

London Academy of Mathematics

Edexcel - AS-level Maths

Pure

Integration

Question Paper

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int \left(\frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) dx$$

giving your answer in its simplest form.

(4)

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(Total for Question 1 is 4 marks)



15.

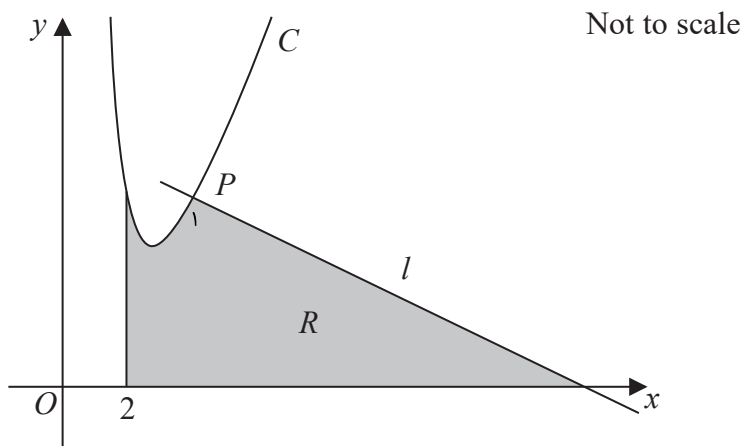


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \quad x > 0$$

The point $P(4, 6)$ lies on C .

The line l is the normal to C at the point P .

The region R , shown shaded in Figure 4, is bounded by the line l , the curve C , the line with equation $x = 2$ and the x -axis.

Show that the area of R is 46

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

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3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) dx$$

simplifying your answer.

(3)

- (b) Hence find the value of k such that

$$\int_{0.5}^2 \left(\frac{4}{x^3} + kx \right) dx = 8$$

(3)



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Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 6 marks)



13.

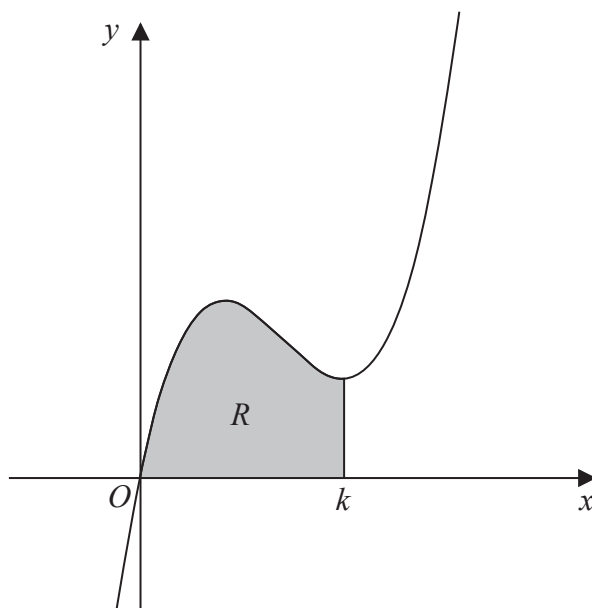
**Figure 3**

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the line with equation $x = k$.

Show that the area of R is $\frac{256}{3}$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(7)



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Question 13 continued

Lined area for writing the answer to Question 13.



7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

(4)



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Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)



3. Find

$$\int \frac{3x^4 - 4}{2x^3} dx$$

writing your answer in simplest form.

(4)

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Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 4 marks)



9. Find the value of the constant k , $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}} dx = 20$$

(4)

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Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 4 marks)



14. A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8$$

(a) Write $f(x)$ in the form

$$a(x + b)^2 + c$$

where a , b and c are constants to be found.

(3)

The curve C has a maximum turning point at M .

(b) Find the coordinates of M .

(2)

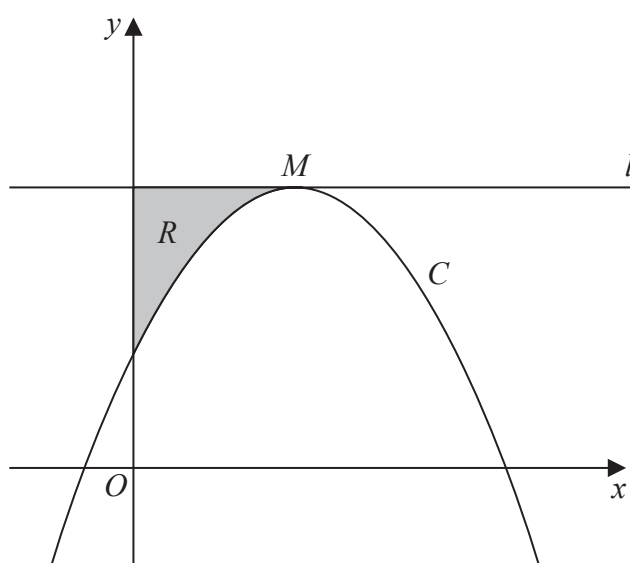


Figure 3

Figure 3 shows a sketch of the curve C .

The line l passes through M and is parallel to the x -axis.

The region R , shown shaded in Figure 3, is bounded by C , l and the y -axis.

(c) Using algebraic integration, find the area of R .

(5)



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Question 14 continued

Handwriting practice area with horizontal lines.



1. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5 \right) dx$$

giving your answer in simplest form.

