Complex Numbers and Argand Diagrams

Mark Scheme

Question	Scheme	Marks	AOs
1(a) (i)	$\{arg(z_1) =\} tan^{-1}\left(\frac{-3}{3}\right)$ or $\{arg(z_1) =\} tan^{-1}(-1)$ or $\{arg(z_1) =\} - tan^{-1}\left(\frac{3}{3}\right)$ or $\{arg(z_1) =\} -\frac{\pi}{4}$ or $\{arg(z_1) =\} 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ or states should be – 3 not 3 on top	B1	2.3
(a) (ii)	States that $\left\{arg\left(\frac{z_1}{z_2}\right) =\right\} arg(z_1) - arg(z_2)$ Or states that the arguments should be subtracted	B1	2.3
		(2)	
(b)	$\left\{arg\left(\frac{z_1}{z_2}\right) = \left(\text{their} -\frac{\pi}{4}\right) - \frac{\pi}{6} =\right\} - \frac{5\pi}{12}$ Or $\left\{arg\left(\frac{z_1}{z_2}\right) = \left(\text{their}\frac{7\pi}{4}\right) - \frac{\pi}{6}\right\} = \frac{19\pi}{12}$	B1ft	2.2a
		(1)	
		(3 marks)	

Notes:

(a) (i)
B1: See scheme, Condone – 45
Any incorrect arguments seen is B0.
$arg(z_1) = tan^{-1}\left(\frac{3}{-3}\right)$ is B0
Note: They used 3 instead of – 3 is B0, there are two 3's in line 1 do they mean both should – 3
It should be negative is B0
(a) (ii)
B1: See scheme
(b)
B1ft: States a correct value for $arg\left(\frac{z_1}{z_2}\right)$ Follow through on their answer to part (a) (i), do not ISW

Question	Scheme	Marks	AOs
2(a)	e.g. $\lvert z_1\rvert=\sqrt{(-4)^2+4^2}$ or $\arg z_1=\pi-\frac{\pi}{4}$ oe	M1	1.1b
	$(z_1=)4\sqrt{2}\left(\cos\frac{3\pi}{4}+\text{i}\sin\frac{3\pi}{4}\right)$ or e.g. $(z_1=)\sqrt{32}\left(\cos\frac{3\pi}{4}+\text{i}\sin\frac{3\pi}{4}\right)$	A1	1.1b
		(2)	
(b)(i)	$\frac{z_1}{z_2}=\frac{"4\sqrt{2}"}{3}\left(\cos\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)+\text{i}\sin\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)\right)=\ldots$ **or** $\frac{z_1}{z_2}=\frac{"4\sqrt{2}"\text{e}^{"\frac{3\pi}{4}\text{i}"}}{3\text{e}^{\frac{17\pi}{12}\text{i}}}=\frac{"4\sqrt{2}"}{3}\text{e}^{\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)\text{i}}$ **or** $\frac{z_1}{z_2}=\frac{-4+4\text{i}}{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\text{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)}\times\frac{\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)+\text{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)}{\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)+\text{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)}=\ldots$	M1	3.1a
	$=-\frac{2\sqrt{2}}{3}-\frac{2\sqrt{6}}{3}\text{i}$ or $-\frac{2\sqrt{2}}{3}-\text{i}\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{2}}{3}+\text{i}\left(-\frac{2\sqrt{6}}{3}\right)$	A1	1.1b
		(2)	

Notes

(a) <u>Correct answer with no working scores both marks in (a)</u>

M1: Any correct expression for $\lvert z_1\rvert$ or $\arg z_1$ e.g. $\lvert z_1\rvert=\sqrt{(-4)^2+4^2}$ or $\arg z_1=\pi-\frac{\pi}{4}$

A1: Correct <u>expression</u>. The "$z_1=$" is not required.

This mark is not for correct modulus and correct argument it is for the complex number written in the required form. Condone the missing closing bracket e.g. $(z_1=)\sqrt{32}\left(\cos\frac{3\pi}{4}+\text{i}\sin\frac{3\pi}{4}\right.$

(b)(i) <u>Correct answer with no working scores no marks in (b)(i)</u>

M1: Employs a correct method to find the quotient. E.g.

- uses modulus argument form and divides moduli and subtracts arguments the right way round
- uses exponential form and divides moduli and subtracts arguments the right way round
- converts z_2 to Cartesian form and multiplies numerator and denominator by the complex conjugate of the denominator. Allow if the "3" is missing for this method. Allow with decimals for this method e.g. $\frac{z_1}{z_2}=\frac{-4+4\text{i}}{-0.258\ldots-0.965\ldots\text{i}}\times\frac{-0.258\ldots+0.965\ldots\text{i}}{-0.258\ldots+0.965\ldots\text{i}}=\ldots$

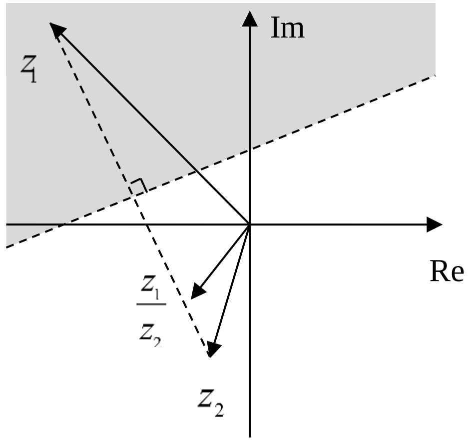
<u>If they convert z_2 to Cartesian form it must be correct as shown or correct decimals.</u>

A1: Correct exact answer in the required form.

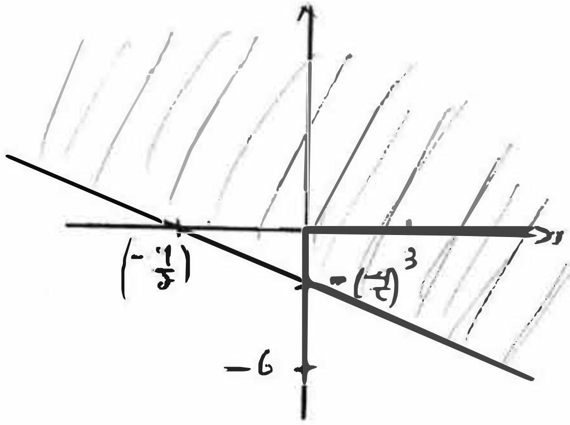
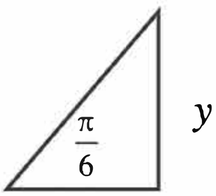
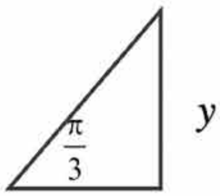
Do not allow e.g. $-\frac{2}{3}\left(\sqrt{2}+\sqrt{6}\text{i}\right)$ or $\frac{-2\sqrt{2}-2\sqrt{6}\text{i}}{3}$ unless a correct form is seen previously then apply isw.

