

R Addition Formulae

Question Paper

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1. (a) Express $2\sin x - 4\cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
 Give the exact value of R and give the value of α , in radians, to 3 significant figures.
 (3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H , by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \quad 0 \leq t \leq 365$$

where t is the number of days since he began recording.

- (b) Using your answer to part (a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answers to 3 significant figures.
 (3)
- (c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(Solutions based entirely on graphical or numerical methods are not acceptable.)
 (6)

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Question 1 continued

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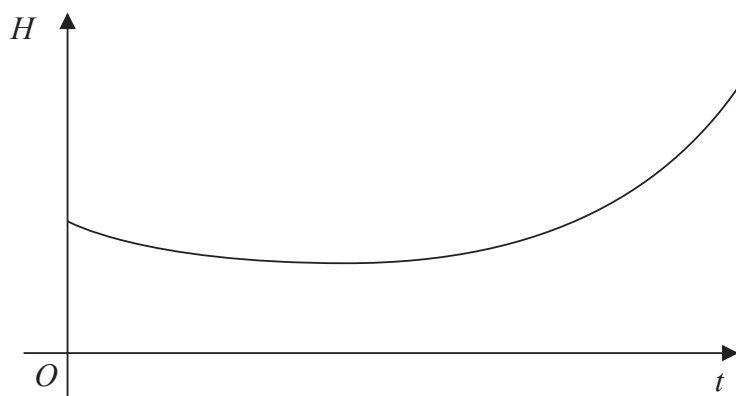


Figure 4

A jet is flying over the sea.

The height above sea level, H metres, of the jet is modelled by the equation

$$H = \frac{8000}{56 + 9\cos t^\circ + 40\sin t^\circ} \quad 0 \leq t \leq 200$$

where t is the time in seconds, measured from when the jet passed over a boat.

Figure 4 is a sketch showing the graph of H against t .

Use the model to answer parts (a), (c) and (d).

(a) Find the height above sea level of the jet as it passed over the boat. (1)

(b) Write $9\cos t^\circ + 40\sin t^\circ$ in the form $R\cos(t - \alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$
Give the exact value of R and the value of α to one decimal place. (3)

(c) Find (i) the minimum height of the jet above sea level,
(ii) the value of t at which this minimum height occurs. (3)

(d) Find the value of t when the jet is 150m above sea level. (4)

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Question 2 continued

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- (3)

$$\theta = 19 + 3 \sin\left(\frac{\pi t}{12} + 4\right) - \cos\left(\frac{\pi t}{12} + 4\right), \quad 0 \leq t < 24$$

(3)

Question 3 continued

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4. (a) Express $1.5 \sin \theta - 1.2 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

(3)

$$H = 3 + 1.5 \sin\left(\frac{\pi t}{6}\right) - 1.2 \cos\left(\frac{\pi t}{6}\right), \quad 0 \leq t < 12$$

(4)

(6)

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Question 4 continued

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5. (a) Express $\cos x + 4 \sin x$ in the form $R \cos(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α , in radians, to 3 decimal places. (3)

A scientist is studying the behaviour of seabirds in a colony.

She models the height above sea level, H metres, of one of the birds in the colony by the equation

$$H = \frac{24}{3 + \cos\left(\frac{1}{2}t\right) + 4\sin\left(\frac{1}{2}t\right)} \quad 0 \leq t \leq 6.5$$

where t seconds is the time after it leaves the nest.

Find, according to the model,

- (b) the minimum height of the seabird above sea level, giving your answer to the nearest cm, (2)

- (c) the value of t , to 2 decimal places, when $H = 10$ (4)

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Question 5 continued

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- Give the exact value of R and give the value of α , in radians, to 4 decimal places.

The height of sea water, H metres, on a harbour wall is modelled by the equation

where t is the number of hours after midday.

- (6)

Question 6 continued

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7. (a) Write $\cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

- (b) Hence solve, for $0 \leq \theta < \pi$, the equation

$$\cos 2\theta + 4 \sin 2\theta = 1.2$$

giving your answers to 2 decimal places.

(5)

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Question 7 continued

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- (3)

- (4)

- (3)

- (2)

Question 8 continued

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9. (a) Express $35 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R , and give the value of α , in radians, to 4 significant figures.
(3)

- (b) Hence solve, for $0 \leq x < 2\pi$,

$$70 \sin x - 24 \cos x = 37$$

(Solutions based entirely on graphical or numerical methods are not acceptable.)
(4)

$$y = \frac{7000}{31 + (35 \sin x - 12 \cos x)^2}, \quad x > 0$$

- (c) Use your answer to part (a) to calculate

- (i) the minimum value of y ,
 (ii) the smallest value of x , $x > 0$, at which this minimum value occurs.

(4)

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Question 9 continued

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$$f(x) = \cos x + 2\sin x$$

- $$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

(3)

$$g(x) = 3 - 7f(2x)$$

- (ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places.

(3)

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Question 10 continued

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11. (a) Express $3 \sin 2x + 5 \cos 2x$ in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α to 3 significant figures.

(3)

- (b) Solve, for $0 < x < \pi$,

$$3 \sin 2x + 5 \cos 2x = 4$$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

$$g(x) = 4(3 \sin 2x + 5 \cos 2x)^2 + 3$$

- (c) Using your answer to part (a) and showing your working,

(i) find the greatest value of $g(x)$,

(ii) find the least value of $g(x)$.

(4)

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Question 11 continued

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Question 12 continued

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- State the value of R and give the value of α to 4 significant figures.

(3)

- $$\sqrt{5} \cos \theta - 2 \sin \theta = 0.5$$

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(4)

$$f(x) = A(\sqrt{5} \cos \theta - 2 \sin \theta) + B \quad \theta \in \mathbb{R}$$

Given that the range of f is

$$-15 \leq f(x) \leq 33$$

- (c) find the value of B and the possible values of A .

