London Academy of Maths

# R Addition Formulae

Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (a) | $R=\sqrt{4+16}=\sqrt{20}$ *or* $2\sqrt{5}$ | B1 |
| | $\tan\alpha=\frac{4}{2}$ | M1 |
| | $\Rightarrow \alpha=1.11$ (awrt) | A1 |
| | | **(3)** |
| (b) | Maximum is 12+2*R* or minimum is 12– 2*R* | M1 |
| | maximum = 20.9 (hours) (20h 57m) and minimum = 3.06 (hours) (3 hours 3 m) | A1 A1 |
| | | **(3)** |
| (c) | $17=12+k\,"R"\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)$ | M1 |
| | $\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)=....$ | dM1 |
| | For proceeding to one value for *t* from $17=12+2"R"\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)$ | M1 |
| | *t* = 99 (days) **or** 212 or 213 (days) | A1 |
| | For finding two values for t | dM1 |
| | *t* = 99 (days) **and** 212 or 213 (days) | A1 |
| | | **(6)** |
| | | **(12 marks)** |

(a)

**B1**: $R=\sqrt{20}$ or $2\sqrt{5}$ no working needed. Condone $R=\pm\sqrt{20}$ oe

**M1**: $\tan\alpha=\pm\frac{4}{2}$ or $\tan\alpha=\pm\frac{2}{4}$ and attempts to find alpha. If *R* is used accept $\sin\alpha=\pm\frac{4}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

**A1**: accept $\alpha=$ awrt 1.11 ; also accept $\sqrt{20}\sin(x-1.11)$ . Answers in degrees are A0

**(b)**

**M1:** Uses Maximum is 12+2*R* or minimum is 12– 2*R* with their value of *R*

**A1:** maximum value or minimum value correct allowing exact value(s) $12\pm2\sqrt{20}$ or $12\pm4\sqrt{5}$

**A1:** maximum **and** minimum value **awrt** 20.9 (20h 57m) 3.06 (3 hours 3 m)
Ignore any units in this part.

**Note:** It is possible to do this by differentiation. To score M1 you would need to see

Differentiation to $\lambda\cos\left(\frac{2\pi t}{365}-'\alpha'\right)=0\Rightarrow\frac{2\pi t}{365}-'\alpha'=\frac{\pi}{2}$ or $\frac{3\pi}{2}\Rightarrow t=...$ and then substitute into *H* and find a value.

(c)

**M1:** For an attempt to interpret the model and writing it in terms of (a), condoning slips

Allow for $17 = 12 + k\,"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$, even $k = 1$ with their value for $R$ and $\alpha$ (Slip on "2")

Allow $17 = 12 + k\,"R"\sin\left(x \pm "\alpha"\right)$ even $k = 1$ with their value for $R$ and $\alpha$ ($x$ instead of $\frac{2\pi t}{365}$ )

**dM1**: For attempting to make $\sin\left(x \pm \text{their } \alpha\right)$ or $\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$ the subject.

**M1**: For the method of finding at least one value for $t$, $0 < t < 365$, from a "**correct**" starting point with $2 \times$ their $R$.

$17 = 12 + 2\,"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) \to \sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) = C$ to $t$ = ... by undoing the operations in the correct order

A good intermediate value to check (for correct $R$) is $\frac{2\pi t}{365} \pm "\alpha" = 0.593....$

Condone slips on the $\frac{2\pi}{365}$ for all M marks. Example you may see $\frac{2\pi}{36}$

**A1:** For one correct value for $t$, either awrt 99 **or** awrt 212/213.

**dM1:** For attempting to find a second value for $t$.

It is dependent upon the previous M mark and it is usually for moving from

$\left(\frac{2\pi t}{365} \pm "\alpha"\right) = \pi - \beta$ (where $\beta$ was the principal value) to $t$ = ...

by undoing the operations in the correct order

A good intermediate value to check (for correct $R$) is $\frac{2\pi t}{365} \pm "\alpha" = 2.548....$

**A1**: awrt 99 **and** awrt 212 or 213 only $0 < t < 365$. Remember to ISW

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 2(a) | $\frac{8000}{56+9+0}=\frac{8000}{65}=\frac{1600}{13}$ | Allow any equivalent fraction or awrt 123m | B1 |
| | | | **(1)** |
| (b) | $9\cos t+40\sin t=R\cos(t-\alpha)$ | | |
| | $R=\sqrt{9^2+40^2}=41$ | 41 only | B1 |
| | $\alpha=\arctan\left(\pm\frac{40}{9}\right)=\ldots$ or $\alpha=\arctan\left(\pm\frac{9}{40}\right)=\ldots$ <br> or <br> $\alpha=\arcsin\left(\pm\frac{40}{"41"}\right)=\ldots$ or $\alpha=\arccos\left(\pm\frac{9}{"41"}\right)=\ldots$ | | M1 |
| | $\alpha=77.3$ | Awrt 77.3 | A1 |
| | | | **(3)** |
| (c)(i) | $\frac{8000}{56+'R'}=\ldots$ m | Attempts $\frac{8000}{56+'R'}$ | M1 |
| | $=\frac{8000}{97}$ | $\frac{8000}{97}$ or awrt 82.5 | A1 |
| (ii) | $t=77.3$ | Awrt 77.3 or follow through their $\alpha$ (ignore what they do in (c)(i)) | B1ft |
| | | | **(3)** |
| (d) | $150=\frac{8000}{56+41\cos(t-77.3)}\Rightarrow\cos(t-77.3)=-0.065$ <br> Uses their part (b) with $H$ = 150 and reaches $\cos(t\pm77.3)=k$ with $-1<k<0$ | | M1 |
| | $\cos(t\pm"77.3")=-\frac{8}{123}$ or awrt $-0.065$ (Follow through their 77.3) | | A1ft |
| | $\cos(t\pm77.3)=-\frac{8}{123}\Rightarrow t\pm77.3=\arccos\left(-\frac{8}{123}\right)\Rightarrow t=\ldots$ <br> Takes arccos and then ±"77.3" and uses the **<u>obtuse</u>** angle leading to a value for $t$ <br> **Dependent on the first M so requires** $-1<k<0$ | | **d**M1 |
| | $(t=)171$ | Awrt 171 **and no other values** | A1 |
| | | | **(4)** |
| | | | **[11 marks]** |

**Note that the use of radians for an otherwise correct solution would normally lose the A mark in (b) and the final A mark in (d). (Values are (a) 1.349 and (d) 2.98)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3** | $3\sin x - \cos x$ | | |
| **(a)** | $R = \sqrt{10}$ | Must be exact. Condone $R = \pm\sqrt{10}$ | B1 |
| | $\tan\alpha = \frac{1}{3} \Rightarrow \alpha = ...$ | Condone $\tan\alpha = \pm\frac{1}{3}$ or $\tan\alpha = \pm\frac{3}{1}$ $\sin\alpha = \pm\frac{1}{"\sqrt{10}"}$ or $\sin\alpha = \pm\frac{3}{"\sqrt{10}"}$ or $\cos\alpha = \pm\frac{1}{"\sqrt{10}"}$ or $\cos\alpha = \pm\frac{3}{"\sqrt{10}"}$ $\Rightarrow \alpha = ...$ Implied by 0.32 or 18.4° | M1 |
| | $\alpha = 0.322$ | Awrt 0.322 following a correct statement | A1 |
| | | | **(3)** |
| **(b)(i)** | $19 - \sqrt{10}$ | $19 - "\sqrt{10}"$ or awrt 15.8 (ft on their $R$) | B1ft |
| **(ii)** | $\frac{\pi t}{12} + 4 - "0.322" = \frac{3\pi}{2} \Rightarrow t = ...$ | Condone $\frac{\pi t}{12} + 4 \pm \text{their } \alpha = \frac{3\pi}{2} \Rightarrow t = ...$ Don't be too concerned by the mechanics of their attempt to solve the equation. Note that 15.95 is evidence that $\frac{5\pi}{2}$ has been selected and scores M0 | M1 |
| | $t = 3.95$ | Awrt 3.95 Condone 3hrs 57 mins or 3:57 am If multiple answers are given (and not rejected) withhold the final A1 | A1 |
| | | | **(3)** |
| | | | **Total 6** |

Extra Note 1:
Although highly unlikely, it is possible to do (b)(ii) in degrees. In such an attempt the 4 would also need to be changed to degrees.