Provided a correct method is shown as above, allow to go from the forms in the main scheme to the correct exact answer with no intermediate step.

(c)(i) and (ii)



Notes:		
(c)(i) **B1:** z_1 and z_2 correctly positioned. Look for <u>correct quadrants</u> with z_1 approximately on $y = -x$ and z_2 below $y = x$ closer to the origin than z_1. Note that the points are usually labelled but mark positively if it is clear which points are which if there is no labelling.	**B1**	1.1b
B1ft: $\frac{z_1}{z_2}$ in the correct quadrant. Follow through their answer to (b)(i). Note that the point is usually labelled but mark positively if it is clear which point it is. It is sometimes labelled as z_3 which is fine.	**B1ft**	1.1b
(ii) **M1:** Draws a line (solid or dashed) that is the perpendicular bisector of z_1z_2 **or** draws a line that crosses z_1z_2 and shades one of the sides of this line.	**M1**	3.1a
A1: A line drawn (solid or dashed) that is the perpendicular bisector of z_1z_2 **<u>with either side shaded</u>** as long as it is clear they are not discounting the upper region. The B1 in part (i) may not have been scored but z_1 must be in quadrant 2 and z_2 in quadrant 3. **Note that some candidates are drawing the region on a separate diagram and this is acceptable. You do not need to see a line joining z_1 to z_2.**	**A1**	1.1b
	(4)	
(10 marks)		

Question	Scheme	Marks	AOs
3(i)		M1 A1 B1	3.1a 1.1b 1.1b
		(3)	
(ii)	$m=\tan\left(\frac{\pi}{3}\right)\left\{=\sqrt{3}\right\}$ and $y-0=m(x-2)$ leads to $y-0=\sqrt{3}(x-2)$ or $y=\sqrt{3}x-2\sqrt{3}$ $m=\tan\left(\frac{\pi}{6}\right)\left\{=\frac{\sqrt{3}}{3}\right\}$ and $y-0=m(x-(-1))$ leads to $y-0=\frac{\sqrt{3}}{3}(x-(-1))$ or $y=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\sqrt{3}x-2\sqrt{3}=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}\Rightarrow x=\ldots$	M1	1.1b
	$y=\sqrt{3}\left(\frac{7}{2}\right)-2\sqrt{3}=\ldots$	M1	1.1b
	$\{w=\}\ \frac{7}{2}+\frac{3\sqrt{3}}{2}\mathrm{i}$	A1	2.1
		(6)	
	Alternative $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}=\frac{y}{x_{-1}}$ and $\tan\left(\frac{\pi}{3}\right)=\sqrt{3}=\frac{y}{x_2}$	M1	1.1b
	 $x_{-1}=\sqrt{3}y$  $x_2=\frac{\sqrt{3}}{3}y$	A1 A1	1.1b 1.1b

	$y\sqrt{3} = y\frac{\sqrt{3}}{3} + 3 \Rightarrow y = ...$	M1	3.1a
	Uses $x = y\sqrt{3} - 1$ or $x = \frac{\sqrt{3}}{3}y + 2$ with their value of *y* leading to a value for *x*	M1	1.1b
	$(w =)\frac{7}{2} + \frac{3\sqrt{3}}{2}\mathrm{i}$	A1	2.1
		(6)	

(9 marks)

Notes:

(i)

M1: Draws a **single** straight line through **both axes** with a negative gradient. Ignore any line joining (3, 0) and (0, -6)

A1: Draws a **single** straight line through **both axes** with a negative gradient which has a negative *y* intercept**.** Ignore any intercept marked on the axes. Ignore any line joining (3, 0) and (0, -6)

B1: Shades the area above their straight line (not a bounded region such as a triangle bounded by the axes and the line)

(ii)

M1: Finds the Cartesian equations for both loci by using the gradient as tan(argument) and correct coordinate. Must be an attempt at both equations but one correct equation scores this mark

A1: One equation correct, need not be simplified

A1: Both equations correct, need not be simplified

M1: Solve simultaneously to find either the real or imaginary component.

M1: Finds the other component to complete the process of finding *w*.

A1: Correct exact answer

Note: If leaves the answer as a coordinate this is A0. If defines $w = a + b\mathrm{i}$ and then states $a = \frac{7}{2}$ and $b = \frac{3\sqrt{3}}{2}$ this is A1

Alternative

M1: Use both arguments to form equations involving *x* and *y*

A1: (One correct triangle) value for *x* in terms of *y*

A1: (Two correct triangles), values for *x* in terms of *y*

M1: Forms and solves an equation $y\sqrt{3} = y\frac{\sqrt{3}}{3} + 3 \Rightarrow y = ...$ must be come from $x_2 = x_{-1} + 3$

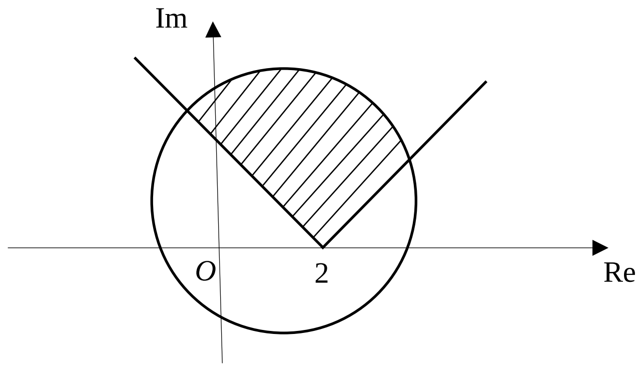
M1: Uses their *y* value and $x = y\sqrt{3} - 1$ or $x = \frac{\sqrt{3}}{3}y + 2$ to find a value for *x*

A1: Correct exact answer

Note: If candidates use decimal instead of exact values throughout allow the method marks $y = 1.73x - 3.46$ and $y = 0.58x + 0.58$

