London Academy of Maths

Implicit Differentiation

Mark Scheme

Question Number	Scheme	Marks
1 (a)	$x = 2y^2 + 5y - 6$	
	$\dfrac{dx}{dy} = 4y + 5 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\dfrac{5}{4}$	M1
	Substitutes their $y = -\dfrac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\dfrac{73}{8}, -\dfrac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2 + 5y - 6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\dfrac{dx}{dy} = 4y + 5 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{4y} + \dfrac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x = 2y^2 + 5y - 6 \Rightarrow 1 = ay\dfrac{dy}{dx} + b\dfrac{dy}{dx}$ followed by an attempt to make $\dfrac{dy}{dx}$ the subject

A1: $\dfrac{dy}{dx} = \dfrac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\dfrac{dx}{dy} = \dfrac{ay+b}{k}$ **or** $\dfrac{dy}{dx} = \dfrac{k}{ay+b}$

So full marks can only be awarded after sight of $\dfrac{dx}{dy} = 4y + 5$ **or** $\dfrac{dy}{dx} = \dfrac{1}{4y+5}$

M1: Sets $'ay + b' = 0 \Rightarrow y = ...$ following sight of $\dfrac{dx}{dy} = \dfrac{ay+b}{k}$ **or** $\dfrac{dy}{dx} = \dfrac{k}{ay+b}$

Ignore/condone incorrect statements such as $\dfrac{dy}{dx} = 0$ here

This can be implied. E.g $\dfrac{dy}{dx} = \dfrac{1}{4y+5}$ followed by $y = -\dfrac{5}{4}$

dM1: Substitutes their solution of their $4y + 5 = 0$ into $x = 2y^2 + 5y - 6$ to find x.

This is dependent upon the previous M

A1: $\left(-\dfrac{73}{8}, -\dfrac{5}{4}\right)$ following $\dfrac{dx}{dy} = 4y + 5$ **or** $\dfrac{dy}{dx} = \dfrac{1}{4y+5}$

The answer may be given separately, for example, as $x = -9.125$, $y = -1.25$

ISW after a correct answer.

Further guidance to highlighted sentence.

If (a) is incorrect or incomplete follow the following guidance.

Incorrect (a): If they incorrectly state $\frac{\mathrm{d}y}{\mathrm{d}x} = 4y + 5$ o.e. and go on to use $4y + 5 = 0 \Rightarrow y = ...$ and write down a correct set of coordinates for P they will score NO MARKS in (b)

Incomplete (a): If they state $\frac{\mathrm{d}x}{\mathrm{d}y} = 4y + 5$ in (a) and go on to use $4y + 5 = 0 \Rightarrow y = ...$ in (b) they will score no marks in (a) but could potentially score full marks in (b)

You can award all 3 marks for $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following either $\frac{\mathrm{d}x}{\mathrm{d}y} = 4y + 5$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4y+5}$

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Alternative approach: Part (a) may well be blank or incorrect. Part(b) done via completion of square

It is possible to do part (b) by not doing any differentiating.

$$x = 2y^2 + 5y - 6 \Rightarrow x = 2\left(y + \frac{5}{4}\right)^2 - \frac{73}{8}$$

M1: An allowable attempt to complete the square followed by a valid attempt at either x or y from their attempt
A1: For a correct x or y
A1: For a correct x and y

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question Number	Scheme	Marks
2	Differentiates wrt x $\quad \frac{dy}{dx} = 16\sec^2(2x)$ oe	M1
	Inverts to get $\frac{dx}{dy} = \frac{1}{16\sec^2 2x} = \frac{1}{16(1+\tan^2 2x)}$ | $\frac{dy}{dx} = 16(1+\tan^2(2x)) = 16\left(1+\left(\frac{y}{8}\right)^2\right)$	dM1
	$\frac{dx}{dy} = \frac{A}{B+y^2}$	ddM1
	$= \frac{4}{64+y^2}$	A1
		(4)
		(4 marks)

M1: Achieves $\frac{dy}{dx} = \lambda\sec^2(2x)$ oe or implicitly $1 = \lambda\sec^2(2x)\frac{dx}{dy}$

If they change $\tan 2x$ to $\frac{\sin 2x}{\cos 2x}$ they can score this mark for $\frac{dy}{dx} = \frac{\alpha\cos 2x\cos 2x \pm \beta\sin 2x\sin 2x}{(\cos 2x)^2}$

If they change $\tan 2x$ to $\frac{2\tan x}{1-\tan^2 x}$ they could never reach the required solution so score M0

dM1: Scored for two of the three processes 1 and 2 (either order) or 2 followed by 3 :

1. The reciprocal must be taken. (The variable cannot change)
2. The identity $1+\tan^2 2x = \sec^2 2x$ must be attempted
3. There must be an attempt to replace $\tan 2x$ by $\frac{y}{8}$

ddM1: Scored for attempting all three processes **and** attempting to eliminate the fractions (seen in at least two of the terms in the expression)

A1: cso

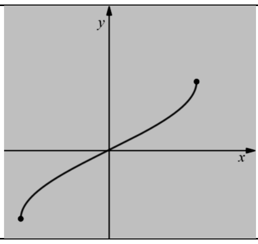
Alternative using arctan

M1: Expresses x as $x = \lambda\arctan\left(\frac{y}{8}\right)$ and attempts some differentiation $\Rightarrow \frac{dx}{dy} = \frac{...}{...+...y^2}$

dM1: As above but achieves $\frac{dx}{dy} = \frac{C}{(1+\left(\frac{y}{8}\right)^2)}$

ddM1: Eliminates fractions (seen in at least two of the terms in the expression) $\frac{dx}{dy} = \frac{A}{B+y^2}$

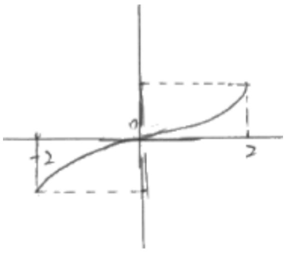
A1: cso $\frac{dx}{dy} = \frac{4}{64+y^2}$