(4)

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Question 13 continued

Lined area for writing the answer to Question 13.



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- (3)**

- (4)

- (2)

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Question 14 continued

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15.

$$f(\theta) = 5 \cos \theta - 4 \sin \theta \quad \theta \in \mathbb{R}$$

- (a) Express $f(\theta)$ in the form $R \cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

The curve with equation $y = \cos \theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

- (b) (i) describe fully the transformation that is a stretch,
 (ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \quad \theta \in \mathbb{R}$$

- (c) find the range of g .

(2)

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Question 15 continued

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- (3)

$$g(\theta) = 10 + 12 \sin\left(2\theta - \frac{\pi}{6}\right) - 5 \cos\left(2\theta - \frac{\pi}{6}\right) \quad \theta > 0$$

(3)

$$h(\beta) = 10 - (12 \sin \beta - 5 \cos \beta)^2$$

- (2)

Question 16 continued

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Question 17 continued

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Solutions relying entirely on calculator technology are not acceptable.

- Give the exact value of R , and give the value of α , in radians, to 4 significant figures.
- (3)**

(ii) the smallest value of x at which this minimum value occurs.

- $$y = 2f(x) - 5 \quad x > 0 \quad (1)$$

- $$y = -f(2x) \quad x > 0 \quad (1)$$

Question 18 continued

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19. (a) Write $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leq 90^\circ$. Give the exact value of R and give the value of α to one decimal place. (3)

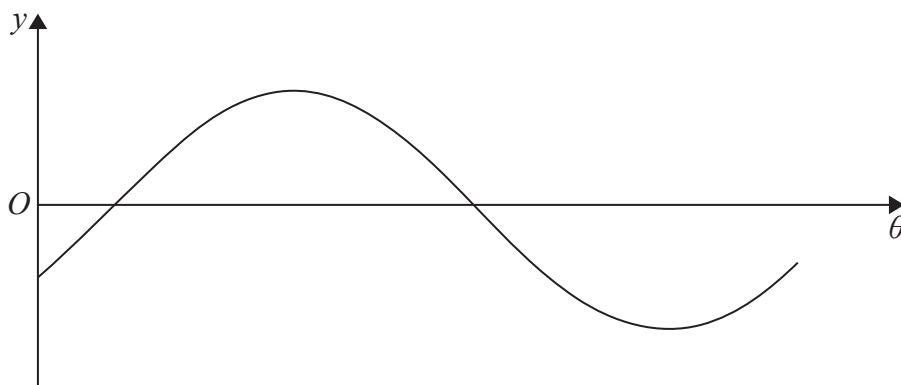


Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta$, $0 \leq \theta < 360^\circ$

- (b) Sketch the graph with equation

$$y = |2\sin\theta - \cos\theta|, \quad 0 \leq \theta < 360^\circ$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.

(3)

The temperature of a warehouse is modelled by the equation

$$f(t) = 5 + |2\sin(15t)^\circ - \cos(15t)^\circ|, \quad 0 \leq t < 24$$

where $f(t)$ is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

- (c) (i) the maximum value of $f(t)$,
(ii) the largest value of t , for $0 \leq t < 24$, at which this maximum value occurs. Give your answer to one decimal place. (3)

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Question 19 continued

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20. (a) Express $2 \sin \theta + \cos \theta$ in the form $R \sin (\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$. Give your value of α to 2 decimal places. (3)

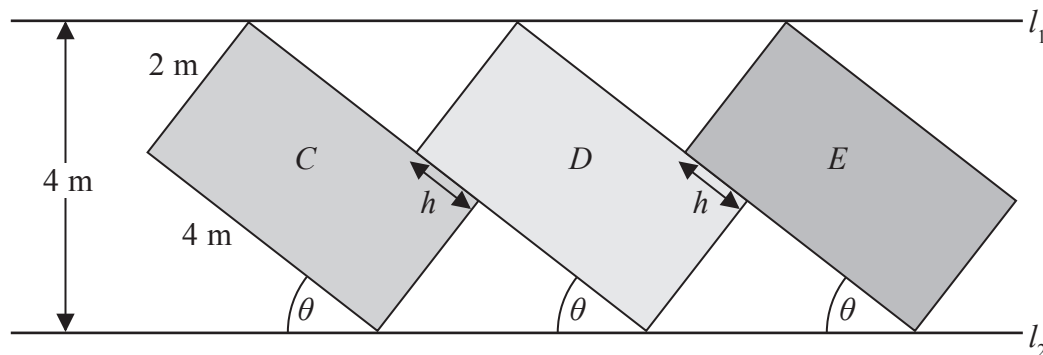


Figure 4

Figure 4 shows the design for a logo that is to be displayed on the side of a large building. The logo consists of three rectangles, C , D and E , each of which is in contact with two horizontal parallel lines l_1 and l_2 . Rectangle D touches rectangles C and E as shown in Figure 4.

Rectangles C , D and E each have length 4 m and width 2 m. The acute angle θ between the line l_2 and the longer edge of each rectangle is shown in Figure 4.

Given that l_1 and l_2 are 4 m apart,

- (b) show that

$$2 \sin \theta + \cos \theta = 2 \quad (2)$$

Given also that $0 < \theta < 45^\circ$,

- (c) solve the equation

$$2 \sin \theta + \cos \theta = 2$$

giving the value of θ to 1 decimal place. (3)

Rectangles C and D and rectangles D and E touch for a distance h m as shown in Figure 4.

Using your answer to part (c), or otherwise,

- (d) find the value of h , giving your answer to 2 significant figures. (3)

Question 20 continued

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