Question	Scheme	Marks	AOs
4(a)	$\lvert z_1 \rvert = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$	B1	1.1b
	$z_1 = \sqrt{13}\left(\cos 0.9828 + \mathrm{i}\sin 0.9828\right)$	B1ft	1.1b
		(2)	
(b)	A complete method to find the modulus of z_2 e.g. $\lvert z_1 \rvert = \sqrt{13}$ and uses $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \times \lvert z_2 \rvert = 39\sqrt{2} \Rightarrow \lvert z_2 \rvert = 3\sqrt{26}$ or $\sqrt{234}$	M1 A1	3.1a 1.1b
	A complete method to find the argument of z_2 e.g. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = \frac{\pi}{4} \Rightarrow \arg(z_2) = \ldots$ $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974\ldots$	M1 A1	3.1a 1.1b
	$z_2 = 3\sqrt{26}\left(\cos('-0.1974...') + \mathrm{i}\sin('-0.1974...')\right)$ or $z_2 = a + b\mathrm{i} \Rightarrow a^2 + b^2 = 234$ and $\tan^{-1}(-0.1974) = \frac{b}{a} \Rightarrow \frac{b}{a} = -0.2$ $\Rightarrow a = \ldots$ and $b = \ldots$	ddM1	1.1b
	Deduces that $z_2 = 15 - 3\mathrm{i}$ only	A1	2.2a
	Alternative $z_1 z_2 = (a + b\mathrm{i})(2 + 3\mathrm{i}) = (2a - 3b) + (3a + 2b)\mathrm{i}$ $(2a - 3b)^2 + (3a + 2b)^2 = (39\sqrt{2})^2$ or 3042 $\Rightarrow a^2 + b^2 = 234$ or $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \times \lvert z_2 \rvert = 39\sqrt{2} \Rightarrow \lvert z_2 \rvert = 3\sqrt{26}$ or $\sqrt{234}$ $\Rightarrow a^2 + b^2 = 234$	M1 A1	3.1a 1.1b
	$\arg\left[(2a - 3b) + (3a + 2b)\mathrm{i}\right] = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{3a + 2b}{2a - 3b}\right) = \frac{\pi}{4} \Rightarrow \frac{3a + 2b}{2a - 3b} = 1$ $\Rightarrow a = -5b$	M1 A1	3.1a 1.1b
	Solves $a = -5b$ and $a^2 + b^2 = 234$ to find values for a and b	ddM1	1.1b
	Deduces that $z_2 = 15 - 3\mathrm{i}$ only	A1	2.2a
		(6)	
			(8 marks)

Notes:
(a) **B1:** Correct exact value for $\|z_1\| = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$. The value for $\arg z_1$ can be implied by sight of awrt 0.98 or awrt 56.3° **B1ft**: Follow through on $r = \|z_1\|$ and $\theta = \arg z_1$ and writes $z_1 = r(\cos\theta + i\sin\theta)$ where r is exact and θ is correct to 4 s.f. do not follow through on rounding errors.
(b) **M1:** A complete method to find the modulus of z_2 **A1:** $\|z_2\| = 3\sqrt{26}$ **M1:** A complete method to find the argument of z_2 **A1:** $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974...$ **ddM1:** Writes z_2 in the form $r(\cos\theta + i\sin\theta)$, dependent on both previous M marks. Alternative forms two equations involving *a* and *b* using the modulus and argument of z_2 and solve to find values for *a* and *b* **A1**: Deduces that $z_2 = 15 - 3i$ only
(b) Alternative: $z_1 z_2 = (a + bi)(2 + 3i) = (2a - 3b) + (3a + 2b)i$ **M1:** A complete method to find an equation involving *a* and *b* using the modulus **A1:** Correct simplified equation $a^2 + b^2 = 234$ o.e. **M1:** A complete method to find an equation involving *a* and *b* using the argument. Note $\tan^{-1}\left(\frac{2a - 3b}{3a + 2b}\right) = \frac{\pi}{4}$ this would score **M0 A0 ddM0 A0** **A1:** Correct simplified equation $a = -5b$ o.e. **ddM1:** Dependent on both the previous method marks. Solves their equations to find values for *a* and *b* **A1**: Deduces that $z_2 = 15 - 3i$ only

Question	Scheme	Marks	AOs
5(a)		M1 M1 A1 M1 A1	1.1b 1.1b 2.2a 3.1a 1.1b
		(5)	
(b)	$(x-1)^2+(y-1)^2=9,\ \ y=x-2 \Rightarrow x=...,$ or $y=...$	M1	3.1a
	$x=2+\frac{\sqrt{14}}{2},\ y=\frac{\sqrt{14}}{2}$	A1	1.1b
	$\lvert w\rvert^2=\left(2+\frac{\sqrt{14}}{2}\right)^2+\left(\frac{\sqrt{14}}{2}\right)^2$	M1	1.1b
	$=11+2\sqrt{14}$	A1	1.1b
		(4)	
			(9 marks)

Notes

(a)

M1: Circle or arc of a circle with centre in first quadrant and with the circle in all 4 quadrants or arc of circle in quadrants 1 and 2

M1: A "V" shape i.e. with both branches above the *x*-axis and with the vertex on the positive real axis. Ignore any branches below the *x*-axis.

A1: Two half lines that meet on the positive real axis where the right branch intersects the circle or arc of a circle in the first quadrant and the left branch intersects the circle or arc of a circle in the second quadrant but not on the *y*-axis.

M1: Shades the region between the half-lines and within the circle

A1: Cso. A fully correct diagram including 2 marked (or implied by ticks) at the vertex on the real axis with the correct region shaded and all the previous marks scored.

(b)

M1: Identifies a suitable strategy for finding the *x* or *y* coordinate of the point of intersection.

Look for an attempt to solve equations of the form $(x\pm1)^2+(y\pm1)^2=9$ or 3 and $y=\pm x\pm 2$

A1: Correct coordinates for the intersection (there may be other points but allow this mark if the correct coordinates are seen). (The correct coordinates may be implied by subsequent work.)

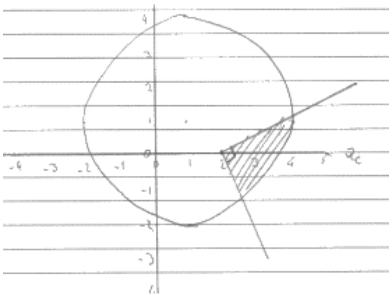
Allow equivalent exact forms and allow as a complex number e.g. $2+\frac{\sqrt{14}}{2}+\frac{\sqrt{14}}{2}\mathrm{i}$

M1: Correct use of Pythagoras on their coordinates (There must be no i's)

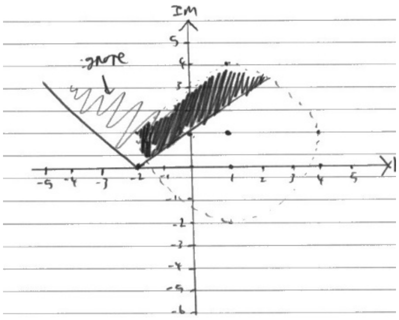
A1: Correct **exact** value by cso

Note that solving $(x-1)^2+(y-1)^2=9,\ \ y=x+2$ gives $x=\frac{\sqrt{14}}{2},\ y=2+\frac{\sqrt{14}}{2}$ and hence the correct answer fortuitously so scores M1A0M1A0

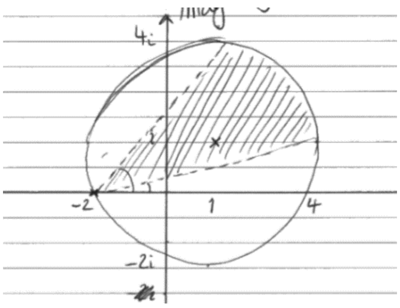
Example marking for 3(a)