M1: $\frac{\pi t}{12} + 4 - "0.322" = \frac{3\pi}{2} \Leftrightarrow \frac{180t}{12} + 4 \times \frac{180}{\pi} - "18.4" = 270$

Extra Note 2:
You may see attempts that rely on differentiation. This is essentially the same and would require candidates selecting the second zero

M1: $A\cos\left(\frac{\pi t}{12} + 4 \pm "0.322"\right) = 0 \Rightarrow \frac{\pi t}{12} + 4 \pm "0.322" = \frac{3\pi}{2} \Rightarrow t = ...$

Extra Note 3: Answers without working can score all marks (even though for (b) they were asked to use part (a))

| Question Number | Scheme | Marks |
|---|---|---|
| **4(a)** | $(R = \sqrt{1.5^2 + 1.2^2}) =$ awrt 1.921 - accept e.g. $\sqrt{3.69}$ or $\dfrac{3\sqrt{41}}{10}$ | B1 |
| | $\tan\alpha = \dfrac{1.2}{1.5} \Rightarrow \alpha = 0.675$ or $0.215\pi$ | M1A1 |
| | | **(3)** |
| **(b)** | $H = 3 + 1.921\sin\left(\dfrac{\pi t}{6} - 0.675\right)$ | |
| | $H_{\min} = 3 - '1.921' =$ awrt 1.08 | M1A1 |
| | $\left(\dfrac{\pi t}{6} - "0.675"\right) = \dfrac{3\pi}{2} \Rightarrow t = 10.29$ | M1A1 |
| | | **(4)** |
| **(c)** | $4 = 3 + 1.921\sin\left(\dfrac{\pi t}{6} - 0.675\right) \Rightarrow \sin\left(\dfrac{\pi t}{6} - 0.675\right) = \dfrac{1}{1.921}$ | M1 |
| | $\dfrac{\pi t}{6} - 0.675 = 0.548 \Rightarrow t =$ awrt 2.33 or 2.34 | dM1A1 |
| | $\dfrac{\pi t}{6} - 0.675 = \pi - 0.548 = 2.594 \Rightarrow t =$ awrt 6.24 or 6.25 | ddM1A1 |
| | Times are 2:20pm and 6:15pm or 6.14pm (14:20 and 18:15 or 18:14) – allow 2 hours 20minutes and 6 hours 15 or 14minutes or 140 minutes and 375 or 374 minutes | A1 |
| | Extra values in the range – lose final A mark. | **(6)** |
| | | **(13 marks)** |

## Notes for Question 4

(a) B1 $R =$ awrt $1.921$ (3dp) – allow any equivalent e.g. $\sqrt{3.69}$ or $\frac{3\sqrt{41}}{10}$

M1 $\tan\alpha = \pm\frac{1.2}{1.5}$ or $\tan\alpha = \pm\frac{1.5}{1.2}$

A1 $\alpha =$ awrt 0.675 (3dp) also allow $0.215\pi$ (must be in radians)

(b) M1 States or attempts to calculate $3 - R$ with their value of $R$

A1 $H_{\min} =$ awrt $1.08$. Or $3 - \frac{3\sqrt{41}}{10}$ o.e. Accept this for both marks as long as no incorrect working is seen.

M1 Attempts $\left(\frac{\pi t}{6} - '\alpha'\right) = \frac{3\pi}{2} \Rightarrow t = ...$

( Putting equal to $-\frac{\pi}{2}$ is M1A0 (outside range) Putting equal to $\frac{\pi}{2}$ is M0A0 (wrong) )

(Allow method mark for using -90 or 270 degrees (not 90 degrees), if alpha was in degrees earlier)

A1 $t =$ awrt $10.29$ (2dp). Accept this for both marks as long as no incorrect working is seen.

(c) M1 $\sin\left(\frac{\pi t}{6} \pm '\alpha'\right) = \frac{4-3}{R}$, where $\left|\frac{4-3}{R}\right| < 1$ (allow for degrees)

dM1 Dependent upon the previous M, using the correct order to find **one value** of $t$ (allow consistent degrees)

NB: sin (1/1.921) = 0.497 Seeing 0.497 is indication that sin instead of arcsin has been used. This indicates the wrong method and so M0.

A1 Accept either awrt 2.33 or 2.34 **or** awrt 6.24 or 6.25 do not need units – ignore wrong units e.g. minutes and seconds for this mark. 2 hours 20 minutes is correct here. Note that 2 minutes 20 seconds could get this mark but would lose the final A1.

ddM1 Dependent upon the previous M, using the correct order to find a **second value** of $t$

A1 Accept awrt 2.33 or 2.34 **and** awrt 6.24 or 6.25- ignore wrong units. So 6 minutes 14 seconds could get this mark but would lose the next.

A1 Times are 2:20pm and 6:15pm (or 6.14pm) (14:20 and 18:15 (or 18.14)) (Need both times) – allow 2 hours 20minutes and 6 hours 15 or 14minutes or 140 minutes and 375 or 374 minutes. Extra values in the range – lose final A mark

Allow method marks for degrees, and accuracy marks if they converted to

$\sin(30t \pm '38.65') = \frac{4-3}{R}$, where $\left|\frac{4-3}{R}\right| < 1$ and continued to correct answers. Using

$\sin\left(\frac{\pi t}{6} \pm '38.65'\right) = \frac{4-3}{R}$, where $\left|\frac{4-3}{R}\right| < 1$ will lose the accuracy marks.

| Question Number | Scheme | Marks |
|---|---|---|
| **5.(a)** | $R=\sqrt{17}$ | B1 |
| | $\tan\alpha = 4 \Rightarrow \alpha = \text{awrt } 1.326$ | M1A1 |
| | | **(3)** |
| **(b)** | Minimum height $= \dfrac{24}{3+"R"} = 3.37$ (metres) | M1 A1 |
| | | **(2)** |
| **(c)** | Uses part (a) $10 = \dfrac{24}{3+\sqrt{17}\cos\left(\frac{1}{2}t-1.326\right)} \Rightarrow \cos\left(\frac{1}{2}t-1.326\right) = \dfrac{-0.6}{\sqrt{17}}$ | M1 A1 |
| | $t = \text{awrt } 6.09$ | M1 A1 |
| | | **(4)** |
| | | **(9 marks)** |

(a)

B1 $R=\sqrt{17}$

Condone $R=\pm\sqrt{17}$ (Do not allow decimals for this mark Eg 4.12 but remember to isw after $\sqrt{17}$ )

M1 $\tan\alpha = \pm 4,\ \tan\alpha = \pm\frac{1}{4} \Rightarrow \alpha = ...$

If $R$ is used to find α accept $\sin\alpha = \pm\frac{4}{R}$ or $\cos\alpha = \pm\frac{1}{R} \Rightarrow \alpha = ...$

A1 $\alpha = \text{awrt } 1.326$ Note that the degree equivalent $\alpha = \text{awrt } 75.96°$ is A0

(b)

M1 Attempts minimum height by stating or finding $\dfrac{24}{3+"R"}$

Attempts via differentiation must be complete methods with correct work up to slips in coefficients. They are unlikely to succeed.

FYI $$\frac{dH}{dt} = -\frac{24\left(2\cos\left(\frac{t}{2}\right)-\frac{1}{2}\sin\left(\frac{t}{2}\right)\right)}{\left(4\sin\left(\frac{t}{2}\right)+\cos\left(\frac{t}{2}\right)+3\right)^2} = 0 \Rightarrow \tan\left(\frac{t}{2}\right) = 4 \Rightarrow H = \frac{24}{3+\cos(1.326)+4\sin(1.326)}$$

A1 3.37 (metres) Assume metres unless otherwise stated, but 3.37 cm is A0. Accept 337 cm as long as the units are stated, but do not accept $\dfrac{24}{3+\sqrt{17}}$ and do not isw if incorrect units are given following 3.37.

(c)

M1 Attempts to use their answer to part (a) (including their $R$ and their $\alpha$ ) AND proceeds to

$\cos(\beta t \pm "1.326") = k, \quad -1<k<1$ and $\beta = 1$ or $\frac{1}{2}$

A1 $\cos(\beta t \pm "1.326") = \dfrac{-0.6}{\sqrt{17}}$ or awrt $-0.146$ where $\beta = 1$ or $\frac{1}{2}$

M1 Full method to make $t$ the subject from an equation of the form $\cos\left(\frac{1}{2}t \pm "1.326"\right) = k, \quad -1<k<1$

Look for $2\times(\text{their } \arccos(k) \pm \text{their } \alpha)$

A1 awrt $t = 6.09$ (Ignore any extra solutions outside the domain, but A0 if extras inside are given.)