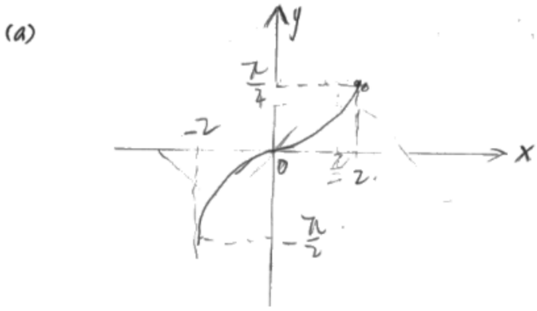
Question Number	Scheme	Marks
3 (a)	 Shape and position	B1
		(1)
(b)	$x=2\sin y \Rightarrow \left(\frac{dx}{dy}=\right)2\cos y$ and attempts to use **both** $\frac{dy}{dx}=1\div\frac{dx}{dy}$ and $\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-...x^2}$	M1
	$\left(\frac{dy}{dx}=\right)\frac{1}{2\sqrt{1-\frac{x^2}{4}}}$	A1
	$\left(\frac{dy}{dx}=\right)\frac{1}{\sqrt{4-x^2}}$	A1cso
		(3)
(c)	Substitutes $x=\sqrt{2}$ into their $\frac{dy}{dx}=\frac{1}{\sqrt{4-x^2}}$ $\left(=\frac{\sqrt{2}}{2}\right)$	M1
	Finds the equation of the tangent at P $y-\frac{\pi}{4}=\text{"}\frac{\sqrt{2}}{2}\text{"}\left(x-\text{"}\sqrt{2}\text{"}\right)$	dM1
	$y=\frac{\sqrt{2}}{2}x-1+\frac{\pi}{4}$	A1
		(3)
		(7 marks)
Alt(b)	$y=\arcsin\left(\frac{x}{2}\right)\Rightarrow\left(\frac{dy}{dx}=\right)\frac{1}{2}\times\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	M1A1
	$\left(\frac{dy}{dx}=\right)\frac{1}{\sqrt{4-x^2}}$	A1

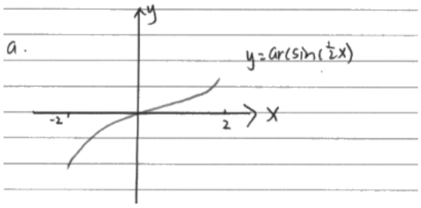
(a)

B1 Correct shape and position: Look for a curve in quadrants 1 and 3 with non zero gradient at the origin and gradient $\to\infty$ at both ends. Ignore values labelled on the axes.
If there is more than one attempt, mark the one which appears in the main body of the work as their fullest attempt. If there is more than one sketch on the same graph, then it must be clearly labelled which is the one to be marked.

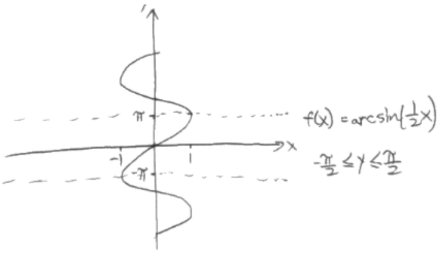
Examples of B1

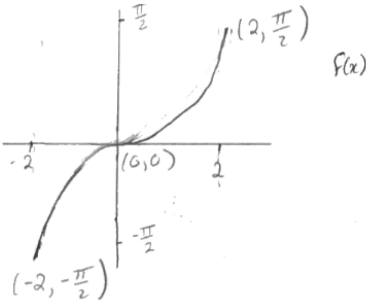






Examples of B0





(b) Note on EPEN it is M1M1A1 which is now marked as M1A1A1

M1 $\quad x = 2\sin y \Rightarrow \dfrac{dx}{dy} = \pm k\cos y$ where k is non zero and attempts to use both $\dfrac{dy}{dx} = 1 \div \dfrac{dx}{dy}$ and $\sin^2 y + \cos^2 y = 1$ in order to obtain $\dfrac{dy}{dx}$ in terms of x. Allow slips with dealing with their "2"

Allow working leading to $\dfrac{...}{...\sqrt{1-...x^2}}$ to score M1.

A1 $\quad$ A correct expression for $\dfrac{dy}{dx}$ as a function of x which may be unsimplified.

eg $\left(\dfrac{dy}{dx} =\right) \dfrac{1}{2\sqrt{1-\dfrac{x^2}{4}}}$ or $\dfrac{1}{\sqrt{\dfrac{4}{4}-\dfrac{x^2}{4}}}$ or $\dfrac{1}{2\sqrt{\dfrac{4-x^2}{4}}}$

A1cso $\quad \left(\dfrac{dy}{dx} =\right) \dfrac{1}{\sqrt{4-x^2}}$

Alt (b)

M1 $\quad y = \arcsin\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{dy}{dx} = k \times \dfrac{1}{\sqrt{1-\left(\dfrac{x}{2}\right)^2}}$ (where k can equal 1) then A1A1 as above

SC $\quad y = \arcsin\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{4-x^2}}$ with no correct intermediate working score as 100.

(c)

M1 Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ at *P*. See scheme but allow sub of $y=\frac{\pi}{4}$ into their $\frac{\mathrm{d}x}{\mathrm{d}y}=\pm k\cos y$

dM1 Full method to find the equation of the tangent at *P*. Their $\frac{\mathrm{d}y}{\mathrm{d}x}$ may not be exact. If they use $y=mx+c$ they must proceed as far as $c=...$

A1 Correct equation and in the correct form. The coefficient of x and the constant must be exact. Isw

Question Number	Scheme	Marks
4		
(ii)	$x=\ln(\sin y)\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{1}{\sin y}\times\cos y$	B1
	Attempts $\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\mathrm{e}^{2x}}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\sin y}{\cos y}$	M1
	Hence $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\sin y}{\cos y}=\dfrac{\mathrm{e}^x}{\sqrt{1-\mathrm{e}^{2x}}}$	dM1 A1
		(4)
		(9 marks)

(ii)

B1 $\dfrac{dx}{dy} = \dfrac{1}{\sin y} \times \cos y$ OR $e^x = \cos y \dfrac{dy}{dx}$ via $e^x = \sin y$

M1 For one of the two operations needed to complete the proof

- Either an attempt to get $\cos y$ in terms of e^x look for an attempt using $\sin^2 y + \cos^2 y = 1$ with $\sin y$ being replaced by e^x. Alternatively allow use of arcsin
- Or taking the reciprocal and making $\dfrac{dy}{dx}$ the subject (variables must be consistent)

dM1 Applies both operations to obtain $\dfrac{dy}{dx}$ in terms of just e^x

A1 $\dfrac{dy}{dx} = \dfrac{e^x}{\sqrt{1-e^{2x}}}$ or $\dfrac{e^x}{\cos\left(\arcsin e^x\right)}$ Allow $\dfrac{1}{\sqrt{1-\left(e^x\right)^2}}$ or states $f(x) = \sqrt{1-e^{2x}}$ following a correct expression for $\dfrac{dy}{dx} = \dfrac{e^x}{\cos y}$ or similar.