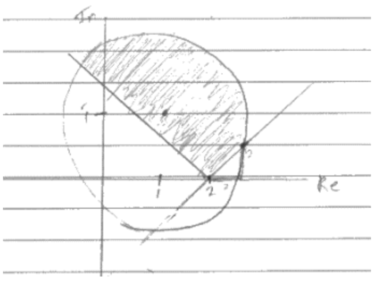
M1: Circle with centre in first quadrant
M0: The branches of the “V” must be above the x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle
A0: Depends on all previous marks



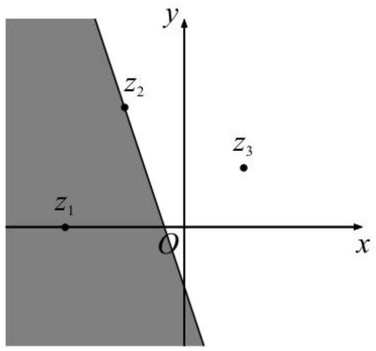
M1: Circle with centre in first quadrant
M0: The vertex of the “V” must be on the positive x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle (BOD)
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M0: The vertex of the “V” must be on the positive x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M1: A “V” shape i.e. with both branches above the x-axis and with the vertex on the positive real axis. Ignore any branches below the x-axis.
A1: Two half lines that meet on the positive real axis where the right branch intersects the circle in the first quadrant and the left branch intersects the circle in the second quadrant.
M1: Shades the region between the half-lines and within the circle
A1: A fully correct diagram including 2 marked at the vertex on the real axis with the correct region shaded and all the previous marks scored.

Question	Scheme	Marks	AOs
6 (a)	Complex roots of a real polynomial occur in conjugate pairs	M1	1.2
	so a polynomial with z_1, z_2 and z_3 as roots also needs z_2* and z_3* as roots, so 5 roots in total, but a quartic has at most 4 roots, so no quartic can have z_1, z_2 and z_3 as roots.	A1	2.4
		(2)	
(b)	$\frac{z_2-z_1}{z_3-z_1}=\frac{-1+2\mathrm{i}-(-2)}{1+\mathrm{i}-(-2)}=\frac{1+2\mathrm{i}}{3+\mathrm{i}}\times\frac{3-\mathrm{i}}{3-\mathrm{i}}=\ldots$	M1	1.1b
	$=\frac{3-\mathrm{i}+6\mathrm{i}+2}{9+1}=\frac{5+5\mathrm{i}}{10}=\frac{1}{2}+\frac{1}{2}\mathrm{i}$ oe	A1	1.1b
	As $\frac{1}{2}+\frac{1}{2}\mathrm{i}$ is in the first quadrant (may be shown by diagram), hence $\arg\left(\frac{z_2-z_1}{z_3-z_1}\right)=\arctan\left(\frac{1/2}{1/2}\right)\left(=\arctan(1)\right)=\frac{\pi}{4}$ *	A1*	2.1
		(3)	
(c)	$\arg\left(\frac{z_2-z_1}{z_3-z_1}\right)=\arg\left(z_2-z_1\right)-\arg\left(z_3-z_1\right)=\arg\left(1+2\mathrm{i}\right)-\arg\left(3+\mathrm{i}\right)$	M1	1.1b
	Hence $\arctan\left(2\right)-\arctan\left(\frac{1}{3}\right)=\frac{\pi}{4}$ *	A1*	2.1
		(2)	
(d)	 Line passing through z_2 and the negative imaginary axis drawn.	B1	1.1b
	Area below and left of their line shaded, where the line must have negative gradient passing through negative imaginary axis but need not pass through z_2	B1	1.1b
	Unless otherwise indicated by the student mark Diagram 1(if used) if there are multiple attempts.		
		(2)	
	(9 marks)		

Notes		
(a)	**M1**	Some evidence that complex roots occur as conjugate pairs shown, e.g. stated as in scheme, or e.g. identifying if $-1+2\text{i}$ is a root then so is $-1-2\text{i}$. Mere mention of complex conjugates is sufficient for this mark.
	A1	A complete argument, referencing that a quartic has at most 4 roots, but would need at least 5 for all of z_1, z_2 and z_3 as roots. There should be a clear statement about the number of roots of a quartic (e.g a quartic has four roots), and that this is not enough for the two conjugate pairs and real root.
(b)	**M1**	Substitutes the numbers in expression and attempts multiplication of numerator and denominator by the conjugate of their denominator or uses calculator to find the quotient. (May be implied.) NB Applying the difference of arguments and using decimals is M0 here.
	A1	Obtains $\frac{1}{2}+\frac{1}{2}\text{i}$. (May be from calculator.) Accepted equivalent Cartesian forms.
	A1*	Uses arctan on their quotient and makes reference to first quadrant or draws diagram to show they are in the first quadrant. to justify the argument.
(c)	**M1**	Applies the formula for the argument of a difference of complex numbers and substitutes values (may go directly to arctans if the arguments have already been established). If used in (b) it must be seen or referred to in (c) for this mark to be awarded. Allow for $\arg\left(z_2-z_1\right)-\arg\left(z_3-z_1\right)$ if z_2-z_1 and z_3-z_1 have been clearly identified in earlier work.
	A1*	Completes the proof clearly by identifying the required arguments and using the result of (b). Use of decimal approximations is A0.
(d)	**B1**	Draws a line through z_2 and passing through negative imaginary axis.
	B1	Correct side of bisector shaded. Allow this mark if the line does not pass through z_2. But it should be an attempt at the perpendicular bisector of the other two points – so have negative gradient and pass through the negative real axis. Ignore any other lines drawn for these two marks.

Question	Scheme		Marks	AOs
7(i) (a)	$\frac{2+3i}{5+i}\times\frac{5-i}{5-i}$	$2+3i=k(1+i)(5+i)=...$	M1	1.1a
	$\frac{10-2i+15i+3}{25+1}$ or $\frac{13+13i}{26}$	$2+3i=k(5+i+5i-1)=...$	dM1	1.1b
	$\frac{1}{2}(1+i)$ cso	$2+3i=k(4+6i)$ therefore $\frac{2+3i}{5+i}=k(1+i)$ where $k=\frac{1}{2}$ cso	A1	2.1
			(3)	
(i)(b)	$n=4$		B1	2.2a
			(1)	
(ii)	$\lvert z\rvert=3$		B1	1.2
	$\arg(z^{10})=10\arg(z)=-\frac{5\pi}{3}\Rightarrow\arg(z)=...\left\{-\frac{\pi}{6}\right\}$ $\arg(z^{10})=10\arg(z)=\frac{\pi}{3}\Rightarrow\arg(z)=...\left\{\frac{\pi}{30}\right\}$		M1	1.1b
	$z=3\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)=...$		M1	2.1
	$z=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ or $a=\frac{3\sqrt{3}}{2}$ and $b=-\frac{3}{2}$		A1	1.1b
			(4)	
	Alternative $a^2+b^2=9$		B1	1.2
	$10\arg\ z=-\frac{5\pi}{3}\Rightarrow\arg\ z=-\frac{5\pi}{3}\div 10$ Or e.g $10\arg(z)=\frac{\pi}{3}\Rightarrow\arg(z)=...\left\{\frac{\pi}{30}\right\}$		M1	1.1b
	Forming and solving simultaneous equations to find a value for a or b $\frac{b}{a}=\arctan\left(-\frac{\pi}{6}\right)\Rightarrow\frac{b}{a}-\frac{\sqrt{3}}{3}\Rightarrow b=-a\frac{\sqrt{3}}{3}$ or $\frac{b}{a}=\arctan\frac{\pi}{30}\Rightarrow b=0.104...a$		M1	2.1
	$z=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ or $a=\frac{3\sqrt{3}}{2}$ and $b=-\frac{3}{2}$		A1	1.1b
			(4)	
				(8 marks)