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="5">5(a)</td><td colspan="2">$5\cos\theta - 3\sin\theta = R\cos(\theta+\alpha)$</td><td rowspan="2">B1</td></tr>
<tr><td>$R=\sqrt{5^2+3^2}=\sqrt{34}$</td><td>$R=\sqrt{34}$ $\left(R=\pm\sqrt{34}\text{ is B0}\right)$</td></tr>
<tr><td colspan="2">$\tan\alpha=\pm\frac{3}{5},\ \tan\alpha=\pm\frac{5}{3}\ \Rightarrow\ \alpha=...$<br>(Also allow $\cos\alpha=\pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}$, $\sin\alpha=\pm\frac{3}{\sqrt{34}}$ or $\pm\frac{5}{\sqrt{34}}$ $\Rightarrow\ \alpha=...$, where "√34" is their $R$.)</td><td>M1</td></tr>
<tr><td>$\alpha=\arctan\left(\frac{3}{5}\right)=\text{awrt }0.5404$</td><td>Anything that rounds to 0.5404<br>(Degrees is 30.96… and scores A0)</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
</table>

| | | | |
|---|---|---|---|
| **6(b)** | $6+2.5\cos\left(\frac{4\pi t}{25}\right)-1.5\sin\left(\frac{4\pi t}{25}\right)=4.6 \Rightarrow \frac{\sqrt{34}}{2}\cos\left(\frac{4\pi t}{25}+0.5404\right)=4.6-6$<br>$\Rightarrow \cos\left(\frac{4\pi t}{25}+"0.5404"\right)=...$<br>Uses part (a) and proceeds as far as<br>$\cos\left(\frac{4\pi t}{25}\pm \text{their } 0.5404\right)=k$ or $\cos(\theta \pm \text{their } 0.5404)=k$ or<br>$\cos(t\pm \text{their } 0.5404)=k$ where $\|k\|<1$. | | M1 |
| | $\cos\left(\frac{4\pi t}{25}+"0.5404"\right)=-0.48$ | Allow:<br>$\cos\left(\frac{4\pi t}{25}\pm \text{their } 0.5404\right)=\text{awrt } -0.48$<br>**or** $\cos(\theta \pm \text{their } 0.5404)=\text{awrt } -0.48$<br>**or** $\cos(t\pm \text{their } 0.5404)=\text{awrt } -0.48$<br>May see $-\frac{7\sqrt{34}}{85}$ or $-\frac{2.8}{\sqrt{34}}$ for $-0.48$ | A1 |
| | $\frac{4\pi t}{25}+"0.5404"=2.07 \Rightarrow t=...$ **or**<br>$\frac{4\pi t}{25}+"0.5404"=2\pi-2.07=4.21 \Rightarrow t=...$<br>NB 2.07… may be seen as π – 1.07 and 4.21… may be seen as π + 1.07<br>$\cos\left(\frac{4\pi t}{25}\pm \text{their } 0.5404\right)=k \Rightarrow t=..$ by first taking invcos then adds or subtracts their 0.5404 and applies $\frac{4\pi t}{25}$ to obtain a value for *t*.<br>**Dependent on the previous method mark and may be implied by obtaining a value for *t* of awrt 3 or awrt 7.** | | **d**M1 |
| | awrt 3.05 or awrt 7.3 | Allow awrt 3.05 **or** awrt 7.3 | A1 |
| | $\frac{4\pi t}{25}\pm"0.5404"=2\pi-2.07 \Rightarrow t=...$ **and** $\frac{4\pi t}{25}\pm"0.5404"=2.07 \Rightarrow t=...$<br>For a correct method to find a different value of *t* in the range<br>**Dependent on both previous method marks.** | | **dd**M1 |
| | 3:03 or 15:03 or 3hrs 3min or 183 minutes<br>**and**<br>7:18 or 19:18 or 7hrs 18min or 438 minutes | | A1 |
| | | | **(6)** |
| | | | **[9 marks]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **7**(a) | $\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ | |
| | $R = \sqrt{1^2 + 4^2} = \sqrt{17}$ | B1 |
| | $\alpha = \arctan 4 = \text{awrt } 1.326$ | M1A1 |
| | | **(3)** |
| (b) | $\sqrt{17}\cos(2\theta - 1.326) = 1.2 \Rightarrow \cos(2\theta - 1.326) = \dfrac{1.2}{\sqrt{17}}$ | M1 |
| | $\Rightarrow (2\theta - 1.326) = \pm 1.275.. \Rightarrow \theta = ...$ | dM1 |
| | $\theta = \text{awrt } 1.30 \text{ or awrt } 0.03$ | A1 |
| | $2\theta - 1.326 = \text{'}1.275...\text{'}..\ \textbf{and}\ \text{'}-1.275...\text{'}$ | ddM1 |
| | $\Rightarrow \theta = \text{awrt } 1.30\ \textbf{and}\ 0.03$ | A1 |
| | | **(5)** |
| | | **[8 marks]** |

(a)

B1 For $R = \sqrt{17}$ . Condone $R = \pm\sqrt{17}$

M1 For $\alpha = \arctan(\pm 4)$ or $\alpha = \arctan\left(\pm\dfrac{1}{4}\right)$ leading to a solution of $\alpha$

It is implied by $\alpha = \text{awrt } 76°$ or awrt $1.3$ rads

Condone any solutions coming from $\cos\alpha = 1, \sin\alpha = 4$

If $R$ has been used to find $\alpha$ award for only $\alpha = \arccos\left(\pm\dfrac{1}{\text{'}R\text{'}}\right)$ $\alpha = \arcsin\left(\pm\dfrac{4}{\text{'}R\text{'}}\right)$

A1 $\alpha = \text{awrt } 1.326$

(b)

M1 Using part (a) and proceeding as far as $\cos(2\theta \pm \text{their}\,1.326) = \frac{1.2}{\text{their}\,R}$.

Condone slips on the 1.2 and miscopying their 1.326

This may be implied by $(2\theta \pm \text{their}\,1.326) = \arccos\left(\frac{1.2}{\text{their}\,R}\right)$

Condone for this mark $\cos(\theta \pm \text{their}\,1.326) = \frac{1.2}{\text{their}\,R}$ or $\cos(2\theta \pm 2 \times \text{their}\,1.326) = \frac{1.2}{\text{their}\,R}$

but $2\cos(\theta \pm \text{their}\,1.326) = \frac{1.2}{\text{their}\,R}$ is M0 and hence dM0…etc

dM1 Dependent upon the first M1. It is for a full method to find one value of $\theta$ within the range 0 to $\pi$ from their principal value. Look for the correct order of operations, that is dealing with the "1.326" before the "2". Condone adding 1.326 instead of subtracting.

$$\cos(2\theta \pm \text{their}\,1.326) = ... \Rightarrow 2\theta \pm \text{their}\,1.326 = \beta \Rightarrow \theta = \frac{\beta \pm \text{their}\,1.326}{2}$$

A1 awrt $\theta = 1.30$ or $\theta =$ awrt 0.03 Only allow 1.3 if it is preceded by an answer that rounds to 1.30

ddM1 For a correct method to find a second value of $\theta$ (**for their** $\alpha$) in the range 0 to $\pi$.
Eg $2\theta \pm 1.326 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 1.326 = 2\pi + '\beta' \Rightarrow \theta =$ THEN MINUS $\pi$

A1 awrt $\theta = 1.30$ and $\theta =$ awrt 0.03. Only allow 1.3 if it is preceded by an answer that rounds to 1.30
Withhold this mark if there are extra solutions **in the range**.

Degree solution: Only lose the first time it occurs.

FYI. In degrees only lose the first A mark awrt (a) $\alpha = 75.964°$ and (b) $\theta_1 = 74.52°, \theta_2 = 1.44°$
Mixing degrees and radians only scores the first M in part (b)

Answers without working.
If $\sqrt{17}\cos(2\theta - 1.326) = 1.2$ is written down then all marks are available. (3 marks for one correct answer)
If there is no initial statement then score SC B1 B1 then 0,0, 0 for a maximum of 2, 1 for each solution.

