b) $x = \dfrac{1}{2y-1}$

$2xy - x = 1$

$2xy = 1 + x$

$y = \dfrac{1+x}{2x} \Rightarrow f^{-1}(x) = \dfrac{1+x}{2x}$

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Question 7 continued

c) Range of $f(x)$: $0 < g(x) < 1$

$\therefore$ Domain of $f^{-1}(x)$ is $0 < x < 1$

d) $f(g(x)) = \frac{1}{7}$

$f(\ln(x+1)) = \frac{1}{7}$

$\frac{1}{2\ln(x+1) - 1} = \frac{1}{7}$

$7 = 2\ln(x+1) - 1$

$8 = 2\ln(x+1)$

$4 = \ln(x+1)$

$e^4 = x+1$

$x = e^4 - 1$

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6. The functions f and g are defined by

$$\mathrm{f} : x \mapsto \mathrm{e}^x + 2, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $\mathrm{f}(2x+3) = 6$

(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

a) Range of $y = e^x$ is $y > 0$

$e^x \rightarrow e^x + 2$ translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$\therefore$ range of $g(x) = e^x + 2$ is $g(x) > 2$

b) $f(g(x)) = f(\ln(x))$

$= e^{\ln(x)} + 2 = x + 2$

c) $f(2x+3) = 6$

$e^{2x+3} + 2 = 6$

$e^{2x+3} = 4$

$2x + 3 = \ln(4)$

$x = \dfrac{\ln(4) - 3}{2}$

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Question 6 continued

d) $x = e^y + 2$

