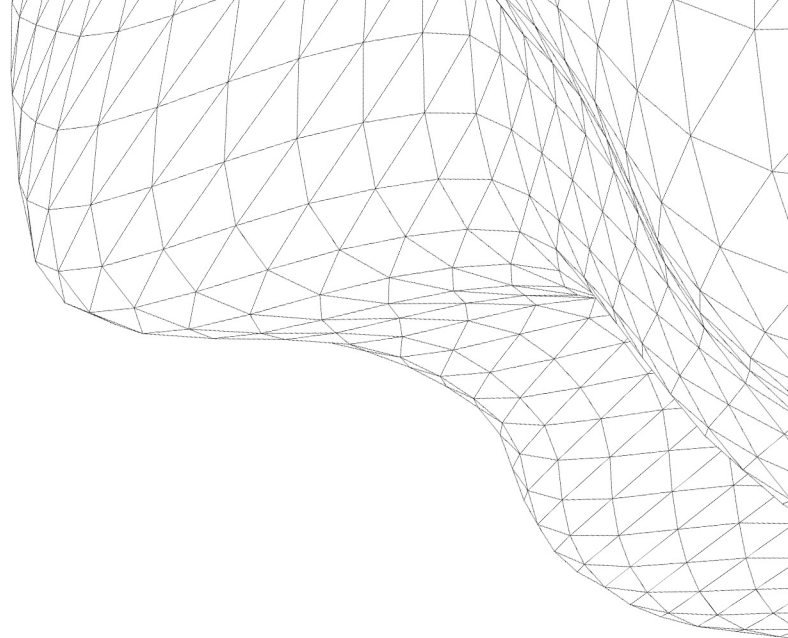
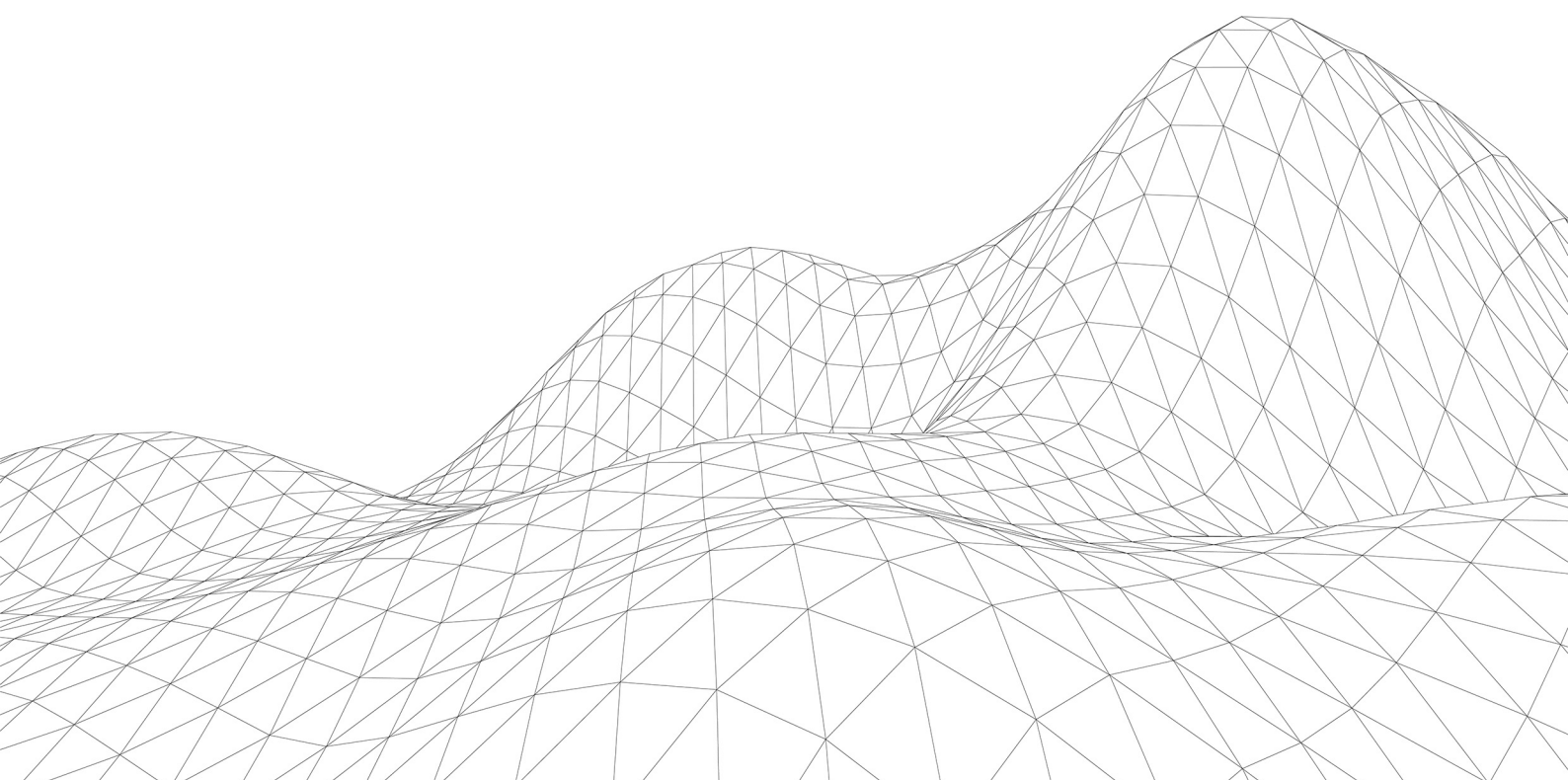