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8(a)** | $R=\sqrt{53}$ | cao | B1 |
| | $\tan\alpha=\dfrac{2}{7}\Rightarrow\alpha=...$<br>$\tan\alpha=\pm\dfrac{2}{7}$ or $\tan\alpha=\pm\dfrac{7}{2}$ or<br>$\sin\alpha=\pm\dfrac{2}{'\sqrt{53}'}$ or $\sin\alpha=\pm\dfrac{7}{'\sqrt{53}'}$ or $\cos\alpha=\pm\dfrac{7}{'\sqrt{53}'}$ or $\cos\alpha=\pm\dfrac{2}{'\sqrt{53}'}$<br>$\Rightarrow\alpha=...$<br>Uses one of these equations to find a value for $\alpha$ | | M1 |
| | $\alpha=15.95^{\circ}$ | Awrt $15.95^{\circ}$ (Allow awrt 0.28 (rad)) | A1 |
| | | | **(3)** |
| **(b)** | $\sqrt{53}\sin(2\theta-15.95^{\circ})=4\Rightarrow\sin(2\theta-15.95^{\circ})=\dfrac{4}{\sqrt{53}}(0.549)$<br>Attempts to use part (a) $"\sqrt{53}"\sin(2\theta-"15.95^{\circ}")=4$ and proceeds to $\sin(2\theta\pm"15.95^{\circ}")=K,\quad \lvert K\rvert<1$<br>Allow the letter α for "15.95" | | M1 |
| | $2\theta-15.95^{\circ}=33.3287...\Rightarrow\theta=24.6^{\circ}$ | Awrt $24.6^{\circ}$ (Allow awrt 0.43 (rad)) | A1 |
| | $2\theta-15.95^{\circ}=180^{\circ}-33.287...\Rightarrow\theta=...$<br>Correct attempt at a second solution in the range.<br>E.g. $2\theta_2\mp'15.95^{\circ}'=180^{\circ}-'33.3287...^{\circ}'\Rightarrow\theta_2=\dfrac{180^{\circ}-'33.3287...^{\circ}'\pm'15.95^{\circ}'}{2}$<br>(May be implied by their $\theta_2$)<br>It is **dependent** upon having scored the previous M.<br>**Do not allow mixing of radians and degrees so if working in radians must be using π not 180** | | dM1 |
| | $\theta=81.3^{\circ}$ | Awrt $81.3^{\circ}$ **only** | A1 |
| | **Ignore extra answers outside range but deduct the final A for extra answers in range.** | | |
| | | | **(4)** |
| **(c)** | $28\sin\theta\cos\theta=a\sin2\theta\Rightarrow a=14$ | $a=14$ | B1 |
| | $8\sin^2\theta=b\left(\pm1\pm2\sin^2\theta\right)+c$ or $8\sin^2\theta=8\left(\dfrac{1}{2}\left(\pm1\pm\cos2\theta\right)\right)$<br>or<br>$8\sin^2\theta=4\sin^2\theta+4\sin^2\theta=4\sin^2\theta+4\left(1-\cos^2\theta\right)=\pm4\cos2\theta\pm4$<br>Attempts to use a $\cos2\theta$ identity e.g. $\cos2\theta=\pm1\pm2\sin^2\theta$ or $\sin^2\theta=\dfrac{1}{2}\left(\pm1\pm\cos2\theta\right)$ at some point in their working and applies it to the given expression. | | M1 |
| | $b=-4,\ c=4$ or $...-4\cos2\theta+4$ | Correct values or correct expression | A1 |
| | | | **(3)** |
| **(d)** | $\left(28\sin\theta\cos\theta+8\sin^2\theta\right)_{\max}=2'\sqrt{53}'+'4'$ | Maximum $=2\times$ their $\sqrt{53}+$ their $c$<br>May be implied e.g. by their decimal answer. | M1 |
| | $2\sqrt{53}+4$ | Cao (must be exact not decimals) | A1 |
| | Attempts to use calculus for the maximum should reach $2R+c$ as above for M1 . | | |
| | | | **(2)** |
| | | | **Total 12** |

| Qu | Scheme | Marks |
|---|---|---|
| **9.** (a) | $R = 37$ | B1 |
| | $\tan\alpha = \frac{12}{35} \Rightarrow \alpha = \text{awrt } 0.3303$ | M1 A1 |
| | | **(3)** |
| (b) | $\sin(x-\alpha) = \frac{37}{2R} \quad (= 0.5...)$ | M1 |
| | $x = \arcsin\left(\frac{37}{2\times \text{their "37"}}\right) + \text{their"0.3303"}$ | M1 |
| | $x = \text{awrt } 0.854$ or awrt $2.95$ | A1 |
| | $x = \text{awrt } 0.854$ and awrt $2.95$ | A1 |
| | | **(4)** |
| (c)(i) | Find $y = \frac{7000}{31+(\pm\text{"}R\text{"})^2} = 5$ | M1 A1 |
| (c)(ii) | $x-\alpha = \frac{\pi}{2} \Rightarrow x = 1.90$ | M1 A1 |
| | | **(4)** |
| | | **(11 marks)** |

(a)

**B1**: $R = 37$ no working needed. Condone $R = \pm 37$

**M1**: $\tan\alpha = \pm\frac{12}{35}$ or $\tan\alpha = \pm\frac{35}{12}$ with an attempt to find alpha. Accept decimal attempts from $\tan\alpha = \text{awrt } \pm 0.343$ or $\tan\alpha = \text{awrt } \pm 2.92$ If $R$ is used allow $\sin\alpha = \pm\frac{12}{R}$ OR $\cos\alpha = \pm\frac{35}{R}$ with an attempt to find alpha

**A1**: $\alpha = \text{awrt } 0.3303$. Answers in degrees are A0

(b)

**M1**: (Uses part (a) to solve equation) $\sin(x \pm \alpha) = \frac{37}{2\times theirR}$

**M1**: operations undone in the correct order to give $x = ...$ Accept $\sin(x\pm\alpha) = k \Rightarrow x = \arcsin k \pm \alpha$

**A1**: one correct answer to within required accuracy. Allow $0.272\pi$ or $0.938\pi$.

Condone for this mark only **both** $\frac{\pi}{6} + 0.3303$ **and** $\frac{5\pi}{6} + 0.3303$

**A1:** both values (and no extra values in the range) correct to within required accuracy. Allow $0.272\pi, 0.938\pi$

(c) (i)

**M1**: For an attempt at $\frac{7000}{31+(\pm\text{"}R\text{"})^2}$

**A1:** 5

(c)(ii)

**M1:** Uses $x - \text{their } \alpha = (2n+1)\frac{\pi}{2}$ to find $x$ This may be implied by $1.57 \pm \text{their } 0.33$ stated or calculated (2dp)

**A1**: Awrt 1.90 but condone 1.9 for this answer

Answers in degrees, withhold the first time seen, usually part (a). FYI (a) $18.92°$ (b) $48.9°, 168.9°$ (c)(ii) $108.9°$

| Question Number | Scheme | Marks |
|---|---|---|
| **10(a)** | $R = \sqrt{5}$ | B1 |
| | $\tan\alpha = \frac{2}{1} \Rightarrow \alpha = ...$ | M1 |
| | $\alpha = 1.107$ | A1 |
| | | **(3)** |
| **(b)(i)** | $\text{Max} = 3 + 7\sqrt{5}$ | B1ft |
| **(b)(ii)** | $(2x - "1.107") = \pi \Rightarrow x = ...$ | M1 |
| | $\Rightarrow x = \frac{\pi + "1.107"}{2} = 2.12$ | A1 |
| | | **(3)** |
| | | **Total 6** |

**(a)**
**B1:** Correct exact value (Condone $R = \pm\sqrt{5}$).
isw after a correct answer. e.g. $R = \sqrt{5} = 2.24$
**M1:** Allow for: $\tan\alpha = \pm\frac{2}{1}$, $\tan\alpha = \pm\frac{1}{2}$, $\cos\alpha = \pm\frac{1}{"R"}$, $\sin\alpha = \pm\frac{2}{"R"}$ leading to a value for $\alpha$
If no method is shown imply by the sight of awrt 1.1 rads or awrt $63°$
**A1:** awrt 1.107
**(b)(i)**
**B1ft:** Award for 3 + 7×their $R$ where $R$ > 0.
Also follow through on decimal answers from (a) e.g.
Condone solutions such as $3 - 7\times -\sqrt{5} = 18.65..$ .
**(b)(ii)**
**M1:** For an attempt to solve $(2x \pm "1.107") = \pi$ or $(2x \pm "1.107") = -\pi$ . May be implied by awrt 2.12
Condone a bracketing slip $\cos(2(x \pm "1.107")) = -1 \Rightarrow 2(x \pm "1.107") = \pi \Rightarrow x = ...$ (condone $-\pi$ as above)
Also condone an attempt to solve $(2x \pm "63°") = 180°$ but not mixed units, e.g. $(2x \pm "1.107") = 180°$
**A1:** For awrt 2.12. This cannot be given in a list, the 2.12 must be selected.

| Question Number | Scheme | Marks |
|---|---|---|
| **11.** (a) | $R = \sqrt{34}$ | B1 |
| | $\tan\alpha = \frac{5}{3}$ | M1 |
| | $\Rightarrow \alpha = 1.03$ awrt 1.03 | A1 |
| | | [3] |
| (b) | $3\sin 2x + 5\cos 2x = 4 \Rightarrow \sqrt{34}\sin(2x+1.03) = 4$ | |
| | $\sin(2x+"1.03") = \frac{4}{"\sqrt{34}"} \quad (= 0.68599...)$ | M1 |
| | One solution in range Eg. $2x+"1.03" = 2\pi + \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x = ...$ | M1 |
| | Either $x =$ awrt 3.0 **or** awrt 0.68 | A1 |
| | Second solution in range Eg $2x+"1.03" = \pi - \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x = ...$ | M1 |
| | Both $x =$ awrt 2sf 3.0 **and** 0.68 | A1 |
| | | [5] |
| (c) | Greatest value is $4\left(\sqrt{34}\right)^2 + 3 \quad = 139$ | M1 A1 |
| | Least value is $4(0)+3=3$ . | M1 A1 |
| | | [4] |
| | | **(12 marks)** |

(a)

B1 $R=\sqrt{34}$ Condone $\pm\sqrt{34}$

M1 For $\tan\alpha=\pm\frac{5}{3}$ or $\tan\alpha=\pm\frac{3}{5}$ This may be implied by awrt 1.0 rads or awrt 59 degrees

If $R$ is used to find α only accept $\cos\alpha=\pm\frac{3}{their\ R}$ or $\sin\alpha=\pm\frac{5}{their\ R}$

A1 accept $\alpha=$ awrt 1.03; also accept $\sqrt{34}\sin(2x+1.03)$.