Alt:

B1 $x = \ln(\sin y) \Rightarrow y = \arcsin\left(e^x\right)$

M1 $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-\left(e^x\right)^2}} \times \ldots$ or $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-(\ldots)^2}} \times e^x$

dM1 both of these

A1 $\dfrac{e^x}{\sqrt{1-\left(e^x\right)^2}}$

Question Number	Scheme	Marks
5(i)	$x=\tan^2 4y \Rightarrow \dfrac{dx}{dy}=8\tan 4y\sec^2 4y$ oe	M1A1
	$\dfrac{dy}{dx}=\dfrac{1}{8\tan 4y\sec^2 4y}, =\dfrac{1}{8\tan 4y(1+\tan^2 4y)}=\dfrac{1}{8\sqrt{x}(1+x)}=\dfrac{1}{8\left(x^{0.5}+x^{1.5}\right)}$	M1,M1A1
		(5)

(i)

M1 Differentiates $\tan^2 4y$ to get an expression equivalent to the form $C\tan 4y\sec^2 4y$

You may see $\tan 4y\times A\sec^2 4y+\tan 4y\times B\sec^2 4y$ from the product rule or versions appearing from $\sqrt{x}=\tan 4y \Rightarrow Ax^{-0.5}\times ...=B\sec^2 4y$ or

$Ax^{-0.5}=B\sec^2 4y\times ...$ $x=\dfrac{\sin^2 4y}{\cos^2 4y}\Rightarrow \dfrac{dx}{dy}=\dfrac{\cos^2 4y\times A\sin 4y\cos 4y-\sin^2 4y\times B\cos 4y\sin 4y}{\left(\cos^2 4y\right)^2}$

from the quotient rule

A1 Any fully correct answer, or equivalent, including the left hand side. $\dfrac{dx}{dy}=2\tan 4y\times 4\sec^2 4y$

Also accept the equivalent by implicit differentiation $1=8\tan 4y\sec^2 4y\dfrac{dy}{dx}$

M1 Uses $\dfrac{dy}{dx}=1\Big/\dfrac{dx}{dy}$ Follow through on their $\dfrac{dx}{dy}$.

Condone issues with reciprocating the '8' but not the trigonometrical terms.

If implicit differentiation is used it is scored for writing $\dfrac{dy}{dx}$ as the subject.

M1 Uses $\sec^2 4y=1+\tan^2 4y$ where $x=\tan^2 4y$ to get their expression for $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ in terms of just x.

If they use other functions it is for using $\sin^2 4y=\dfrac{x}{1+x}$ and $\cos^2 4y=\dfrac{1}{1+x}$ where $x=\tan^2 4y$ to get their expression for $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ in terms of just x.

A1 Correct answer and solution. Accept $\dfrac{1}{8\left(x^{0.5}+x^{1.5}\right)}, \dfrac{1}{8\left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)}$ or A=8, p =0.5and q =1.5

Candidates do not have to explicitly state the values of A, p and q. Remember to isw after the sight of an acceptable answer.

Alt (i) using $y = \frac{1}{4}\arctan\left(\sqrt{x}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4}\left(\frac{1}{1+\left(\sqrt{x}\right)^2}\right) \times \frac{1}{2}x^{-\frac{1}{2}}$

M1 Changes the subject of the formula to get $y = D\arctan\left(\sqrt{x}\right)$ and proceeds to $\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{1}{1+\left(\sqrt{x}\right)^2}\right) \times \ldots$

A1 Achieves $y = \frac{1}{4}\arctan\left(\sqrt{x}\right)$ and proceeds to $\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{1}{1+\left(\sqrt{x}\right)^2}\right) \times \ldots$

M1 Correctly proceeds to $\frac{\mathrm{d}y}{\mathrm{d}x} = E\left(\frac{1}{1+\left(\sqrt{x}\right)^2}\right) \times x^{-\frac{1}{2}}$

M1 Writes $x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$ and multiplies out bracket to get $\frac{\mathrm{d}y}{\mathrm{d}x} = E\left(\frac{1}{x^{\frac{1}{2}} + x^{\frac{3}{2}}}\right)$

A1 Correct answer and solution. Accept $\frac{1}{8\left(x^{0.5} + x^{1.5}\right)}, \frac{1}{8\left(x^{\frac{1}{2}} + x^{\frac{3}{2}}\right)}$

Question Number	Scheme	Marks
6(a)	$\frac{dx}{dy}=12\sec^2 2y\tan 2y$	M1, A1
		(2)
(b)	$\frac{dx}{dy}=12\left(\frac{x}{3}\right)\sqrt{\sec^2 2y-1}\Rightarrow\frac{dx}{dy}=12\left(\frac{x}{3}\right)\sqrt{\frac{x}{3}-1}$	M1, A1ft
	$\frac{dy}{dx}=\frac{\sqrt{3}}{4x\sqrt{x-3}}$	A1
		(3)
(c)	$y=\frac{\pi}{12}\Rightarrow x=4$	B1
	$\frac{dx}{dy}=12\times\frac{4}{3}\times\frac{1}{\sqrt{3}}$ or $\frac{dy}{dx}=\frac{1}{12\left(\frac{4}{3}\right)\sqrt{\frac{4}{3}-1}}$	M1
	Correct $m_N=-\frac{16}{\sqrt{3}}$ o.e	A1
	$y-\frac{\pi}{12}=-\frac{16}{\sqrt{3}}(x-4)$	dM1
	$y=-\frac{16\sqrt{3}}{3}x+\frac{64\sqrt{3}}{3}+\frac{\pi}{12}$	A1
		(5)
		Total 10

(a)
M1: Differentiates to a form on the rhs of $\alpha\sec^2 2y\tan 2y$ which may be written $...\sec 2y\times...\sec 2y\tan 2y$

Note that the same scheme can also be applied to students who adapt $x=3\sec^2 2y$ to $x=\pm3\tan^2 2y\pm3$

A1: $\frac{dx}{dy}=12\sec^2 2y\tan 2y$. If the lhs is included it must be correct. So $\frac{dy}{dx}=12\sec^2 2y\tan 2y$ is M1 A0

Condone this to be unsimplified $\frac{dx}{dy}=6\sec 2y\times 2\sec 2y\tan 2y$ ISW after sight of correct answer

(b)
M1: For an attempt to

- replace $\sec^2 2y$ with αx
- use the identity $\pm1\pm\tan^2 2y=\pm\sec^2 2y$ and replaces $\tan 2y=b\sqrt{\pm1\pm d\,x}$

to obtain an expression for $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of x only.