Notes:

(i) (a)

M1: Selects the process $\frac{2+3\text{i}}{5+\text{i}}\times\frac{5-\text{i}}{5-\text{i}}$

dM1: Evidence of multiplying out brackets

A1: Achieves $\frac{1}{2}(1+\text{i})$ or $\frac{13}{26}(1+\text{i})$ with no errors cso, isw.

Note: Correct answer from no working score no marks

Note: Going from $\frac{13+13\text{i}}{26}$ and then stating $k=\frac{1}{2}$ is A0, they have not shown the form asked for

Alternative

M1: Multiplies across by $(5+\text{i})$ and expands the brackets

dM1: Collects terms

A1: Achieves $2+3\text{i}=k(4+6\text{i})$ and draws the conclusion that therefore $\frac{2+3\text{i}}{5+\text{i}}=k(1+\text{i})$ where $k=\frac{1}{2}$

(i) (b)

B1: Deduces $n=4$ only

(ii)

Note: Send to review any attempts where they are finding additional solutions such as arguments of z is $\frac{(6k-5)\pi}{30}$ For example correctly uses $\arg(z)=\frac{\pi}{30}$

B1 (M1 on ePen): $|z|=3$ can be implied by $a^2+b^2=9$ isw

M1: Uses $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ to find $\arg(z)=-\frac{5\pi}{3}\div 10$ or $\arg(z)=\frac{\pi}{3}\div 10$

M1: Uses $z=\text{their}|z|(\cos(\text{their arg})+\text{i}\sin(\text{their arg}))$ to find the complex number z or values for a or b.
As long as the modulus has changed.

A1: Correct complex number or values for a and b.

Alternative

B1: $a^2+b^2=9$ isw

M1: Uses $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ to find $\arg(z)=-\frac{5\pi}{3}\div 10$

M1: Uses the argument of z find an equation in a and b . Then solve simultaneously to find a value for a or b.
As long as $\sqrt{a^2+b^2}\neq 59049$

A1: Correct complex number or values for a and b.

Note there are other correct answers

$z_1=\frac{3\sqrt{3}}{2}-\frac{3}{2}\text{i}$	$z_6=-\frac{3\sqrt{3}}{2}+\frac{3}{2}\text{i}$
$z_2=\text{awrt } 2.98+\text{awrt } 0.314\text{i}$	$z_7=\text{awrt } -2.98+\text{awrt } -0.314\text{i}$
$z_3=\text{awrt } 2.23+\text{awrt } 2.01\text{i}$	$z_8=\text{awrt } -2.23+\text{awrt } -2.01\text{i}$
$z_4=\text{awrt } 0.624+\text{awrt } 2.93\text{i}$	$z_9=\text{awrt } -0.624+\text{awrt } -2.93\text{i}$
$z_5=\text{awrt } -1.22+\text{awrt } 2.74\text{i}$	$z_{10}=\text{awrt } 1.22+\text{awrt } -2.74\text{i}$

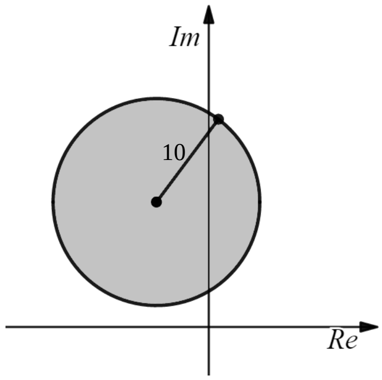
Question	Scheme	Marks	AOs
8(a)	$z* = a - bi$ then $zz* = (a+bi)(a-bi) = \ldots$	M1	1.1b
	$zz* = a^2 + b^2$ therefore, a real number	A1	2.4
		(2)	
(b)	$\frac{z}{z*} = \frac{a+bi}{a-bi} = \frac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \frac{(a^2-b^2)+2abi}{a^2+b^2} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$ or $\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow$ $z^2 = 14 + 8\sqrt{2}i$ or $a + bi = \left(\frac{7}{9} + \frac{4\sqrt{2}i}{9}\right)(a - bi) = ..+..i$	M1	1.1b
	Forms two equations from $a^2 + b^2 = 18$ or $\frac{a^2-b^2}{18} = \frac{7}{9}$ or $\frac{a^2-b^2}{a^2+b^2} = \frac{7}{9}$ or $\frac{2ab}{18} = \frac{4\sqrt{2}}{9}$ or $\frac{2ab}{a^2+b^2} = \frac{4\sqrt{2}}{9}$ or $a = \frac{7}{9}a + \frac{4\sqrt{2}}{9}b$ oe	M1 A1	3.1a 1.1b
	Solves the equations simultaneously e.g. $a^2 + b^2 = 18$ and $a^2 - b^2 = 14$ leading to a value for *a* or *b*	dM1	1.1b
	$z = \pm(4 + \sqrt{2}i)$	A1	2.2a
		(5)	
			(7 marks)

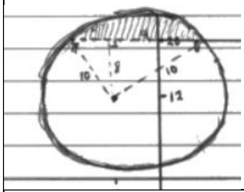
Notes:

(a)(i)
M1: States or implies $z* = a - bi$ and finds an expression for $zz*$
A1: Achieves $zz* = a^2 + b^2$ and draws the conclusion that $zz*$ is a real number. Accept $\in \mathbb{R}$ as conclusion, but not just "no imaginary part".

(b)
M1: Starts the process of solving by using the conjugate to form an equation with real denominators, and without z^* or i^2 in the equation. Accept as shown in scheme, or may multiply through by $a - bi$ and expand and gather terms. May be implied by correct extraction of equation(s).
M1: Uses the given information to form two equations involving *a* and *b* at least one of which includes both. It must involve equating real or imaginary parts of $\frac{z}{z*} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$
A1: Any two correct equations arising from use of both given facts. (Note: if multiplying through by $a - bi$ then equating real and imaginary terms gives the same equation.)
dM1: Dependent on previous method mark, solves the equations to find a value for either *a* or *b*.
A1: Deduces the correct complex numbers and no extras. Do not accept $\pm 4 \pm \sqrt{2}i$
Note: it is possible to solve via polar coordinates, but unlikely to succeed. If you see responses you think are worthy of credit but are unsure how to mark, use review. Example solutions shown below.