If the question is done in degrees only the first accuracy mark is withheld. The answer in degrees (59.04) is A0

(b) On epen this is marked up M1M1M1A1A1. We are scoring it M1M1A1M1A1

M1 For reaching $\sin(2x\pm\text{their }\alpha)=\frac{4}{\text{their }R}$ (Uses part (a) to solve equation)

It may be implied by $(2x\pm\text{their }\alpha)=\arcsin\left(\frac{4}{\text{their }R}\right)=0.75$ rads

M1 For an attempt at one solution in the range. It is acceptable to find the negative solution, -0.14 and add $\pi$

Look for $2x\pm\text{their }\alpha=2\pi+\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$ (correct order of operations)

Alternatively $2x\pm\text{their }\alpha=\pi-\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$

A1 Awrt 3.0 **or** awrt 0.68. Condone 3 for 3.0. In degrees accept awrt 38.8 or 172.1

M1 For an attempt at a second solution in the range. This can be scored from their "$\arcsin\left(\frac{4}{\text{their }R}\right)$"

Look for $2x\pm\text{their }\alpha=\pi-\text{their }\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$ (correct order of operations)

Or $2x\pm\text{their }\alpha=2\pi+\text{their }\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$

A1 Awrt 3.0 **AND** awrt 0.68 in radians or awrt 38.8 and awrt 172.1 in degrees. Condone 3 for 3.0

(c) (i)

M1 Attempts to find $4(R)^2+3$

A1 139 cao

(c)(ii)

M1 Uses 0 for minimum value. Accept $4(0)^2+3$

A1 3

| Question Number | Scheme | Marks |
|---|---|---|
| **12(a)** | $R = \sqrt{109}$ | B1 |
| | $\tan\alpha = \dfrac{3}{10} \Rightarrow \alpha = \text{awrt}16.70°$ | M1A1 |
| | | **(3)** |
| **(b)(i)** | Max height = 12+ ‘ $\sqrt{109}$ ’= 22.44 m | M1A1 |
| **(ii)** | Occurs when $30t + '16.70' = 180 \Rightarrow t = 5.44$ | M1A1 |
| | | **(4)** |
| **(c)** | $18 = 12 - \sqrt{109}\cos(30t + 16.70) \Rightarrow \cos(30t + 16.70) = -\dfrac{6}{\sqrt{109}} \quad (-0.57..)$ | M1A1 |
| | $\Rightarrow 30t + 16.70 = \arccos\left(-\dfrac{6}{\sqrt{109}}\right) \Rightarrow t = ..$ | dM1 |
| | $t = \text{awrt } 3.61$ (2dp) | A1 |
| | | **(4)** |
| **(d)** | Attempting $30t = 360 \Rightarrow t = ..$ or $30t = 720 \Rightarrow t = ..$ | M1 |
| | 2 revolutions in 24 **minutes** | A1 |
| | | **(2)** |
| | | **(13 marks)** |

(a)

B1 Accept $R = \pm\sqrt{109}$ . Remember to isw after a correct answer. Eg $R = \sqrt{109} = 10.....$

M1 For $\tan\alpha = \pm\frac{3}{10}$ or $\tan\alpha = \pm\frac{10}{3}$. If $R$ is used to find $\alpha$, only accept $\cos\alpha = \pm\frac{10}{'R'}$ or $\sin\alpha = \pm\frac{3}{'R'}$

A1 $\alpha = \text{awrt}\,16.70°$ (2dp). Condone $\alpha = 16.7°$

Note that the answer of $\alpha = \text{awrt}\,0.29$ radians scores A0.

(b)(i)

M1 For 12 +their $R$

A1 Awrt 22.44 m. Accept 12+ $\sqrt{109}$

(b) (ii)

M1 For arriving at a solution for $t$ from $30t \pm '16.70' = 180 \Rightarrow t = ..$

If radians were used in part a then accept $30t \pm '0.29' = \pi \Rightarrow t = ..$

A1 $t = \text{awrt}\,5.44$ only.

If multiple solutions are found, 5.44 must be referred to as their 'chosen' solution

……………………………………………………………………………………………………

**Answers from calculus** will be rare. They can be scored as follows.

From the original function:

For (b)(ii) M1 $\frac{dH}{dt} = 0 \rightarrow 30t = 180 - '\arctan\left(\frac{3}{10}\right)' \Rightarrow t = ..$ A1 $t = \text{awrt}\,5.44$ only

(b)(i) M1 Sub their $t = \text{awrt}\,5.44$ obtained from $\frac{dH}{dt} = 0$ A1 Awrt 22.44 m

From the adapted function:

For (b)(ii) M1 $\frac{dH}{dt} = 0 \rightarrow (30t + '16.70') = 180 \Rightarrow t = ..$ A1 $t = \text{awrt}\,5.44$ only

(b)(i) M1 Sub their $t = \text{awrt}\,5.44$ obtained from $\frac{dH}{dt} = 0$ A1 Awrt 22.44 m

……………………………………………………………………………………………………

(c)

M1 Attempts to substitute $H = 18$ into $H = 12 - 10\cos 30t + 3\sin 30t$ and use their answer to part (a) to proceed to $\cos(30t \pm \mathit{their}\,'16.70') = ...$

A1 $\cos(30t + \mathit{their}\,'16.70') = -\frac{6}{\sqrt{109}}$ or awrt $-0.57$ . It may be implied by $30t + '16.70' = \mathit{awrt}\,125°$

dM1 Dependent upon previous M. Score for $\cos(30t \pm '16.70') = -... \Rightarrow t = ..$

The cos(..) must be negative, the order of operations must be seen to be correct with the 'invcos' being attempted first and the second quadrant must be chosen for their calculation.

A1 $t = 3.61$.

The answer with no incorrect working scores all 4 marks.

If multiple solutions are found, 3.61 must be referred to as their 'chosen' solution

(d)

M1 Attempting $30t = 360 \Rightarrow t = ..$ or $30t = 720 \Rightarrow t = ..$

A1 24 minutes. **Both** 24 **and** minutes are required.

| Question Number | Scheme | Notes | Mar |
|---|---|---|---|
| **13.** | $\sqrt{5}\cos q -\ 2\sin q\ ^{\circ}\ R\cos(q + a)$ | | |
| (a) | $R = 3$ | $R = 3$, **cao** ($\pm 3$ is B0) ($\sqrt{9}$ is B0) | B1 |
| | $\tan\alpha = \pm\frac{2}{\sqrt{5}}$, $\tan\alpha = \pm\frac{\sqrt{5}}{2} \Rightarrow \alpha = ...$<br>(Also allow $\cos\alpha = \pm\frac{\sqrt{5}}{3}$ or $\pm\frac{2}{3}$, $\sin\alpha = \pm\frac{2}{3}$ or $\pm\frac{\sqrt{5}}{3} \Rightarrow \alpha = ...$, where "3" is their $R$.) | | M1 |
| | $\alpha = 0.7297276562... \Rightarrow \alpha = 0.7297$ (4 sf) | Anything that rounds to 0.7297 (Degrees is 41.81 and scores A0) | A1 |
| | {**Note**: $\sqrt{5}\cos q -\ 2\sin q = 3\cos(q + 0.7297)$ } | | **[3]** |
| | | | |
| (b) | $\sqrt{5}\cos q -\ 2\sin q = 0.5$ | | |
| | $3\cos(\theta + 0.7297) = 0.5$<br>$\Rightarrow \cos(\theta + 0.7297) = \frac{0.5}{3}$ | Attempts to use part (a) $"3"\cos(\theta \pm "0.7297") = 0.5$<br>9and proceeds to $\cos(\theta \pm "0.7297") = K$, $\lvert K \rvert < 1$<br>May be implied by $\theta \pm "0.7297" = 1.4033$<br>or $\theta \pm "0.7297" = \cos^{-1}\left(\frac{0.5}{\text{their } 3}\right) (= 1.4033...)$ | M1 |
| | $\theta_1 = 0.673648... \Rightarrow \theta_1 = 0.674$ (3 sf) | Anything that rounds to 0.674 | A1 |
| | $\theta_2 + "0.7297" = "-1.4033" \Rightarrow \theta_2 = ...$ | **dependent on the previous M mark**<br>Correct attempt at a second solution in the range.<br>Usually given for:<br>$\theta_2$ + their 0.7297 = – their 1.4033 $\Rightarrow \theta_2 = ...$ | dM1 |
| | $\theta_2 = -2.133048... \Rightarrow \theta_2 = -2.13$ (3 sf) | Anything that rounds to – 2.13 | A1 |
| | For solutions in (b) that are otherwise fully correct, if there are extra answers in the range, deduct the final A mark. | | |
| | For candidates who work consistently in degrees in (a) and (b) allow awrt 38.6° and awrt – 122° in part (b) as the A mark will be lost in part (a) | | |
| | | | **[4]** |
| (c) | $f(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B, \theta \in \mathbb{R}; -15 \leqslant f(x) \leqslant 33$<br>$\Rightarrow -15 \leqslant 3A\cos(\theta + 0.730) + B \leqslant 33$ | | |
| | **Note that part (c) is now marked as B1M1A1A1** | | |
| | $B = 9$ | Correct value for $B$ | B1 |
| | $\begin{array}{l} 3A + B = 33 \\ -3A + B = -15 \end{array}$ or $\begin{array}{l} 3A + B = -15 \\ -3A + B = 33 \end{array}$ | Writes down at least one pair of simultaneous equations (or inequalities) of the form $\begin{array}{l} RA + B = 33 \\ -RA + B = -15 \end{array}$ or $\begin{array}{l} RA + B = -15 \\ -RA + B = 33 \end{array}$ and finds at least one value for $A$ | M1 |
| | $A = 8$ **or** $A = -8$ | One correct value for $A$ | A1 |
| | $A = 8$ **and** $A = -8$ | Both values correct | A1 |
| | | | **[4]** |
| | | | **11** |