The expression in part (a) must have had $\frac{dx}{dy}$ as a function of both $\sec 2y$ and $\tan 2y$ o.e.

A1ft: Requires a substitution of both $\sec^2 2y$ with $\frac{x}{3}$ and $\tan 2y=\sqrt{\frac{x}{3}-1}$ to obtain a correct expression for $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of x. Follow through on their $\frac{dx}{dy}=\alpha\sec^2 2y\tan 2y$

For a correct $\frac{dx}{dy}$ it is awarded for $\frac{dy}{dx} = \frac{1}{4x\sqrt{\frac{1}{3}x - 1}}$

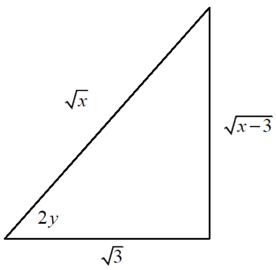
A1: Correct answer in the required form.

Allow equivalents e.g. $\frac{3\sqrt{3}}{12x\sqrt{x-3}}$. Form required is $\frac{p}{qx\sqrt{x-3}}$ where p is irrational and q is an integer

Alt method for (a) and (b) which can be marked in a similar way

(a) M1 A1: $x = 3(\cos 2y)^{-2} \Rightarrow \frac{dx}{dy} = 12(\cos 2y)^{-3}\sin 2y$

(b) If $x = 3(\cos 2y)^{-2} \Rightarrow \cos 2y = \frac{\sqrt{3}}{\sqrt{x}}$



Score in a similar way to the main scheme

M1 A1: $\frac{dx}{dy} = \frac{12\sin 2y}{(\cos 2y)^3} = \frac{12\sqrt{\frac{x-3}{x}}}{\left(\sqrt{\frac{3}{x}}\right)^3}$

Alt (b) via arccos

$$y = \frac{1}{2}\arccos\sqrt{\frac{3}{x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\frac{1}{\sqrt{1-\left(\sqrt{\frac{3}{x}}\right)^2}} \times \frac{\sqrt{3}}{2}x^{-\frac{3}{2}}$$

M1: For $\frac{dy}{dx} = \lambda\frac{1}{\sqrt{1-\left(\sqrt{\frac{\alpha}{x}}\right)^2}} \times -x^{-\frac{3}{2}}$ A1: Correct and unsimplied A1: Correct and in the required form

(c)

B1: Correct value for x

M1: Attempts to find the value of $\frac{dx}{dy}$ using their part (a) with $y = \frac{\pi}{12}$

the value of $\frac{dy}{dx}$ from an inverted $\frac{dx}{dy}$ using their part (a) with $y = \frac{\pi}{12}$

or the value of $\frac{dy}{dx}$ using their part (b) with their value of x found using $y = \frac{\pi}{12}$.

These may be called m or f ' and not identified as $\frac{dx}{dy}$ or $\frac{dy}{dx}$

A1: Correct normal gradient

dM1: Attempt at the equation of normal at $y = \frac{\pi}{12}$.

The gradient should be either an attempt at the value of $-\frac{dx}{dy}$ at $y = \frac{\pi}{12}$ for their $\frac{dx}{dy}$

or an attempt at the negative reciprocal of their $\frac{dy}{dx}$ at their "4" which must have been found from $y = \frac{\pi}{12}$

A1: Fully correct equation in the required form. ISW after a correct answer

Allow equivalent exact forms e.g. $y=-\frac{16}{\sqrt{3}}x+\frac{64}{\sqrt{3}}+\frac{\pi}{12}$, $y=-\frac{16}{\sqrt{3}}x+\frac{256\sqrt{3}+\pi}{12}$

Question Number	Scheme	Marks
7(a)	$x = 3\tan\left(y - \frac{\pi}{6}\right) \Rightarrow \frac{dx}{dy} = 3\sec^2\left(y - \frac{\pi}{6}\right)$	B1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{3\sec^2\left(y - \frac{\pi}{6}\right)}$	M1
	$\frac{1}{3\left(1 + \tan^2\left(y - \frac{\pi}{6}\right)\right)} = \frac{1}{3\left(1 + \left(\frac{x}{3}\right)^2\right)}$	dM1
	$= \frac{3}{x^2 + 9}$	A1
		(4)
(b)	$y = \frac{\pi}{3} \Rightarrow x = 3\tan\frac{\pi}{6} = \sqrt{3}$	B1
	$x = \sqrt{3} \Rightarrow \frac{dy}{dx} = \frac{3}{\left(\sqrt{3}\right)^2 + 9}$ or $y = \frac{\pi}{3} \Rightarrow \frac{dy}{dx} = \frac{1}{3\sec^2\left(\frac{\pi}{3} - \frac{\pi}{6}\right)} = \ldots$	M1
	$y - \frac{\pi}{3} = \frac{1}{4}\left(x - \sqrt{3}\right)$	dM1
	$y = 0 \Rightarrow 0 - \frac{\pi}{3} = \frac{1}{4}\left(x - \sqrt{3}\right) \Rightarrow x = \ldots$	ddM1
	$x = \sqrt{3} - \frac{4\pi}{3}$	A1
		(5)
		Total 9

(a)

B1: Correct derivative, including correct lhs. Condone $\frac{dx}{dy} = k\sec^2\left(y - \frac{\pi}{6}\right)$ where k is a constant.

M1: Either (1) attempts to apply the reciprocal rule $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Don't be too concerned by the position of the constant, it is the function and variable that is important.