(b) **Alt**	$\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow z^2 = 14 + 8\sqrt{2}i$ or let arg $z = \theta$. then $\frac{z}{z*} = \frac{re^{i\theta}}{re^{-i\theta}} = e^{2i\theta} = \cos 2\theta + \text{i}\sin 2\theta$	M1	1.1b
	$z^2 = 18(\cos\alpha + \text{i}\sin\alpha)$where $\tan\alpha = \frac{4\sqrt{2}}{7} \Rightarrow z = \pm\sqrt{18}\left(\cos\frac{1}{2}\alpha + \text{i}\sin\frac{1}{2}\alpha\right)$Or $\cos 2\theta + \text{i}\sin 2\theta = \frac{7}{9} + \frac{4\sqrt{2}i}{9} \Rightarrow 2\cos^2\theta - 1 = \frac{7}{9}, 2\sin\theta\cos\theta = \frac{4\sqrt{2}}{9}$	M1 A1	1.1b 1.1b
	$\cos\frac{1}{2}\alpha = \sqrt{\frac{1}{2}(1+\cos\alpha)} = \sqrt{\frac{1}{2}\left(1+\frac{7}{9}\right)} =\dots$ and $\sin\frac{1}{2}\alpha = \sqrt{\frac{1}{2}(1-\cos\alpha)} = \sqrt{\frac{1}{2}\left(1-\frac{7}{9}\right)} =\dots$or $\Rightarrow \cos\theta = \frac{2\sqrt{2}}{3}, \sin\theta = \frac{1}{3}, r = \lvert z\rvert = \sqrt{zz^*} = \sqrt{18}$	dM1	3.1a
	$z = \pm\left(4 + \sqrt{2}i\right)$	A1	2.2a
		(5)	

Question	Scheme	Marks	AOs
9(a)(i)	$(-5, 12)$ or $-5 + 12\text{i}$	B1	1.1b
(ii)	$r = 10$	B1	1.1b
		(2)	
(b)		B1ft	1.1b
		(1)	
(c)	$OC = \sqrt{5^2 + 12^2}$	M1	1.1b
	$\|z\|_{\max} = \sqrt{5^2 + 12^2} + 10$	M1	3.1a
	$= 23$	A1	1.1b
		(3)	
	Alternative $y = -\frac{12}{5}x$ and $(x+5)^2 + (y-12)^2 = 10^2$ $(x+5)^2 + \left(-\frac{12}{5}x - 12\right)^2 = 10^2 \Rightarrow x = ...$ Or $\tan\theta = \frac{5}{12} \Rightarrow \theta = ...\{22.61..\}\quad x = -5 - 10\sin\theta = ...$	M1	1.1b
	$x = '-\frac{115}{13}' \Rightarrow y = ...\left\{\frac{276}{13}\right\}$ $\|z\|_{\max} = \sqrt{\left(-\frac{115}{13}\right)^2 + \left(\frac{276}{13}\right)^2}$ Or $x = '-\frac{15}{13}' \Rightarrow y = ...\left\{\frac{36}{13}\right\}$ $\|z\|_{\max} = \sqrt{\left(-\frac{15}{13}\right)^2 + \left(\frac{36}{13}\right)^2} + 2\times 10$	M1	3.1a
	$= 23$	A1	1.1b

		(3)	
(d)	$\{z : 0 \leqslant \arg(z+5-20\text{i}) \leqslant \pi\} \Rightarrow y = 20$ $\Rightarrow (x+5)^2 + 8^2 = 100 \Rightarrow x = \ldots$ **AND finds an angle** $\cos\theta = \dfrac{10^2+10^2-12^2}{2\times10\times10} = 0.28$ Or $a^2 = 10^2 - 8^2 \Rightarrow a = \ldots\{6\}\ \ \sin\left(\dfrac{1}{2}\theta\right) = \dfrac{6}{10}$ Or $\cos\left(\dfrac{1}{2}\theta\right) = \dfrac{8}{10}$  	M1	3.1a
	$\theta = 1.287\ldots \text{or } 73.7^\circ$ or $\dfrac{1}{2}\theta = 0.6435\ldots \text{or } 36.9^\circ$	A1	1.1b
	Area $= \dfrac{1}{2}\times10^2\times\theta - \dfrac{1}{2}\times12\times8$ angle in radians Area $= \pi\times10^2\times\dfrac{\theta}{360} - \dfrac{1}{2}\times12\times8$ angle in degrees or Area $= \dfrac{1}{2}\times10^2\times\theta - \dfrac{1}{2}\times10\times10\times\sin\theta$ angle in radians Area $= \pi\times10^2\times\dfrac{\theta}{360} - \dfrac{1}{2}\times10\times10\times\sin\theta$ angle in degrees Or Area $= 2\left[\dfrac{1}{2}\times10^2\times\theta - \dfrac{1}{2}\times8\times6\right]$	M1	3.1a
	= awrt 16.4	A1	1.1b
		(4)	
			(10 marks)

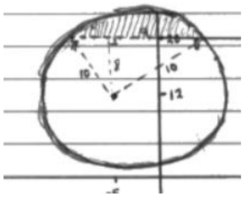
Notes

(a)(i)
B1: Correct centre, condone $(-5, 12\text{i})$

(a)(ii)
B1: Correct radius

(b)
B1ft: A circle drawn with the inside shaded. Follow through their centre and radius.
The centre must be in the correct quadrant and intercept the axes as appropriate.
If they have the correct centre and radius then the centre must be in the second quadrant and the circle must **only** intercept the imaginary-axis. If diagram is correct consider this a restart B1.

(c)
M1: Calculates the distance from *O* to the centre of their circle.
M1: Fully correct strategy for the maximum. E.g. Finds distance from *O* to centre of their circle and adds their radius.
A1: Correct answer.
Correct answer with no working and following a correct centre and radius scores M1M1A1
Alternative
M1: Finds the equation of the line from the origin to centre and the Cartesian equation of the circle. Solves simultaneously to find the *x* values (or *y*) where the line intersects the circle.
M1: Selects the *x* coordinate to give the largest distance, find the corresponding *y* value and then the distance from the origin.
Selects the *x* coordinate to give the smallest distance, find the corresponding *y* value and then adds on 2 times the radius.
A1: Correct answer
(d)
M1: Recognises that $\{z : 0 \leqslant \arg(z+5-20\text{i}) \leqslant \pi\}$ represents the line $y = 20$ and uses this with the circle in an attempt to find the angle or half angle at the centre.