| | | | |
|---|---|---|---|
| (c) **Alt 1** | $B = 9$ | Correct value for $B$ | B1 |
| | $(2)(A)(3) = 33 - -15$ | $(2)(A)(\text{their } R) = 33 - -15 \Rightarrow A = ...$ | M1 |
| | $A = 8$ **or** $A = -8$ | One correct value for $A$ | A1 |
| | $A = 8$ **and** $A = -8$ | Both values correct | A1 |
| | | | **[4]** |
| (c) **Alt 2** | $B = \dfrac{33-15}{2} = 9$ | Correct value for $B$ | B1 |
| | $3A = 33 - 9 \Rightarrow A = 8$ | $(\text{their } R)A = 33 - \text{their } B \Rightarrow A = ...$ | M1 |
| | $A = 8$ **or** $A = -8$ | One correct value for $A$ | A1 |
| | $A = 8$ **and** $A = -8$ | Both values correct | A1 |
| | | | **[4]** |

| | | |
|---|---|---|
| | **Question 6 Notes** | |
| (c) | **Note** | The M mark may be implied by correct answers so obtaining A = 8 implies M1A1 |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **14.(a)** | $R=\sqrt{34}$ | Cao (Must be exact but score when first seen and ignore decimal value (5.83…)) | B1 |
| | $\tan\alpha=\pm\frac{5}{3}, \tan\alpha=\pm\frac{3}{5}\Rightarrow\alpha=...$ (Allow $\cos\alpha=\pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}$, $\sin\alpha=\pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}\Rightarrow\alpha=...$) Where $\sqrt{34}$ is their $R$ | | M1 |
| | $\alpha = 59.04°$ | awrt 59.04° | A1 |
| | | | **(3)** |
| **(b)** | $\sqrt{34}\cos(\theta-59.04)=2\Rightarrow\cos(\theta-59.04)=\frac{2}{\sqrt{34}}(0.343)$ Attempts to use part (a) "$\sqrt{34}$" $\cos(\theta-$"59.04"$)=2$ and proceeds to $\cos(\theta\pm$"59.04"$)=K$, $\lvert K\rvert\leq 1$ May be implied by $\theta-$"59.04"$=69.94…°$ or $\theta-$"59.04"$\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)$ The $\theta-$"59.04" must be seen here or implied later | | M1 |
| | $\theta_1-59.04=69.94\Rightarrow\theta_1=\text{awrt }129.0°$ | | A1 |
| | $\theta_2\pm59.04=360-$'69.94'$\Rightarrow\theta_2=...$ Correct attempt at a second solution in the range. It is **dependent** upon having scored the previous M. Usually for $\theta-$their $59.04=360-$their '69.94'$\Rightarrow\theta=...$ | | **d**M1 |
| | $\theta_2=349.1°$ | awrt 349.1° | A1 |
| | For solutions in (b) that are otherwise fully correct, if there are extra answers in range, deduct the final A mark. | | |
| | | | **(4)** |
| **(c)** | $\theta+\text{their }59.04=\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)\Rightarrow\theta=...$ Allow $\theta-\text{their }59.04=\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)\Rightarrow\theta=...$ if they have $\theta+$ .. in (b) Evidence that use is being made of parts (a) and (b) to obtain a value for $\theta$. This can be implied by the use of their answers to part (b). | | M1 |
| | $\theta=10.9°$ | awrt 10.9 | A1 |
| | | | **(2)** |
| | | | **(9 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **15.(a)** | $R=\sqrt{41}$ | B1 |
| | $\tan\alpha=\frac{4}{5}\Rightarrow\alpha=$ awrt 0.675 | M1A1 |
| | | (3) |
| **(b)** | (i) Describes stretch: stretch in the $y$ direction by "$\sqrt{41}$" | B1 ft |
| | (ii) Describes translation: E.g. translate by $\begin{pmatrix}-\arctan\frac{4}{5}\\0\end{pmatrix}$ | B1 ft |
| | | (2) |
| **(c)** | Attempts either $g(\theta)=\frac{90}{4+\left(\sqrt{41}\right)^2}$ OR $g(\theta)=\frac{90}{4}$ | M1 |
| | Range $2\leqslant g(\theta)\leqslant 22.5$ | A1 |
| | | (2) |
| | | **7 marks** |

(a)

B1 $R=\sqrt{41}$

Condone $R=\pm\sqrt{41}$ (Do not allow decimals for this mark Eg 6.40 but remember to isw after $\sqrt{41}$ )

M1 $\tan\alpha=\pm\frac{4}{5},\ \tan\alpha=\pm\frac{5}{4}\Rightarrow\alpha=...$ Condone $\sin\alpha=4,\ \cos\alpha=5\Rightarrow\tan\alpha=\frac{4}{5}$

If $R$ is used to find α accept $\sin\alpha=\pm\frac{4}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=...$

A1 $\alpha=$ awrt 0.675 Note that the degree equivalent $\alpha=$ awrt $38.7°$ is A0

(b)(i)

B1ft Fully describes the stretch. Follow through on their *R*. **Requires the size and the direction**

**Allow** responses such as

- stretch in the $y$ direction by "$\sqrt{41}$"
- multiplies all the $y$ coordinates/values by "$\sqrt{41}$"
- stretch in ↑ direction by "$\sqrt{41}$"
- vertical stretch by "$\sqrt{41}$"
- Scale Factor "$\sqrt{41}$" in just the $y$ direction

Do **not** award for $y$ is translated/transformed by "$\sqrt{41}$"

(b)(ii)

B1 ft Fully describes the translation. **Requires the size and the direction**

Follow through on their 0.675 or $\alpha =$ awrt $38.7°$ or $\arctan\frac{4}{5}$

**Allow** responses such as

- translates left by 0.675
- horizontal by $-0.675$
- condone "transforms" left by 0.675. (question asks for the translation)
- moves $\leftarrow$ by $38.7°$
- $x$ values move back by 0.675
- shifts in the negative $x$ direction by $\arctan\frac{4}{5}$
- $\begin{pmatrix}-0.675\\0\end{pmatrix}$

Do **not** award for translates left by $-0.675$ (double negative...wrong direction)
horizontal shift of 0.675 (no direction)

If there are no labels score in the order given but do allow these to be written in any order as long as the candidate clearly states which one they are answering. For example it is fine to write ….
translation is..............
stretch is ..................
If the candidate does not label correctly, or states which one they are doing, but otherwise gets both completely correct then award SC B1 B0

(c)

M1 Score for either end achieved by a correct method

Look for $\frac{90}{4}$ (implied by 22.5) , $\frac{90}{4+\text{their}\left(\sqrt{41}\right)^2}$ , $g \geqslant 22.5$ or $g \geqslant 2$ etc

A1 See scheme but allow 22.5 to be written as $\frac{90}{4}$

Accept equivalent ways of writing the interval such as $[2, 22.5]$

Condone $2 \leqslant \mathrm{g}(x) \leqslant 22.5$ or $2 \leqslant y \leqslant 22.5$

| Question Number | Scheme | Marks |
|---|---|---|
| **16(a)** | $R=13$ | B1 |
| | $\tan\alpha=\frac{5}{12}\Rightarrow\alpha=\text{awrt } 0.395$ | M1A1 |
| | | **(3)** |
| **(b)** | $g(\theta)=10+13\sin\left(2\theta-\frac{\pi}{6}-0.395\right)$ | |
| **(i)** | (i) Minimum value is $-3$ | B1 ft |
| **(ii)** | $2\theta-\frac{\pi}{6}-0.395=\frac{3\pi}{2}\Rightarrow\theta=\text{awrt } 2.82$ | M1 A1 |
| | | **(3)** |
| **(c)** | $h(\beta)=10-169\sin^2(\beta-0.395)$ | |
| | $-159\leqslant h\leqslant 10$ | M1 A1 |
| | | **(2)** |
| | | **(8 marks)** |

(a)

B1: $R=13$ ($R=\pm13$ is B0)

M1: $\tan\alpha=\pm\frac{5}{12}$, $\tan\alpha=\pm\frac{12}{5}\Rightarrow\alpha=...$

If $R$ is used to find $\alpha$ accept $\sin\alpha=\pm\frac{5}{R}$ or $\cos\alpha=\pm\frac{12}{R}\Rightarrow\alpha=...$

A1: $\alpha=\text{awrt } 0.395$ Note that the degree equivalent $\alpha=\text{awrt } 22.6°$ is A0

(b)(i)

B1ft: States the value of $10-R$ following through their $R$.