Or (2) attempts to apply $\sec^2\left(y - \frac{\pi}{6}\right) = 1 + \tan^2\left(y - \frac{\pi}{6}\right)$ with $\tan\left(y - \frac{\pi}{6}\right)$ being replaced by $\frac{x}{3}$ to get $\frac{dx}{dy}$ in terms of x

dM1: Attempts both (1) and (2) to obtain $\frac{dy}{dx}$ in terms of x

A1: Correct answer

(b)

B1: Correct value for x. Condone awrt 1.73. This can be awarded from the sight of this value in an equation

M1: Uses a correct method to find the value of $\frac{dy}{dx}$ which may be a decimal.

Note that this can be scored either from answer to part (a) e.g $x = "\sqrt{3}" \Rightarrow \dfrac{dy}{dx} = "\dfrac{3}{\left(\sqrt{3}\right)^2 + 9}"$

Or via their $\dfrac{dx}{dy} = 3\sec^2\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right)$ followed by an attempt at the reciprocal

dM1: Correct straight line method for the tangent at $\left("\sqrt{3}", \dfrac{\pi}{3}\right)$ using their correctly found *m*.

It is dependent upon having found the gradient and *x* value using a correct method.
If they use the form $y = mx + c$ they must proceed as far as $c = ...$
ddM1: Uses $y = 0$ to find *x*. It is dependent upon having scored the previous mark

A1: Correct value or exact equivalent for example $\dfrac{3\sqrt{3} - 4\pi}{3}$

Alt (a) via arctan

B1: $x = 3\tan\left(y - \dfrac{\pi}{6}\right) \Rightarrow y = \dfrac{\pi}{6} + \arctan\left(\dfrac{x}{3}\right) \Rightarrow \dfrac{dy}{dx} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{...}\right)^2} \times ...$ where ... could be any values even 1

M1: $\dfrac{dy}{dx} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{3}\right)^2} \times ...$ where ... could be any value even 1

dM1: $\dfrac{dy}{dx} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{3}\right)^2} \times \dfrac{1}{3}$

A1*: $\dfrac{dy}{dx} = \dfrac{3}{x^2 + 9}$

Alt (a) via compound angle identity they could pick up the first two marks

$$x = \frac{3\left(\tan y - \tan\frac{\pi}{6}\right)}{1 + \tan\frac{\pi}{6}\tan y} \Rightarrow \frac{dx}{dy} = \frac{\left(1 + \tan\frac{\pi}{6}\tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan\frac{\pi}{6}\right) \times \tan\frac{\pi}{6}\sec^2 y}{\left(1 + \tan\frac{\pi}{6}\tan y\right)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(1 + \tan\frac{\pi}{6}\tan y\right)^2}{\left(1 + \tan\frac{\pi}{6}\tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan\frac{\pi}{6}\right) \times \tan\frac{\pi}{6}\sec^2 y}$$

Question Number	Scheme	Marks
8	$x=6\sin^2 2y \Rightarrow \frac{dx}{dy}=24\sin 2y\cos 2y$	M1 A1
	Attempts to use $\frac{dy}{dx}=1\div\frac{dx}{dy}$	M1
	Attempts to use both $\sin 2y=\sqrt{\frac{x}{6}}$ and $\cos 2y=\sqrt{1-\sin^2 2y}=\sqrt{1-\frac{x}{6}}$	M1
	$\frac{dy}{dx}=\frac{1}{24\sin 2y\cos 2y}=\frac{1}{24\times\sqrt{\frac{x}{6}}\times\sqrt{1-\frac{x}{6}}}=\frac{1}{4\sqrt{6x-x^2}}$	A1
		(5)
Way 2	$x=6\sin^2 2y=3-3\cos 4y\Rightarrow\frac{dx}{dy}=12\sin 4y$	M1A1
	Attempts to use $\frac{dy}{dx}=1\div\frac{dx}{dy}$	M1
	Attempts to use $\sin 4y=\frac{\sqrt{6x-x^2}}{3}$	M1
	$\frac{dy}{dx}=\frac{1}{12\sin 4y}=\frac{1}{12\times\frac{\sqrt{6x-x^2}}{3}}=\frac{1}{4\sqrt{6x-x^2}}$	A1
Way 3	$x=6\sin^2 2y=24\sin^2 y\cos^2 y\Rightarrow\frac{dx}{dy}=48\sin y\cos^3 y-48\sin^3 y\cos y$	M1A1
	Attempts to use $\frac{dy}{dx}=1\div\frac{dx}{dy}$	M1
	$\frac{dy}{dx}=\frac{1}{48\sin y\cos^3 y-48\sin^3 y\cos y}=\frac{1}{48\sin y\cos y\left(\cos^2 y-\sin^2 y\right)}$ $\frac{dy}{dx}=\frac{1}{24\sin 2y\cos 2y}=\frac{1}{24\times\sqrt{\frac{x}{6}}\times\sqrt{1-\frac{x}{6}}}$	M1
	$=\frac{1}{4\sqrt{6x-x^2}}$	A1
		(5 marks)

In general, apply the following marking guidance for this question:

M1: Attempts to differentiate to obtain $\frac{dx}{dy}$ in a correct form

A1: Correct derivative

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

M1: Change fully to a function of x.

A1: All correct

E.g.

M1: Attempts to use the chain rule on the rhs to achieve $k\sin 2y \cos 2y$

A1: Fully correct derivative $\frac{dx}{dy} = 24\sin 2y\cos 2y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy} = 24\sin 2y\cos 2y \Rightarrow \frac{dy}{dx} = \frac{24}{\sin 2y\cos 2y}$

Do not condone slips/errors on the variable. E.g. $\frac{dx}{dy} = 24\sin 2y\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{24\sin 2x\cos 2x}$

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x.

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1-qx}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

Way 2

M1: Differentiates to obtain $k\sin 4y$

A1: Fully correct derivative $\frac{dx}{dy} = 12\sin 4y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy} = 12\sin 4y \Rightarrow \frac{dy}{dx} = \frac{12}{\sin 4y}$

M1: Attempts to change both $\sin 4y$ to a function in x.

Expect to see $\sin 4y = p\sqrt{6x-x^2}$ or equivalent e.g. $\sin 4y = p\sqrt{9-\left(9-6x+x^2\right)}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

Way 3

M1: Differentiates to obtain $A\sin y\cos^3 y + B\sin^3 y\cos y$

A1: Fully correct derivative $\frac{\mathrm{d}x}{\mathrm{d}y} = 48\sin y\cos^3 y - 48\sin^3 y\cos y$

M1: Attempts to use $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \frac{\mathrm{d}x}{\mathrm{d}y}$.