(4)

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Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 4 marks)



10.

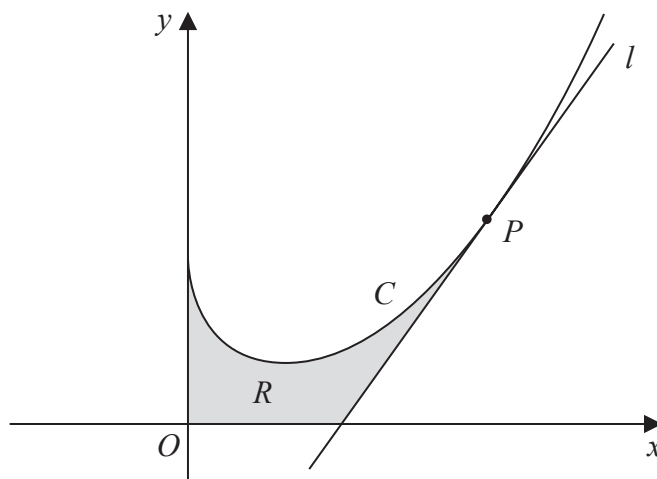


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \quad x \geq 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P .

(a) Show that l has equation

$$13x - 6y - 26 = 0 \quad (5)$$

The region R , shown shaded in Figure 2, is bounded by the y -axis, the curve C , the line l and the x -axis.

(b) Find the exact area of R . (5)



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Question 10 continued

Lined area for writing the answer to Question 10.



5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

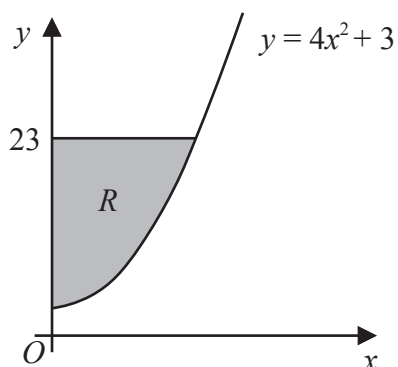


Figure 2

The finite region R , shown shaded in Figure 2, is bounded by the curve with equation $y = 4x^2 + 3$, the y -axis and the line with equation $y = 23$

Show that the exact area of R is $k\sqrt{5}$ where k is a rational constant to be found.

(5)

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Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 5 marks)



8.

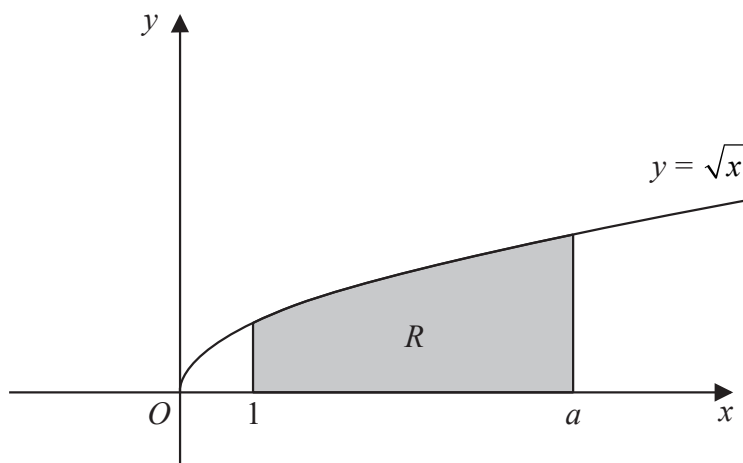


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{x}$, $x \geq 0$

The region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 1$, the x -axis and the line with equation $x = a$, where a is a constant.

Given that the area of R is 10

(a) find, in simplest form, the value of

(i) $\int_1^a \sqrt{8x} \, dx$

(ii) $\int_0^a \sqrt{x} \, dx$

(4)

(b) show that $a = 2^k$, where k is a rational constant to be found.

(4)



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Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)



14.

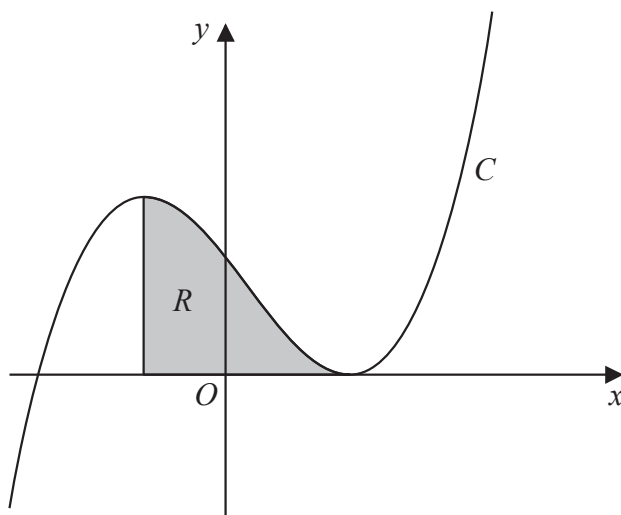


Figure 5

Figure 5 shows a sketch of the curve C with equation $y = (x - 2)^2(x + 3)$

The region R , shown shaded in Figure 5, is bounded by C , the vertical line passing through the maximum turning point of C and the x -axis.

Find the exact area of R .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(9)

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Question 14 continued

Lined area for writing the answer to Question 14.



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