(b)(ii)

M1: Attempts to solve $2\theta-\frac{\pi}{6}\pm\text{"}0.395\text{"}=\frac{3\pi}{2}\Rightarrow\theta=...$

A1: $\theta=\text{awrt } 2.82$. No other values should be given

(c)

M1: Achieves one of the end values, either $-159$ (or $10$ – (their $R$)$^2$ evaluated) or $10$

A1: Fully correct range $-159\leqslant h\leqslant 10$, $-159\leqslant h(\beta)\leqslant 10$, $-159\leqslant \text{range}\leqslant 10$, $-159\leqslant h(x)\leqslant 10$, $[-159,10]$ or equivalent correct ranges.

| Question Number | Scheme | Marks |
|---|---|---|
| | $\mathrm{f}(x) = 8\sin x\cos x + 4\cos^2 x - 3$ | |
| **17(a)** | States or uses $\sin 2x = 2\sin x\cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$ | M1 |
| | Uses $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in f $(x)$ | dM1 |
| | $(\mathrm{f}(x) =)\ 8\sin x\cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$ | A1 |
| | | **(3)** |
| **(b)** | $R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20}$ or $2\sqrt{5}$ | B1ft |
| | $\tan\alpha = \frac{b}{a} \Rightarrow \alpha = ...\ (= \text{"awrt } 0.464\text{"})$ | M1 |
| | $(\mathrm{f}(x) =)\ 2\sqrt{5}\sin(2x + 0.464) - 1$ | A1 |
| | | **(3)** |
| **(c)** | (i) Maximum value = "$2\sqrt{5} - 1$" | B1 ft |
| | (ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = ...$ | M1 |
| | $(x =)$ awrt 3.69 (or $(x =)$ awrt 3.70 ) | A1 |
| | | **(3)** |
| | | **(9 marks)** |

(a) For the method marks condone working in mixed variables provided the intention is clear

M1: **Either** states one of the correct identities $\sin 2x = 2\sin x\cos x$ or $\cos 2x = 2\cos^2 x - 1$

**or** uses $\sin 2x = 2\sin x\cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$ (may be implied by use of e.g. $\cos^2 x = \pm 1 \pm \sin^2 x$ and $\cos 2x = \pm 1 \pm 2\sin^2 x$ or $\cos 2x = \pm\cos^2 x \pm \sin^2 x$ )

Can be implied by **either of**

- $a = 4$
- $b = \pm 2$ **and** $c = -5$ or $-1$

dM1: **Uses** $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ (which may be implied as above for the first method mark) in f $(x)$ to produce an expression of the required form $a\sin 2x + b\cos 2x + c$
Condone slips when substituting in.

Can be implied by $a = 4$ **and** $b = \pm 2$ **and** $c = -5$ or $-1$

A1: $4\sin 2x + 2\cos 2x - 1$ Correct answer scores all three marks. Must be in terms of $x$.

(b) **Full marks can only be scored provided correct values for *a*, *b* and *c* are found in (a).**

B1ft: Finds the exact value of $R$ using $R^2 = a^2 + b^2$ (or may use trigonometry using their value for $\alpha$ )

Can be implied by a correct exact value.

Follow through on their $a$ or $b$ but the correct values should give $R = \sqrt{20}$ or $2\sqrt{5}$

M1: Uses a "correct " method to find $\alpha$ using their *a* and *b*. Accept for example $\tan\alpha = \pm\frac{b}{a} \Rightarrow \alpha = ...$ or $\tan\alpha = \pm\frac{a}{b} \Rightarrow \alpha = ...$ (You may need to check this on your calculator if only the angle is seen)

May also be seen using e.g. $\sin\alpha = \pm\frac{b}{"R"} \Rightarrow \alpha = ...$ or $\cos\alpha = \pm\frac{a}{"R"} \Rightarrow \alpha = ...$ Allow the angle to be found in degrees for this mark. $\alpha =$"awrt $26.6°$" (1dp)

A1cso: $\left(\text{f}(x) =\right)\ 2\sqrt{5}\sin\left(2x + \text{awrt}\, 0.464\right) - 1$ o.e. e.g. $\left(\text{f}(x) =\right)\ \sqrt{20}\sin\left(2x + 0.464\right) - 1$ (may be awarded if seen in (c)) Can only be scored provided correct values for *a*, *b* and *c* are found in (a).

(c) (i)

B1ft: $"2\sqrt{5} - 1"$ o.e. e.g. $"\sqrt{20} - 1"$ but follow through on their $R + c$ as long **as *R* has been correctly found** (ie awarded in (b)). Ft on their *c* which may be different in (a) and (b) but **cannot be 0**

Allow to be a decimal if *R* was given as a decimal in (b) at any point. isw following a correct answer.

(c)(ii)

M1: Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = ...$ using their $\alpha$ found in (b) (note that this is still the equation which will need to be solved if they differentiate f($x$) first and set equal to 0). You may need to check this on your calculator.

Allow this mark to be scored even if there are additional equations formed (and possibly solved).

They must be consistent in their use of degrees or radians in an equation that they are solving.

$\frac{5\pi}{2} = 7.85...$ may be used in their working.

A1: $(x =)$ awrt $3.69$ but also allow $(x =)$ awrt $3.70$. If several angles are found then they must indicate which angle is their final answer by e.g. underlining, circling

| Question | Scheme | Marks |
|---|---|---|
| **18(a)** | $R = 17$ | **B1** |
| | $\tan\alpha = \frac{15}{8}$ | **M1** |
| | $\alpha = 1.081$ | **A1** |
| | | **(3)** |
| **(b)** | (i) Min f($x$) $= \frac{15}{41+2\times"17"}$ | **M1** |
| | $= \frac{1}{5}$ | **A1** |
| | (ii) Occurs when $\sin(x-1.081)=1 \Rightarrow x-1.081=\frac{\pi}{2} \Rightarrow x=...$ | **M1** |
| | $x =$ awrt 2.65 | **A1** |
| | | **(4)** |
| **(c)** | $-\frac{23}{5}$ (or $-4.6$) | **B1ft** |
| | | **(1)** |
| **(d)** | Awrt 1.33 | **B1ft** |
| | | **(1)** |
| | | **(9 marks)** |

**Notes:**

**(a)**

**B1:** For 17 only.

**M1:** Attempts an equation in $\alpha$. Accept $\tan\alpha = \pm\frac{15}{8}$ or $\tan\alpha = \pm\frac{8}{15}$. If using $R$ accept $\cos\alpha = \pm\frac{8}{"17"}$ or $\sin\alpha = \pm\frac{15}{"17"}$. Implied by a correct value for $\alpha$

**A1:** Awrt 1.081. Must be in radians.

**(b)(i)**

**M1:** Attempts to apply the result from (a) to find the minimum. Allow for $\frac{15}{41\pm 2\times"\text{their } R"}$

**A1:** cao.

**(ii)**

**M1:** Attempts to solve $x \pm "\alpha" = \frac{\pi}{2}$

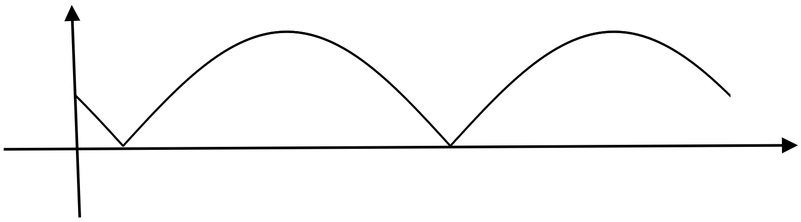
**A1:** cao

**(c)**

**B1ft:** Correct answer or follow through $2\times\left(\text{their } \frac{1}{5}\right)-5$ and no other solutions.

**(d)**

**B1ft:** For awrt 1.33 or follow through $0.5\times\text{their } 2.65$ and no other solutions.

| Question Number | Scheme | Marks |
|---|---|---|
| **19(a)** | $R = \sqrt{5}$ | B1 |
| | $\tan\alpha = \frac{1}{2} \Rightarrow (\alpha =) 26.6°$ | M1,A1 |
| | | (3) |
| **(b)** |  Shape | B1 |
| | $(0,1)$ | B1 |
| | $("26.6",0)$ and $("206.6",0)$ (Allow in radians i.e. their α and π+α | B1ft |
| | | **(3)** |
| **(c)(i)** | $5 + 'R' = 5 + \sqrt{5}$ | B1ft |
| | | |
| **(c)(ii)** | $15t - '26.6' = 270 \Rightarrow t = 19.8$ | M1,A1 |
| | | **(3)** |
| | | **(9 marks)** |