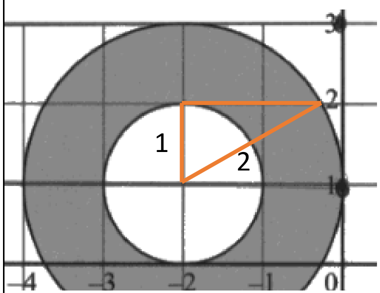
A1: Correct value for the angle or half angle at the centre.
M1: Fully correct strategy for the area of the segment using their values.
A1: Awrt 16.4
Note: finding the area of the major segment using $100\pi - 16.4 = ...$ scores M1A1M1A0

Question	Scheme	Marks	AOs
10	$(x-3)^2+(y-5)^2=(2r)^2$ and $y=-x+2$	B1	1.1b
	$(x-3)^2+(-x+2-5)^2=(2r)^2$ or $(-y+2-3)^2+(y-5)^2=(2r)^2$	M1	3.1a
	$2x^2+18-4r^2=0$ or $2y^2-8y+26-4r^2=0$	A1	1.1b
	$b^2-4ac>0 \Rightarrow 0^2-4(2)(18-4r^2)>0 \Rightarrow r>...$ or $x^2=9-2r^2 \Rightarrow 9-2r^2>0 \Rightarrow r>...$ or $b^2-4ac>0 \Rightarrow (-8)^2-4(2)(26-4r^2)>0 \Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2 \Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
	Alternative Using a circle with centre (3, 5) and radius $2r$ and $y=-x+2$	B1	1.1b
	$y-5=1(x-3) \Rightarrow y=x+2$ $x+2=-x+2 \Rightarrow x=...$	M1	3.1a
	$(0,2)$	A1	1.1b
	$2r>\sqrt{(3-0)^2+(5-2)^2} \Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2 \Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
		(7)	
			(7 marks)

Notes:

B1: Correct equations for each loci of points
M1: A complete method to find a 3TQ involving one variable using equations of the form
$(x\pm3)^2+(y\pm5)^2=(2r)^2$ or $2r^2$ or r^2 and $y=\pm x\pm2$
A1: Correct quadratic equation

dM1: Dependent on previous method mark. A complete method uses $b^2-4ac>0$ or rearranges to find $x^2=\mathrm{f}(r)$and uses $\mathrm{f}(r)>0$ to the minimum value of *r*. **M1:** Realises there will be an upper limit for *r* and uses Pythagoras theorem $(2r)^2=(y\text{ coord of centre})^2+(x\text{ coord of centre}-2)^2$ condone$(r)^2=(y\text{ coord of centre})^2+(x\text{ coord of centre}-2)^2$ **A1:** One correct limit, either $\frac{3\sqrt{2}}{2}<r$ or $r<\frac{\sqrt{26}}{2}$ o.e. **A1:** Fully correct inequality
Alternative **B1:** Using a circle with centre (3, 5) and radius 2*r* and $y=-x+2$ **M1:** A complete method to find the point of intersection of the line $y=\pm x\pm 2$ and circle where the line is a tangent to the circle. **A1:** Correct point of intersection **dM1:** Finds the distance between the point of intersection and the centre and uses this to find the minimum value of *r*. Condone radius of r. **M1:** Realises there will be an upper limit for *r* and uses Pythagoras theorem $(2r)^2=(y\text{ coord of centre})^2+(x\text{ coord of centre}-2)^2$ **A1:** One correct limit, either $\frac{3\sqrt{2}}{2}<r$ or $r<\frac{\sqrt{26}}{2}$ o.e. **A1:** Fully correct inequality

Question	Scheme	Marks	AOs
11(a)	$a = 1, d = 2$	B1	1.1b
	$b = 2$	B1	1.1b
	$c = -1$	B1	1.1b
		(3)	
(b)	$\lvert z - \mathrm{i}\rvert = \lvert z - 3\mathrm{i}\rvert \Rightarrow y = 2$	B1	2.2a
	Area between the circles $= \pi \times 2^2 - \pi \times 1^2$	M1	1.1a
	 Angle subtended at centre = $2 \times \cos^{-1}\left(\frac{1}{2}\right)$ Alternatively $(x+2)^2 + (y-1)^2 = 4,\ y = 2 \Rightarrow x = ...$ Or $x = \sqrt{2^2 - 1^2}$ Leading to Angle subtended at centre = $2 \times \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$	M1	3.1a
	Segment area = $\frac{1}{2} \times \frac{2\pi}{3} \times 2^2 - \frac{1}{2} \times 2^2 \times \sin\left(\text{"}\frac{2\pi}{3}\text{"}\right)\left\{= \frac{4}{3}\pi - \sqrt{3}\right\}$	M1 A1	2.1 1.1b
	Area of Q: $\pi \times 2^2 - \pi \times 1^2 - \left(\frac{1}{2} \times \frac{2\pi}{3} \times 2^2 - \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right)\right)$	M1	3.1a
	$= \frac{5\pi}{3} + \sqrt{3}$	A1	1.1b
		(7)	
			(10 marks)

Notes

(a)

B1: Correct values for *a* and *d*

B1: Correct value for *b*

B1: Correct value for *c*

(b)

B1: Deduces that $\lvert z - \mathrm{i}\rvert = \lvert z - 3\mathrm{i}\rvert$ is a perpendicular bisector with equation $y = 2$, this may be drawn on a diagram.

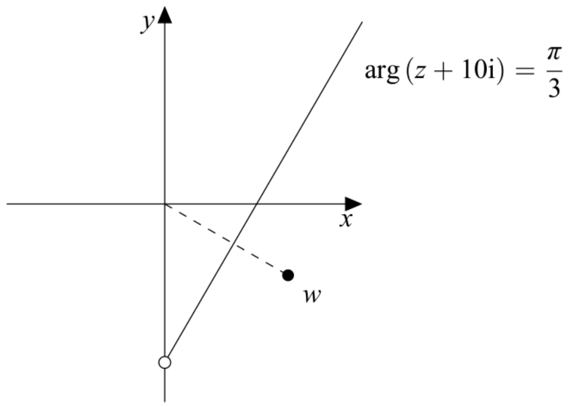
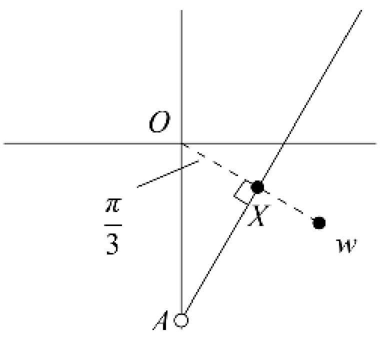
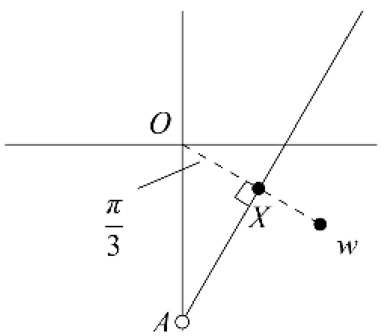
M1: Selects the correct procedure to find the area of the large circle – the area of the small circle.