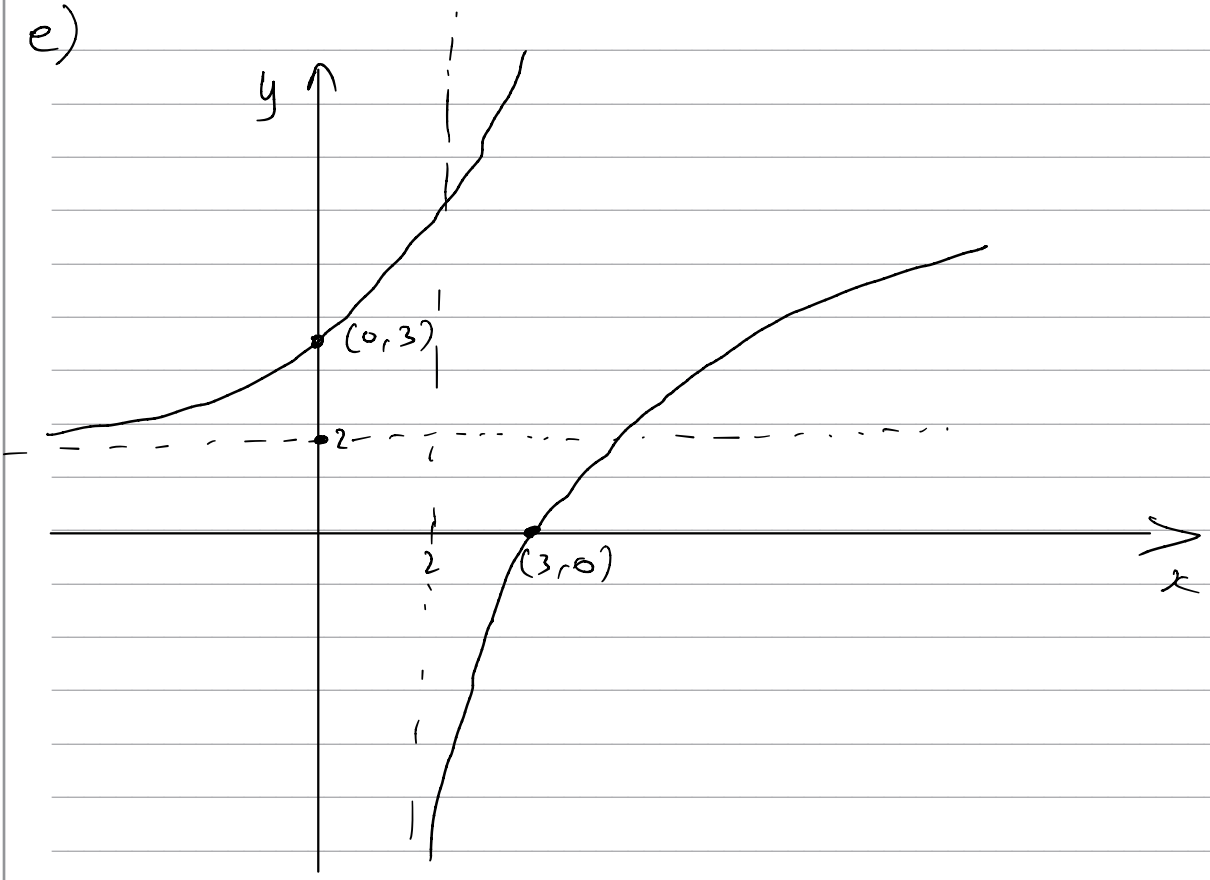
$x - 2 = e^y$

$y = \ln(x-2) \Rightarrow f^{-1}(x) = \ln(x-2)$

Domain of $f^{-1}(x)$ = Range of $f(x)$

$\Rightarrow$ Domain of $f^{-1}(x)$ is $x > 2$

e)



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7. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form. **(3)**

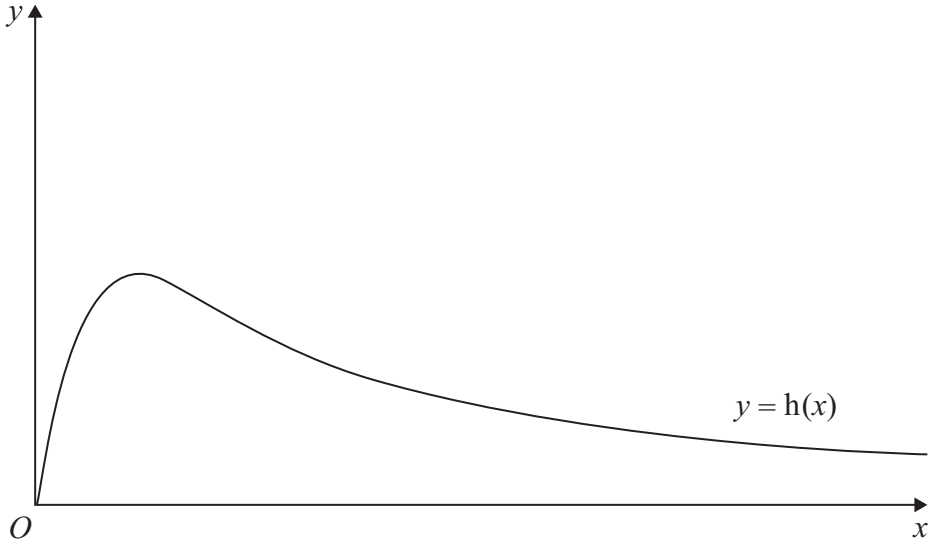

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$. **(5)**

7a) $$\frac{2(x^2+5)+4(x+2)-18}{(x^2+5)(x+2)}$$

$$= \frac{2x^2+10+4x+8-18}{(x^2+5)(x+2)}$$

$$= \frac{2x^2+4x}{(x^2+5)(x+2)}$$

$$= \frac{2x\cancel{(x+2)}}{(x^2+5)\cancel{(x+2)}} = \frac{2x}{x^2+5}$$

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Question 7 continued

b) $u = 2x$ $\quad u' = 2$

$v = x^2+5$ $\quad v' = 2x$

$$h'(x) = \frac{2(x^2+5) - (2x)^2}{(x^2+5)^2}$$

$$= \frac{-2x^2+10}{(x^2+5)^2}$$

c) Find (local) maximum:

$h'(x) = 0 \implies -2x^2+10 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$ ignore $-\sqrt{5}$ as x is +ve by inspection

$$h(\sqrt{5}) = \frac{2\sqrt{5}}{5+5} = \frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$$

as $x \to \infty$, $h(x) \to 0$ since x^2 grows faster than $2x$.