(a)
B1: $R = \sqrt{5}$
M1: For $\tan\alpha = \pm\frac{1}{2}$ or $\tan\alpha = \pm\frac{2}{1}$ or $\sin\alpha = \pm\frac{1}{"\sqrt{5}"}$ or $\cos\alpha = \pm\frac{2}{"\sqrt{5}"}$
A1: Awrt $\alpha = 26.6°$
(b)
B1: Correct shape including cusps. A curve that starts downwards from the positive *y*-axis with two maxima. This mark is essentially for realising that the parts of the curve under the *x*-axis are reflected in the *x*-axis and for cusps that look “pointed” and not rounded.
B1: $(0,1)$ may be seen on the diagram or in the body of the script as coordinates or seen as $x = 0$, $y = 1$. If there is any ambiguity, the sketch takes precedence. Allow (1, 0) as long as it is marked in the correct place on the sketch.
B1**ft**: $(26.6,0)$ and $(206.6,0)$ or their 26.6 and 180 + their 26.6. May be seen on the sketch or in the body of the script as coordinates or seen as $y = 0$, $\theta(\text{or } x) = 26.6$, $\theta(\text{or } x) = 206.6$. If there is any ambiguity, the sketch takes precedence. Allow awrt 26.6 and awrt 207 or their ft values.
(c)(i)
B1ft: Follow through on $5 + 'R'$ including decimal answers (NB 5 + √5 = 7.24…)
(c)(ii)
M1: Attempts $15t - "26.6" = 90$ or $270 \Rightarrow t = ...$ (Allow π/2, 3π/2 for 90, 270 if working in radians)
A1: $t = 19.8$ **only**
**(c)(ii) Alternative:**
$\text{f}(t) = 5 + 2\sin(15t) - \cos(15t) \Rightarrow \text{f}'(t) = 30\cos(15t) + 15\sin(15t)$
M1: Attempts $\text{f}'(t) = 0 \Rightarrow 15t = 180 - 63.43...$ or $360 - 63.43$
A1: $t = 19.8$ **only**

| Question Number | Scheme | Marks |
|---|---|---|
| **20.** (a) | $R = \sqrt{5} = 2.23606...$ (must be given in part (a)) | B1 |
| | $\tan\alpha = \frac{1}{2}$ or $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\cos\alpha = \frac{2}{\sqrt{5}}$ ( see notes for other values which gain M1) | M1 |
| | $\Rightarrow \alpha = 26.56505...^\circ$ (must be given in part (a)) | A1 |
| | | **[3]** |
| (b) | Way 1:<br>Uses distance between two lines is 4 (or half distance is 2) with correct trigonometry **may** state $4\sin\theta + 2\cos\theta = 4$ **or** show sketch | M1 |
| | Need sketch **and** $4\sin\theta + 2\cos\theta = 4$ **and** deduction that $2\sin\theta + \cos\theta = 2$ or $\cos\theta + 2\sin\theta = 2$ * | A1 * |
| | | **[2]** |
| | Way 2:<br>**Alternative method:** Uses diagonal of rectangle as hypotenuse of right angle triangle and obtains $\sqrt{20}\sin(\theta+\alpha) = 4$ | M1 |
| | So from (a) $2\sin\theta + \cos\theta = 2$ or $\cos\theta + 2\sin\theta = 2$ | A1 |
| | | **[2]** |
| | Way 3: They may state and verify the result provided the work is correct and accurate See notes below. **Substitution of 36.9** (obtained in (c) is a circular argument and is **M0A0**) | |
| (c) | Way1: Uses $\sqrt{5}\sin(\theta + 26.57) = 2$ to obtain $\sin(\theta + "26.57") = \frac{2}{"\sqrt{5}"}$ $(= 0.8944...)$ — Way 2 $\cos^2\theta + 4\cos\theta\sin\theta + 4\sin^2\theta = 4$ See notes for variations $4\cos\theta\sin\theta - 3\cos^2\theta = 0$ $\cos\theta(4\sin\theta - 3\cos\theta) = 0$ so $\tan\theta = \frac{3}{4}$ | M1 |
| | $\theta = \arcsin\left(\frac{2}{\text{their } "\sqrt{5}"}\right) - "26.57"$ — $\theta = \arctan\frac{3}{4}$ or equivalent | M1 |
| | Hence, $\theta = 36.8699...^\circ$ | A1 |
| | | **[3]** |
| (d) | Way 1: $"x" = \frac{2}{\tan"36.9"}$ — Way 2: $"y" = \frac{4}{\sin\theta}$ | B1 |
| | $\{h + x = 4 \Rightarrow\}$ $h + \frac{2}{\tan"36.9"} = 4$ — $\{h + y = 8 \Rightarrow\}$ $h + \frac{4}{\sin"36.9"} = 8$ | M1 |
| | $h = 4 - \frac{2}{\tan 36.9} = 1.336...$ or $\frac{4}{3}$ or $\underline{1.3}$ (2sf) — $h = 8 - \frac{4}{\sin 36.9} = \frac{4}{3}$ or $\underline{1.3}$ (2sf) | <u>A1</u> **cao** |
| | | **[3]** |
| | | **11** |

**Notes**

(a) **B1**: $R = \sqrt{5}$ or awrt 2.24 no working needed – must be in part (a)

**M1**: $\tan\alpha = \frac{1}{2}$ **or** $\tan\alpha = 2$ or $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\sin\alpha = \frac{2}{\sqrt{5}}$ or $\cos\alpha = \frac{2}{\sqrt{5}}$ or $\cos\alpha = \frac{1}{\sqrt{5}}$ and attempt to find alpha. Method mark may be implied by correct alpha.

**A1**: accept $\alpha =$ awrt 26.57; also accept $\sqrt{5}\sin(\theta + 26.57)$ - must be in part (a)

Answers in radians (0.46) are A0

(b) **Way 1:**

**M1**: Uses distance between two lines is 4 (or half distance is 2) states $4\sin\theta + 2\cos\theta = 4$ **or** shows sketch (may be on Figure 4 on question paper) with some trigonometry

**A1***: **Shows sketch** with implication of two right angled triangles (may be on Figure 4 on question paper) **and follows** $4\sin\theta + 2\cos\theta = 4$ by stating printed answer or equivalent (given in the mark scheme) and no errors seen.

Way 2:
on scheme (not a common method)

Way 3:
They may state and verify the result provided the work is correct and accurate.

M1: Verification with correct accurate work e.g. $2\times\frac{x}{4} + \frac{4-x}{2} = 2$, with $x$ shown on figure

A1: Needs conclusion that $2\sin\theta + \cos\theta = 2$

**Substitution of 36.9** (obtained in (c) is a circular argument and is **M0A0**)

**(c) Way 1:**

**M1**: $\sin(\theta + \text{their } \alpha) = \frac{2}{\text{their } R}$ (Uses part (a) to solve equation)

**M1**: $\theta = \arcsin\left(\frac{2}{\text{their } R}\right) - \text{their } \alpha$ (operations undone in the correct order with subtraction)

**A1**: awrt 36.9 (answer in radians is 0.644 and is A0)

**Way 2**:

**M1**: Squares both sides, uses appropriate trig identities and reaches $\tan\theta = \frac{3}{4}$ or $\sin\theta = \frac{3}{5}$ or $\cos\theta = \frac{4}{5}$ or $\sin 2\theta = \frac{24}{25}$

**{One example is shown in the scheme**. Another popular one is
$2\sin\theta = 2 - \cos\theta \to 4(1-\cos^2\theta) = 4 - 4\cos\theta + \cos^2\theta \to 5\cos^2\theta - 4\cos\theta = 0$ and so $\cos\theta = \frac{4}{5}$ for M1}

**M1**: $\theta = \arctan\frac{3}{4}$ or other correct inverse trig value e.g. $\arcsin\theta\ (\frac{3}{5})$ or $\arccos\theta\ (\frac{4}{5})$

**A1**: awrt 36.9 (answer in radians is 0.644 and is A0)

(d) Way 1: (Most popular)

**B1** : States $x = \frac{2}{\tan\theta}$, where $x$ (not defined in the question) is the non-overlapping length of rectangle

**M1**: Writes equation $h + \frac{2}{\tan\theta} = 4$ - must be this expression or equivalent e.g. $\tan\theta = \frac{2}{4-h}$ gets B1 M1

**A1**: accept decimal which round to 1.3 or the exact answer i.e. $\frac{4}{3}$ (may follow slight inaccuracies in earlier angle being rounded wrongly)

**N.B. There is a variation which states** $\sin\theta = \frac{2\cos\theta}{4-h}$ or $\frac{\sin\theta}{2} = \frac{\sin(90-\theta)}{4-h}$ for B1 M1 then A1 as before

**Way 2**: (Less common)

**B1** : States $y = \dfrac{4}{\sin\theta}$, where $y$ (not defined in question) is the non-overlapping length of two rectangles

**M1**: Writes equation $h + \dfrac{4}{\sin\theta} = 8$ - must be this expression or equivalent e.g. $\sin\theta = \dfrac{4}{8-h}$ gets B1 M1

**A1**: as in Way 1

**There are other longer trig methods – possibly using Pythagoras for showing that *h* = 1.3 to 2sf. If the method is clear award B1M1A1 – otherwise send to review.**