

Complex Numbers and Argand Diagrams

Question Paper

1. A student was asked to answer the following:

For the complex numbers $z_1 = 3 - 3i$ and $z_2 = \sqrt{3} + i$, find the value of $\arg\left(\frac{z_1}{z_2}\right)$

The student's attempt is shown below.

Line 1	→	$\arg(z_1) = \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$
Line 2	→	$\arg(z_2) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$
Line 3	→	$\arg\left(\frac{z_1}{z_2}\right) = \frac{\arg(z_1)}{\arg(z_2)}$
Line 4	→	$= \frac{\left(\frac{\pi}{4}\right)}{\left(\frac{\pi}{6}\right)} = \frac{3}{2}$

The student made errors in line 1 and line 3

Correct the error that the student made in

(a) (i) line 1

(ii) line 3

(2)

(b) Write down the correct value of $\arg\left(\frac{z_1}{z_2}\right)$

(1)

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Question 1 continued

Lined area for writing the answer to Question 1.



Solutions relying on calculator technology are not acceptable.

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- $$(ii) \text{ the region defined by } \{z \in \mathbb{C} : |z - z_1| < |z - z_2|\}$$
- (4)

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Question 2 continued

Lined area for writing the answer to Question 2.

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3. (i) Shade, on an Argand diagram, the set of points for which

$$|z - 3| \leq |z + 6i| \quad (3)$$

- (ii) Determine the exact complex number w which satisfies both

$$\arg(w - 2) = \frac{\pi}{3} \quad \text{and} \quad \arg(w + 1) = \frac{\pi}{6} \quad (6)$$

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Question 3 continued

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4. Given that

$$\begin{aligned}z_1 &= 2 + 3i \\|z_1 z_2| &= 39\sqrt{2} \\ \arg(z_1 z_2) &= \frac{\pi}{4}\end{aligned}$$

where z_1 and z_2 are complex numbers,

(a) write z_1 in the form $r(\cos \theta + i \sin \theta)$

Give the exact value of r and give the value of θ in radians to 4 significant figures.

(2)

(b) Find z_2 giving your answer in the form $a + ib$ where a and b are integers.

(6)

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Question 4 continued

Lined area for writing the answer to Question 4.

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5. (a) Shade on an Argand diagram the set of points

$$\left\{z \in \mathbb{C} : |z - 1 - i| \leq 3\right\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leq \arg(z - 2) \leq \frac{3\pi}{4}\right\}$$

(5)

The complex number w satisfies

$$|w - 1 - i| = 3 \text{ and } \arg(w - 2) = \frac{\pi}{4}$$

- (b) Find, in simplest form, the exact value of $|w|^2$
- (4)

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Question 5 continued

Handwriting practice area with 30 horizontal lines.

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6.

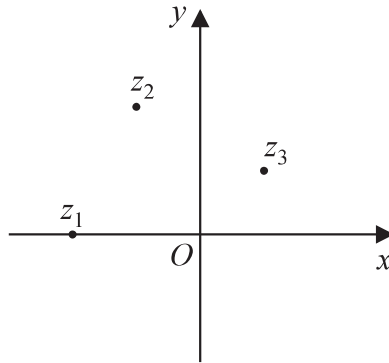


Figure 1

The complex numbers $z_1 = -2$, $z_2 = -1 + 2i$ and $z_3 = 1 + i$ are plotted in Figure 1, on an Argand diagram for the complex plane with $z = x + iy$

- (a) Explain why z_1 , z_2 and z_3 cannot all be roots of a quartic polynomial equation with real coefficients.

(2)

- (b) Show that $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \frac{\pi}{4}$

(3)

- (c) Hence show that $\arctan(2) - \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$

(2)

A copy of Figure 1, labelled Diagram 1, is given on page 12.

- (d) Shade, on Diagram 1, the set of points of the complex plane that satisfy the inequality

$$|z + 2| \leq |z - 1 - i|$$

(2)

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Question 6 continued

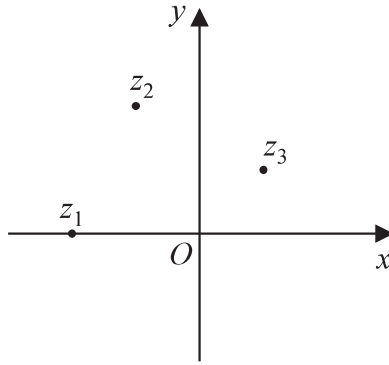


Diagram 1

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7. (i) (a) Show that

$$\frac{2 + 3i}{5 + i} = k(1 + i)$$

where k is a constant to be determined.

(Solutions relying on calculator technology are not acceptable.)