Hyperbolic Equations and Identities

Question Paper





1.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve the equation

$$7 \cosh x + 3 \sinh x = 2e^x + 7$$

Give your answers as simplified natural logarithms.

(5)

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Question 1 continued

Lined area for writing the answer to Question 1.





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2. Solve the equation

$$18\cosh x + 14\sinh x = 11 + e^x$$

Give your answers in the form $\ln a$, where a is rational.

(5)

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Question 2 continued

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Question 3 continued

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4. Solve the equation

$$15 \operatorname{sech}^2 x + 7 \tanh x = 13$$

Give your answers in terms of simplified natural logarithms.

(6)

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Question 4 continued

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5. Solve the equation

$$4 \tanh x - \operatorname{sech} x = 1$$

giving your answer in the form $x = \ln k$ where k is a fully simplified rational number.

(6)

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Question 5 continued

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6. Find the exact values of x for which

$$\coth^2 x + 5 \operatorname{cosech} x + 5 = 0$$

giving your answers in terms of natural logarithms.

(6)

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Question 6 continued

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- $$1 - \tanh^2 x \equiv \operatorname{sech}^2 x \quad (3)$$

- $$2 \operatorname{sech}^2 x + 3 \tanh x = 3$$

giving your answer as an exact logarithm. (3)

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Question 7 continued

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Question 8 continued

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Question 9 continued

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- [illegible]

Question 10 continued

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11. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that
- (i) $\cosh 2x \equiv 2 \cosh^2 x - 1$
- (ii) $\sinh 2x \equiv 2 \sinh x \cosh x$
- (4)

- (b) Solve the equation

$$\cosh 2x - 7 \cosh x = -7$$

giving your answers as exact logarithms.

(5)

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Question 11 continued

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12. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64 \cosh^4 x - 64 \cosh^2 x - 9 = 0$$

Give your answers in the form $q \ln 2$ where q is rational and in simplest form.

(4)

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Question 12 continued

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- (3)

- (5)

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14. Given that

$$\cosh y = x \quad \text{and} \quad y < 0$$

use the definition of $\cosh y$ in terms of exponential functions to prove that

$$y = \ln(x - \sqrt{x^2 - 1})$$

(6)

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Question 14 continued

Handwriting practice area with 30 horizontal lines.

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15. (a) Use the definitions of hyperbolic functions in terms of exponentials to show that

$$\sinh(A + B) \equiv \sinh A \cosh B + \cosh A \sinh B \quad (3)$$

- (b) Hence express $10 \sinh x + 8 \cosh x$ in the form $R \sinh(x + \alpha)$ where $R > 0$, giving α in the form $\ln p$ where p is an integer. (4)

- (c) Hence solve the equation

$$10 \sinh x + 8 \cosh x = 18\sqrt{7}$$

giving your answer in the form $\ln(\sqrt{7} + q)$ where q is a rational number to be determined. (2)

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Question 15 continued

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Answer ALL questions. Write your answers in the spaces provided.

16. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \quad -k < x < k$$

stating the value of the constant k .

(5)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5)

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Question 16 continued

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Question 17 continued

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