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x.

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1-qx}$

A1: CSO $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4\sqrt{6x - x^2}}$

A possible alternative:

$$x = 6\sin^2 2y \Rightarrow \sin 2y = \sqrt{\frac{x}{6}} \Rightarrow 2y = \sin^{-1}\sqrt{\frac{x}{6}} \Rightarrow y = \frac{1}{2}\sin^{-1}\sqrt{\frac{x}{6}}$$

M1: For a correct attempt to make y or $2y$ the subject

A1: Correct rearrangement

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times \frac{1}{\sqrt{1-\frac{x}{6}}} \times \frac{1}{12}\left(\frac{x}{6}\right)^{-\frac{1}{2}}$$

M1: For $\frac{A}{\sqrt{1-\frac{x}{6}}} \times \ldots$ M1: For $\ldots \times B\left(\frac{x}{6}\right)^{-\frac{1}{2}}$

A1: CSO $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4\sqrt{6x - x^2}}$

Question Number	Scheme	Marks
9 (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow\left(\dfrac{dx}{dy}=\right)\dfrac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{6y^2-12y-18}{9(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\dfrac{dx}{dy}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0\Rightarrow 2(y-3)(y+1)=0\Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\dfrac{2y^2+6}{3y-3}\Rightarrow x=..$	dM1
	Achieves $x=-\dfrac{4}{3}$ and $x=4$	A1cso
		(4) **8 marks**

Notes

(a)

M1 Attempts the quotient rule. Condone slips on the coefficients - look for $\dfrac{Ay(3y-3)-B(2y^2+6)}{(3y-3)^2}$ $A,B>0$. Allow a product rule attempt:
$x=(2y^2+6)(3y-3)^{-1}\Rightarrow\left(\dfrac{dx}{dy}=\right)Ay(3y-3)^{-1}+(2y^2+6)\times -B(3y-3)^{-2}$

A1 Correct differentiation which may be unsimplified. Allow if the $\dfrac{dx}{dy}$ is missing or called $\dfrac{dy}{dx}$ for this mark. By product rule $4y(3y-3)^{-1}+(2y^2+6)\times -3(3y-3)^{-2}$ Condone missing brackets if recovered.

dM1 Requires an attempt to get a single fraction with some attempt to simplify.
For the quotient rule look for a simplification of the numerator with like terms collected giving a 3TQ.
Attempts via the product rule will require a correct method to put as a single fraction.

A1 $\left(\dfrac{dx}{dy}=\right)\dfrac{2y^2-4y-6}{3y^2-6y+3}$ or exact simplified equivalent such as $\dfrac{2(y-3)(y+1)}{3(y-1)^2}$ isw after a correct simplified answer. Common factor 3 must have been cancelled. Must be seen in part (a). A0 if called $\dfrac{dy}{dx}$ but allow A1 if LHS is not stated.

Attempts at $\dfrac{dy}{dx}$ can score the first 3 marks if correct. Allow use of x in place of y for the Ms.

(b)

B1 Indicates P and Q are where $\dfrac{dx}{dy}=0$ or where their $2y^2-4y-6=0$ (which may be the denominator of $\dfrac{dy}{dx}$ if they found this instead).

M1 Solves their 3TQ from an attempt at $\dfrac{dx}{dy}=0$ (or denominator of their $\dfrac{dy}{dx}$ =0), usual rules.

dM1 Substitutes both their solutions to $2y^2-4y-6=0$ into $x=\dfrac{2y^2+6}{3y-3}$. Condone slips if the attempt is clear. At least one should be correct if no method is shown.

A1cso Achieves $x=-\dfrac{4}{3}$ and $x=4$ only. Must be equations not just values but isw after correct equations seen as long as no contrary work is shown (such as giving horizontal lines). Accept equivalents. Must have come from a correct derivative - though allow from an isw form if a numerical factor was lost in the numerator. Must be exact.

Answers from no working score 0/4 as the question instructs use of part (a), so must see the attempt at setting $\dfrac{dx}{dy}=0$

Alt (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow 3xy-3x=2y^2+6\Rightarrow 3x+3y\dfrac{dx}{dy}-3\dfrac{dx}{dy}=4y$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{4y-3x}{3(y-1)}$	dM1, A1
		(4)
(b) First 2 marks.	States that P and Q are where $\dfrac{dx}{dy}=0$ or where $4y-3x=0$	B1
	$\Rightarrow \dfrac{4}{3}y=\dfrac{2y^2+6}{3y-3}\Rightarrow 4y^2-4y=2y^2+6\Rightarrow$ as main scheme	M1
Alt II (a)	$x=\dfrac{2y^2+6}{3y-3}=\dfrac{2y}{3}+\dfrac{2}{3}+\dfrac{8}{3(y-1)}\Rightarrow\dfrac{dx}{dy}=\dfrac{2}{3}-\dfrac{8}{3(y-1)^2}$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{2(y-1)^2-8}{3(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ oe	dM1, A1
		(4)

Notes

(a)

M1 Attempts long division or other method to achieve $Ay+B+\dfrac{C}{3y-3}$ oe and differentiates.

A1 Correct differentiation.

dM1 Attempts to get a single fraction and simplifies numerator to 3TQ or uses difference of squares to factorise.

A1 Correct answer.