So the range of $h(x)$ is $0 \le h(x) \le \frac{\sqrt{5}}{5}$

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7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

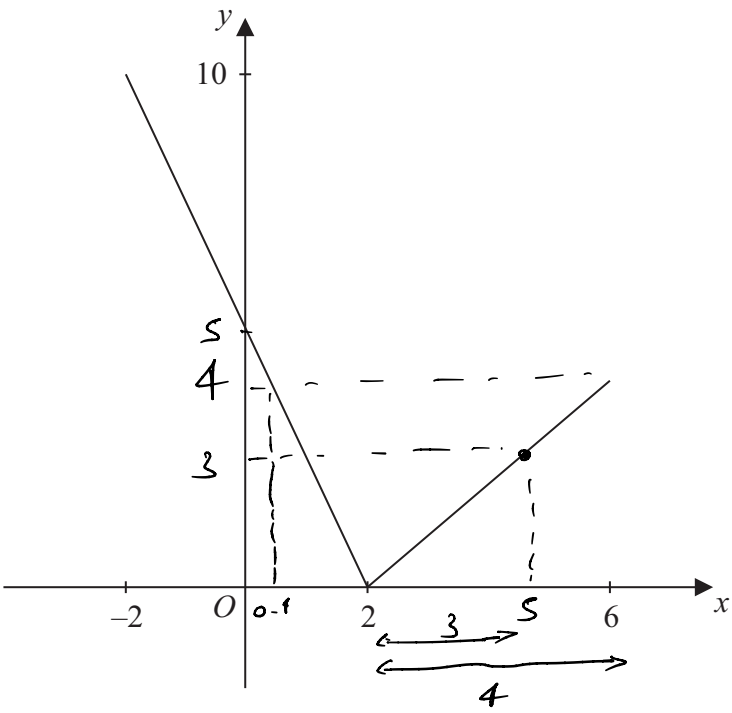

Figure 1

(a) Write down the range of f.

(1)

(b) Find ff(0).

(2)

The function g is defined by

$$\mathrm{g} : x \to \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$

(3)

(d) Solve the equation $\mathrm{gf}(x) = 16$

(5)

a) $0 \leq f(x) \leq 10$

b) $f(0) = 5$ since $(0,5)$ is midpoint of $(-2,10)$ & $(2,0)$

$f(f(0)) = f(5) = \frac{3}{4} \times 4 = 3$

see diagram above

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Question 7 continued

$x = \frac{4+3y}{5-y}$

$5x - xy = 4 + 3y$

$5x - 4 = 3y + xy$

$y(3+x) = 5x - 4$

$y = \frac{5x-4}{3+x} \Rightarrow g^{-1}(x) = \frac{5x-4}{3+x}$

Domain of $g^{-1}(x)$: $x \in \mathbb{R}, x = -3$

d) $g(f(x)) = 16$

$\cancel{g^{-1}}(\cancel{g}(f(x))) = g^{-1}(16)$

$f(x) = g^{-1}(16)$

$= \frac{5(16)-4}{3+16} = \frac{76}{19} = 4$

$f(x) = 4 \Rightarrow x = 6$ or $x = 0.4$

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5.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$

(4)

(b) Find the range of g.

(2)

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$.