(3)

Given that

- n is a positive integer
- $\left(\frac{2 + 3i}{5 + i}\right)^n$ is a real number

(b) use the answer to part (a) to write down the smallest possible value of n .

(1)

(ii) The complex number $z = a + bi$ where a and b are real constants.

Given that

- $|z^{10}| = 59\,049$
- $\arg(z^{10}) = -\frac{5\pi}{3}$

determine the value of a and the value of b .

(4)

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Question 7 continued

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8. Given that $z = a + bi$ is a complex number where a and b are real constants,

(a) show that zz^* is a real number.

(2)

Given that

- $zz^* = 18$

- $\frac{z}{z^*} = \frac{7}{9} + \frac{4\sqrt{2}}{9}i$

(b) determine the possible complex numbers z

(5)

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Question 8 continued

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- (ii) the radius of this circle.

(b) Show, by shading on an Argand diagram, the set of points defined by

(1)

- (3)

(d) Determine the area of the region defined by A , giving your answer to 3 significant figures.

(4)

Question 9 continued

Handwriting practice area with 30 horizontal lines.

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10. Given that there are two distinct complex numbers z that satisfy

$$\{z: |z - 3 - 5i| = 2r\} \cap \left\{z: \arg(z - 2) = \frac{3\pi}{4}\right\}$$

determine the exact range of values for the real constant r .

(7)

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Question 10 continued

Lined area for writing the answer to Question 10.

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11.

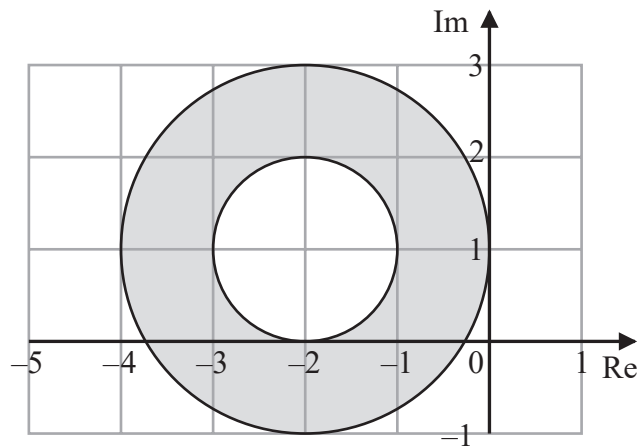


Figure 1

Figure 1 shows an Argand diagram.

The set P , of points that lie within the shaded region including its boundaries, is defined by

$$P = \{z \in \mathbb{C} : a \leq |z + b + ci| \leq d\}$$

where a , b , c and d are integers.

(a) Write down the values of a , b , c and d .

(3)

The set Q is defined by

$$Q = \{z \in \mathbb{C} : a \leq |z + b + ci| \leq d\} \cap \{z \in \mathbb{C} : |z - i| \leq |z - 3i|\}$$

(b) Determine the exact area of the region defined by Q , giving your answer in simplest form.

(7)

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Question 11 continued

Lined area for writing the answer to Question 11.

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12. (a) Express the complex number $w = 4\sqrt[3]{-4i}$ in the form $r(\cos \theta + i \sin \theta)$ where $r > 0$ and $-\pi < \theta \leq \pi$

(4)

(b) Show, on a single Argand diagram,

(i) the point representing w

(ii) the locus of points defined by $\arg(z + 10i) = \frac{\pi}{3}$

(c) Hence determine the minimum distance of w from the locus $\arg(z + 10i) = \frac{\pi}{3}$

(3)

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Question 12 continued

Handwriting practice area with horizontal lines.

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Question 13 continued

Lined area for writing the answer to Question 13.



Question 14 continued

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15. In an Argand diagram, the points A , B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2i$.
- (a) Find the complex numbers represented by the points B and C , giving your answers in the form $x + iy$, where x and y are real and exact. (6)

The points D , E and F are the midpoints of the sides of triangle ABC .

- (b) Find the exact area of triangle DEF . (3)

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Question 15 continued

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16. (i) The point P is one vertex of a regular pentagon in an Argand diagram.
The centre of the pentagon is at the origin.

Given that P represents the complex number $6 + 6i$, determine the complex numbers that represent the other vertices of the pentagon, giving your answers in the form $r(\cos \theta + i \sin \theta)$, where $r \in \mathbb{R}$, $r > 0$ and $0 \leq \theta < 2\pi$

(5)

- (ii) (a) On a single Argand diagram, shade the region, R , that satisfies both

$$|z - 2i| \leq 2 \quad \text{and} \quad \frac{1}{4}\pi \leq \arg z \leq \frac{1}{3}\pi$$

(2)

- (b) Determine the exact area of R , giving your answer in simplest form.

(4)

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Question 16 continued

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