Question Number	Scheme	Marks
10 (a)	$(k=)\ 4\sin^2\left(\frac{\pi}{3}\right)-1=2$ *	B1*
		(1)
(b)	(i) $x=4\sin^2 y-1 \Rightarrow \frac{dx}{dy}=8\sin y\cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y=\frac{\pm x\pm 1}{4}$ or $\cos^2 y=\pm 1\pm\frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx}=\frac{1}{8\sin y\cos y}=\frac{1}{8\times\sqrt{\frac{x+1}{4}}\times\sqrt{\pm 1\pm\frac{x+1}{4}}}=\frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x=2$, gradient of curve $=\frac{1}{2\sqrt{3}}\Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of $\cancel{y}-\frac{\pi}{3}=-2\sqrt{3}(x-2)\Rightarrow x=2+\frac{\pi}{6\sqrt{3}}$ $(=2.3...)$	dM1
	Area $=\frac{1}{2}\times\left(2+\frac{\pi}{6\sqrt{3}}\right)\times\frac{\pi}{3}=\frac{\pi}{3}+\frac{\pi^2}{36\sqrt{3}}$	A1
		(3) **(10 marks)**

(a)

B1*: Verifies that $k=2$ with no errors seen. (Condone $x=2$) Do not allow this mark if rounded numbers are seen in their solution for e.g. $\frac{\pi}{3}$. They may work in degrees and substitute in $60°$ which is acceptable. Look for as a minimum e.g. $(k=)\ 4\sin^2\frac{\pi}{3}-1=2$ Do not allow $k=4\sin\left(\frac{\pi}{3}\right)^2-1=2$

If they rearrange the equation to e.g. $y=\sin^{-1}\left(\sqrt{\frac{x+1}{4}}\right)$ and substitute in $x=2$ proceeding to e.g. $\frac{\pi}{3}$ then they must conclude that $k=2$ (or a preamble followed by a minimal conclusion e.g. QED, tick)

(b) (i)

M1: Differentiates to a form $p\sin y\cos y$ or equivalent. Indices do not need to be processed for this mark. e.g. $...\sin^{2-1} y\cos y$

May use the identity $\cos 2y=\pm 1\pm 2\sin^2 y\Rightarrow\ x=4\sin^2 y-1=1-2\cos 2y\Rightarrow\frac{dx}{dy}=q\sin 2y$

A1: Correct lhs and rhs. For $\frac{dx}{dy}=8\sin y\cos y$ or $4\sin 2y$. Note that $\frac{dy}{dx}=...$ is A0. isw for this mark after a correct answer is seen. You may see $\frac{dx}{dy}$ (or allow x') on an earlier line which is fine for this mark. May be seen at the start of (b)(ii). Indices must be processed for this mark.

(b)(ii) **On EPEN this is M1A1dM1A1* we are marking this as M1dM1ddM1A1***
Note this is a given answer so full working must be seen.

M1: Attempts to find either $\sin y$ (or $\sin^2 y$) **or** $\cos y$ (or $\cos^2 y$) in terms of x (or possibly multiples of these) Typically look for $\sin^2 y = \frac{\pm x \pm 1}{4}$ or $\sin y = \sqrt{\frac{\pm x \pm 1}{4}}$ Also allow e.g. $4\sin^2 y = \pm x \pm 1$

May attempt to use $\pm\sin^2 y \pm \cos^2 y = \pm 1 \Rightarrow \cos^2 y = \pm 1 \pm \sin^2 y \Rightarrow \cos^2 y = \pm 1 \pm \frac{x \pm 1}{4}$ or

$\cos y = \sqrt{\pm 1 \pm \frac{x \pm 1}{4}}$ Ignore $\pm\sqrt{\ldots}$ for this mark. (may be seen within their expression for $\frac{dy}{dx}$ or $\frac{dx}{dy}$)

dM1: Attempts to find both $\sin y$ (or $\sin^2 y$) **and** $\cos y$ (or $\cos^2 y$) or possibly multiples of these in terms of x. (See previous method mark for guidance) It is dependent on the previous method mark.

ddM1: Full method to get $\frac{dy}{dx}$ of the form $\frac{1}{p \sin y \cos y}$ o.e. in terms of x e.g. $\frac{dy}{dx} = \frac{1}{"8"\sqrt{\frac{\pm x \pm 1}{4}}\sqrt{\pm 1 \pm \frac{\pm x \pm 1}{4}}}$

Condone square root notation which does not fully go over any fraction provided it is implied by further work which is not the given answer. Condone missing arguments for sin and cos provided the intention is clear or implied. Condone missing brackets to be recovered **before** achieving the given answer. If they have rearranged or simplified incorrectly for $\cos y$ before substituting, ddM1 can still be scored.

A1*: Achieves the given answer **provided all previous method marks have been scored in (b) and no errors.** Look for

- a correct expression for $\cos^2 y$ or $\cos y$ in x which has not already been simplified to $\ldots\sqrt{3-x}$
- a correct expression for $\sin^2 y$ or $\sin y$ in x which has not already been simplified to $\ldots\sqrt{x+1}$

$\frac{dy}{dx}$ (or y') may appear on an earlier line which is acceptable. Note that if they work backwards by substituting $x = 4\sin^2 y$ into the given answer there must be a conclusion that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ isw

(c)

M1: Full method for the gradient of the normal using the x or y value. Condone arithmetical slips but the gradient must be the negative reciprocal of the gradient of the tangent to the curve.

Note that they may use an earlier incorrect line in part (b) for $\frac{dy}{dx}$ (or even $\frac{dx}{dy}$).

Allow use of an earlier line provided it is an attempt at a correct method substituting in x or y and proceeding from e.g. $\frac{dx}{dy} = \alpha \Rightarrow$ gradient of the normal $= -\alpha$

May be implied by awrt -3.5 If no method is shown where the gradient of the tangent or normal has come from, then it must be correct.

dM1: Attempts to find the x coordinate of N. It is dependent on the previous method mark. Uses their gradient of the normal with $\left(2, \frac{\pi}{3}\right)$, sets $y = 0$ in $y - y_1 = m(x - x_1)$ and proceeds to find x. May also use $y = mx + c$ to find the equation for the straight line and then sets $y = 0$ and rearranges to find x. Do not be concerned by the mechanics of the rearrangement for this mark.

A1: $\frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$ or exact equivalent such as $\frac{\pi}{3} + \frac{\pi^2\sqrt{3}}{108}$ which is in the form $a\pi + b\pi^2$

Question Number	Scheme	Marks
11(a)	$\frac{1}{4}=\sin^2 4y \Rightarrow y=\frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin 4y\cos 4y$	M1A1
		(2)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin 4y\cos 4y \rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x=\frac{1}{2}$	B1ft
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4}$	B1ft
		(2)
		(9 marks)

Notes

(a)

M1: Attempts to substitute in $x=\frac{1}{4}$ and rearranges to find an exact value for y. Look for $y=M\arcsin\left(\pm\frac{1}{2}\right)\rightarrow k\pi$ or $k(30°)$ allowing errors in dividing by the 4, or equivalents criteria via a double angle identity, correct up to sign error.