(4)

a) $g(x) = \dfrac{x}{x+3} + \dfrac{3(2x+1)}{(x+3)(x-2)}$

$= \dfrac{x(x-2) + 3(2x+1)}{(x+3)(x-2)}$

$= \dfrac{x^2+4x+3}{(x+3)(x-2)}$

since $x \neq -3$

$= \dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-2)} = \dfrac{x+1}{x-2}$

b) as $x \to \infty$, $g(x) \to 1$

$g(3) = \dfrac{4}{1} = 4$ so $1 < g(x) < 4$

c) $g(a) = g^{-1}(a)$

$g(g(a)) = \cancel{g(g^{-1}}(a)$

$g(g(a)) = a$

$g\left(\dfrac{a+1}{a-2}\right) = a$

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Question 5 continued

$$\frac{a-2}{a-2} \times \frac{\frac{a+1}{a-2}+1}{\frac{a+1}{a-2}-2} = a$$

$$\frac{a+1+a-2}{a+1-2(a-2)} = a$$

$$\frac{2a-1}{-a+5} = a$$

$$2a-1 = -a^2+5a$$

$$a^2-3a-1=0$$

$$a = \frac{3 \pm \sqrt{(-3)^2-4(1)(-1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$ ignore $\frac{3-\sqrt{13}}{2}$ since domain of $g(x)$ is $x > 3$

$$\boxed{a = \frac{3+\sqrt{13}}{2}}$$

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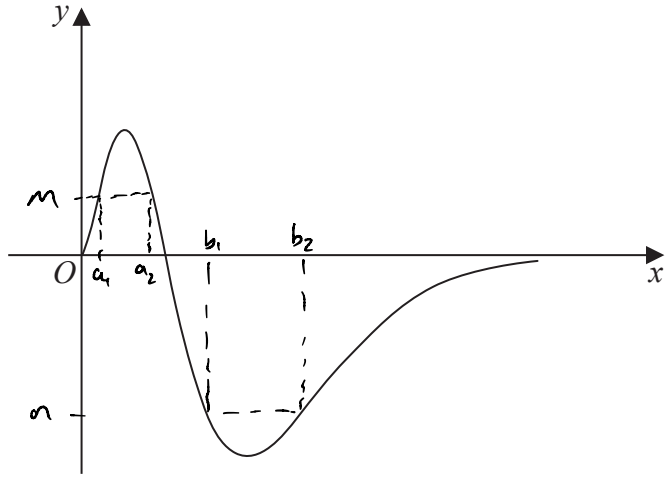

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geqslant 0$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g.

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

a) $g(x) = (x^2 - x^3)e^{-2x}$ ← Common trick of partially expanding so that you only have to apply product rule once

$u = x^2 - x^3 \qquad u' = 2x - 3x^2$

$v = e^{-2x} \qquad v' = -2e^{-2x}$

$g'(x) = (x^2 - x^3)(-2e^{-2x}) + (2x - 3x^2)e^{-2x}$

$= e^{-2x}\left((x^2 - x^3)(-2) + 2x - 3x^2\right)$

$= e^{-2x}(-2x^2 + 2x^3 + 2x - 3x^2)$

$= e^{-2x}(2x^3 - 5x^2 + 2x)$

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Question 7 continued

$g'(x) = 0$

$e^{-2x}(2x^3 - 5x^2 + 2x) = 0$

$e^{-2x} \neq 0 \qquad x(2x^2 - 5x + 2) = 0$

$x = 0 \quad , (2x-1)(x-2) = 0$

$x = \frac{1}{2}, \; x = 2$

$g(\frac{1}{2}) = \frac{1}{4}(\frac{1}{2})e^{-1} = \frac{1}{8e}$

$g(2) = 4(-1)e^{-4} = \frac{-4}{e^4}$

$\frac{-4}{e^4} \leq g(x) \leq \frac{1}{8e} \qquad$ or $-4e^{-4} \leq g(x) = \frac{1}{8}e^{-1}$

(c) For an inverse to exist a function needs to be one to one.
By inspection, this function isn't.
see diagram for context of examples below

$f(a_1) = f(a_2) = m$

$f(b_1) = f(b_2) = n$

<u>Note</u> that this is for the understanding of the reader. Just the first sentence would get you the mark

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3.