A1: $\frac{\pi}{24}$ Ignore extra solutions outside the domain.

(b)

M1: Attempts to differentiate achieving a form $\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)A\sin 4y\cos 4y$ or $A\sin 8y$ oe Accept alternative forms e.g. via implicit differentiation $1=A\sin 4y\cos 4y\frac{\mathrm{d}y}{\mathrm{d}x}$. The $\frac{\mathrm{d}x}{\mathrm{d}y}$ may be missing, or labelled $\frac{\mathrm{d}y}{\mathrm{d}x}$ for this mark.

A1: $\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin 4y\cos 4y$ oe eg $\frac{\mathrm{d}x}{\mathrm{d}y}=4\sin 8y$ Coefficients must be simplified. Must include the $\frac{\mathrm{d}x}{\mathrm{d}y}$

(c)

M1: Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=1\div\frac{\mathrm{d}x}{\mathrm{d}y}$ (allow for the reciprocal of their answer to (b) if left hand side is omitted). Variables must be consistent.

M1: Attempts to use $\sin 4y = \pm\sqrt{x}$ and $\cos 4y = \pm\sqrt{1-x}$ to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x only with no trig terms.

A1: $\frac{dy}{dx} = \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$ and isw after correct answer. Allow with terms under the square root reversed.

(d) Note to score marks in (d) they must have a derivative which does have a minimum and is of a completed square form (the *r* may be inside the bracket) or in some other way clearly uses their (c) (e.g. minimising their quadratic).

(i) B1ft: $(x=)\frac{1}{2}$ ft their $-s$ provided their $q>0$, their $r<0$ and their x is in the range $0 \leqslant x \leqslant 1$

(ii) B1ft: $\left(\frac{dy}{dx}=\right)\frac{1}{4}$ ft their q provided it is positive and their r was negative.

(b) Alt	$x = \sin^2 4y \Rightarrow y = \frac{1}{4}\arcsin\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{4}\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1
	$\frac{dx}{dy} = 8\sqrt{x}\sqrt{1-x}$	A1
		(2)
(c)	$\frac{dx}{dy} = 8\sin 4y\cos 4y \rightarrow \frac{dy}{dx} = \frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{dy}{dx} = \frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx} = \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)

Notes

Alt by finding *y* first. Mark (b) and (c) together via such approaches. But note that part (c) says "Hence" and the reciprocal law must have been used at some stage to score full marks.

(b)

M1: Makes y the subject and differentiates to reach form $\frac{dy}{dx} = A\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}} \times \ldots$ where … is a function of x

A1: For $\frac{dx}{dy} = 8\sqrt{x}\sqrt{1-x}$ oe with square roots combined. Coefficients must have been gathered.

(c)

M1: For use of the reciprocal rule of derivatives evidenced in the working – allow for it being used in (b) to find $\frac{dx}{dy}$ from $\frac{dy}{dx}$.

M1: Award for the correct procedure of having made y the subject and differentiating to reach the correct form for the answer, ie $\frac{dy}{dx} = A\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}} \times Bx^{-\frac{1}{2}}$

A1: As main scheme.

Question Number	Scheme	Marks
12 (a)	$x = y\mathrm{e}^{2y} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$	M1, A1
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{x}{y} + 2x$	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\frac{x}{y} + 2x}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x + 2xy} = \dfrac{y}{x(1+2y)}$ *	A1*
		(4)
(b)	Deduces $y = -\dfrac{1}{2}$	B1
	Substitutes $y = -\dfrac{1}{2} \Rightarrow x = -\dfrac{1}{2\mathrm{e}}$ Range for k $-\dfrac{1}{2\mathrm{e}} < k < 0$	M1, A1
		(3)
		(7 marks)

(a)

M1: For attempting to differentiate with respect to y.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \mathrm{e}^{2y} + ...y\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

A1: Correct differentiation E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$

dM1: Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of just x and y.

This requires, in any order

- a correct attempt to invert, e.g. not inverting each term in a sum of terms
- e^{2y} being fully replaced. You should see e^{2y} being replaced by $\dfrac{x}{y}$ or equivalent and $y\mathrm{e}^{2y}$ being replaced by x

A1*: Correct proof. All relevant steps should be shown and there should be no errors.

If you feel that it hasn't been fully shown then please award M1 A1 dM1 A0

(b) Now being marked B1 M1 A1. On epen it is set up M1 A1 A1

B1: For deducing left hand end occurs when $y = -\dfrac{1}{2}$.

M1: For attempting to find x when $y = -\dfrac{1}{2} \Rightarrow x = ...$ This may be implied by k = awrt -0.183 or -0.184

A1: $-\dfrac{1}{2\mathrm{e}} < k < 0$ or **exact** equivalent such as e.g. $-\dfrac{1}{2}\mathrm{e}^{-1} < k < 0$, $k > -\dfrac{1}{2}\mathrm{e}^{-1}$ **and** $k < 0$, $\left\{k : k > -\dfrac{1}{2\mathrm{e}}\right\} \cap \{k : k < 0\}$

This must be correct so the candidate cannot have two separate inequalities with an "**or**" between

This must be the range for k not x

....

Alt via ln's

$x = y\mathrm{e}^{2y} \Rightarrow \ln x = \ln y + 2y$ o.e.

M1: Attempts to differentiate wrt x . Consider just rhs $\ln y + 2y \to \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + ...\frac{\mathrm{d}y}{\mathrm{d}x}$

Alternatively attempts to differentiate wrt y . Consider both sides $...\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{y} + 2$

A1: Correct differentiation $\frac{1}{x} = \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + 2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{y} + 2$

....

Alt via differentiating wrt x

$$x = y\mathrm{e}^{2y} \Rightarrow \quad 1 = \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + y \times 2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$1 = \frac{x}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$1 = \left(\frac{x + 2xy}{y}\right)\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x(1+2y)} \quad *$$

M1: For attempting to differentiate wrt x.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \quad \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + ...y\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

Uses the quotient rule on $y = \frac{x}{\mathrm{e}^{2y}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{2y} - 2x\mathrm{e}^{2y} \times \frac{\mathrm{d}y}{\mathrm{d}x}}{\mathrm{e}^{4y}}$ condoning slips on the coefficient

A1: Correct differentiation including the lhs

dM1: Full method to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.