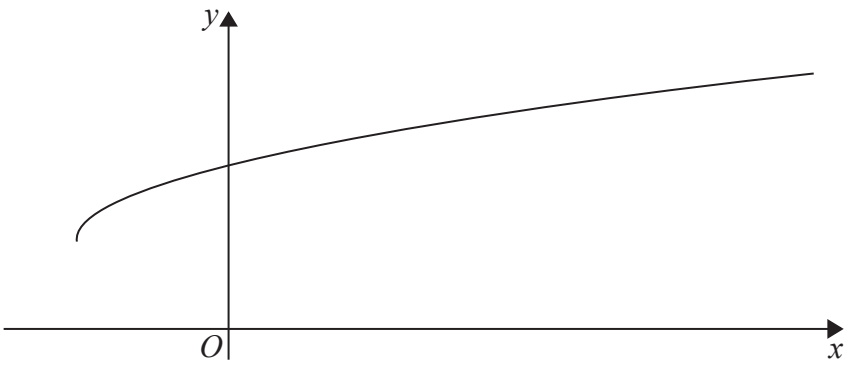

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = 3 + \sqrt{x + 2}\,, \qquad x \geqslant -2$$

(a) State the range of g.

(1)

(b) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(c) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(d) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

a) $g(-2) = 3 + \sqrt{-2+2} = 3$

so $g(x) \geqslant 3$

b) $x = 3 + \sqrt{y+2}$

$(x-3)^2 = y+2$

$y = (x-3)^2 - 2$

$g^{-1}(x) = (x-3)^2 - 2 \qquad x \geqslant 3$.

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Question 3 continued

c) $3+\sqrt{x+2} = x$

$\sqrt{x+2} = x-3$

$x+2 = (x-3)^2$

$x+2 = x^2-6x+9$

$0 = x^2-7x+7$

$x = \frac{7\pm\sqrt{21}}{2}$ ignore $\frac{7-\sqrt{21}}{2}$

Since if $3+\sqrt{x+2} = x$
then $x \geqslant 3$ as $\sqrt{x+2} \geqslant 0$.

d) $g\left(\frac{7+\sqrt{21}}{2}\right) = \frac{7+\sqrt{21}}{2}$

$\cancel{g^{-1}}\left(\cancel{g}\left(\frac{7+\sqrt{21}}{2}\right)\right) = g^{-1}\left(\frac{7+\sqrt{21}}{2}\right)$

$\Rightarrow g\left(\frac{7+\sqrt{21}}{2}\right) = \frac{7+\sqrt{21}}{2} = g^{-1}\left(\frac{7+\sqrt{21}}{2}\right)$

So $a = \frac{7+\sqrt{21}}{2}$

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2. The function f is defined by

$$f(x) = \frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25}, \qquad x > 4$$

(a) Show that $f(x) = \dfrac{A}{Bx+C}$ where A, B and C are constants to be found. **(4)**

(b) Find $f^{-1}(x)$ and state its domain. **(3)**

2. $f(x) = \dfrac{6(2x-5)+2(2x+5)+60}{(2x+5)(2x-5)}$

$= \dfrac{12x-30+4x+10+60}{(2x+5)(2x-5)}$

$= \dfrac{16x+40}{(2x+5)(2x-5)}$

$= \dfrac{8\cancel{(2x+5)}}{\cancel{(2x+5)}(2x-5)} = \dfrac{8}{2x-5}$

$A=8$, $B=2$, $C=-5$

b) $x = \dfrac{8}{2y-5}$

$2xy - 5x = 8$

$2xy = 8+5x$

$y = \dfrac{8+5x}{2x} \Rightarrow f^{-1}(x) = \dfrac{8+5x}{2x}$

Range of $f(x)$: $f(4) = \frac{8}{3} \Rightarrow 0 < f(x) < 8/3$

So Domain of $f^{-1}(x)$: $0 < x < 8/3$

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Question 2 continued

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