M1: Correct method to find the angle at the centre (or half this angle).

Recognises that the hypotenuse is the radius of the larger circle and the adjacent is the radius if the smaller circle and using cosine

Alternatively find where the perpendicular bisector intersects the larger circle so uses their $y = 2$ and the equation of the larger circle in an attempt to establish the x values for the intersection points or uses geometry and Pythagoras to identify the required length and then uses tangent.

M1: Correct method for the area of the minor segment (allow equivalent work)

Question	Scheme	Marks	AOs
12(a)	$\lvert w\rvert=\sqrt{\left(4\sqrt{3}\right)^2+\left(-4\right)^2}=8$	B1	1.1b
	$\arg w=\arctan\left(\dfrac{\pm 4}{4\sqrt{3}}\right)=\arctan\left(\pm\dfrac{1}{\sqrt{3}}\right)$	M1	1.1b
	$=-\dfrac{\pi}{6}$	A1	1.1b
	So $(w=)8\left(\cos\left(-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{6}\right)\right)$	A1	1.1b
		(4)	
(b)	 (i) w in 4th quadrant with either $\left(4\sqrt{3},-4\right)$ seen or $-\dfrac{\pi}{4}<\arg w<0$	B1	1.1b
	(ii) half line with positive gradient emanating from imaginary axis.	M1	1.1b
	The half line should pass between O and w starting from a point on the imaginary axis below w	A1	1.1b
		(3)	
(c)	 ΔOAX is right angled at X so $OX=10\sin\dfrac{\pi}{6}=5$ (oe)	M1	3.1a
	So shortest distance is $WX=OW-OX=$ '8' $-5=\ldots$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 1  A complete method to find the coordinates of X. Finds the equation of the line from O to w, $y=-\dfrac{1}{\sqrt{3}}x$ and the equation of the half line $y=\sqrt{3}x-10$, solves to find the point of intersection $X\left(\dfrac{5\sqrt{3}}{2},-\dfrac{5}{2}\right)$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\dfrac{5\sqrt{3}}{2}\right)^2+\left(-4--\dfrac{5}{2}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 2	M1	3.1a

	Finds the length $AW = \sqrt{\left(4\sqrt{3}-0\right)^2 + \left(-4--10\right)^2} = \ldots\left\{\sqrt{84}\right\}$ Finds the angle between the horizontal and the line AW $= \tan^{-1}\left(\dfrac{-4--10}{4\sqrt{3}}\right) = \ldots\left\{0.7137\ldots\text{radians or } 40.89\ldots^\circ\right\}$		
	Finds the length of $WX = \sqrt{84}\times\sin\left(\dfrac{\pi}{3}-0.7137\right)=\ldots$ Or $=\sqrt{84}\times\sin\left(60-40.89\right)=\ldots$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 3 Vector equation of the half line $r = \begin{pmatrix}0\\-10\end{pmatrix} + \lambda\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$ $XW = \begin{pmatrix}4\sqrt{3}-\lambda\\-4-\lambda\sqrt{3}-(-10)\end{pmatrix}$ Then either $\begin{pmatrix}4\sqrt{3}-\lambda\\6-\lambda\sqrt{3}\end{pmatrix}\bullet\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = 4\sqrt{3}-\lambda+6\sqrt{3}-3\lambda = 0 \Rightarrow \lambda = \ldots\left\{\dfrac{5}{2}\sqrt{3}\right\}$ $r = \begin{pmatrix}0\\-10\end{pmatrix} + \dfrac{5}{2}\sqrt{3}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = \ldots$ Or $XW^2 = \left(4\sqrt{3}-\lambda\right)^2 + \left(6-\lambda\sqrt{3}\right)^2 = 48-8\lambda\sqrt{3}+\lambda^2+36-12\lambda\sqrt{3}+3\lambda^2$ $xw^2 = 84-20\lambda\sqrt{3}+4\lambda^2$ leading to $\dfrac{\mathrm{d}\left(XW^2\right)}{\mathrm{d}\lambda} = -20\sqrt{3}+8\lambda = 0 \Rightarrow \lambda = \ldots$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\dfrac{5\sqrt{3}}{2}\right)^2 + \left(-4--\dfrac{5}{2}\right)^2}$ Or $XW = \sqrt{\left(4\sqrt{3}-'\dfrac{5}{2}\sqrt{3}'\right)^2 + \left(6-'\dfrac{5}{2}\sqrt{3}'\sqrt{3}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
		(3)	
(10 marks)			

Notes:

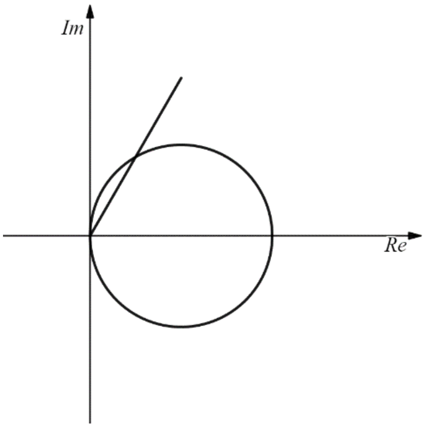
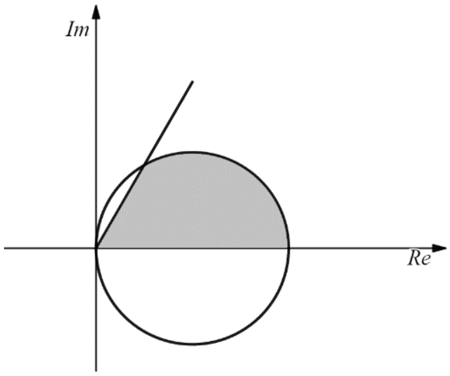
(a)

B1: Correct modulus

M1: Attempts the argument. Allow for $\arctan\left(\dfrac{\pm 4}{\pm 4\sqrt{3}}\right)$ or equivalents using the modulus (may be in wrong quadrant for this mark).

A1: Correct argument $-\dfrac{\pi}{6}$ (must be in fourth quadrant but accept $\dfrac{11\pi}{6}$ or other difference of 2π for this mark).

A1: Correct expression found for w, in the correct form, must have positive r= 8 and $\theta = -\frac{\pi}{6}$. **Note:** using degrees B1 M1 A0 A0
(b)(i)&(ii) **B1:** w plotted in correct quadrant with either the correct coordinate clearly seen or above the line $y = -x$ **M1:** Half line drawn starting on the imaginary axis away from O with positive gradient (need not be labelled) **A1:** Sketch on **one diagram**– both previous marks must have been scored and the half line should pass between O and w starting from a point on the imaginary axis below w. (You may assume it starts at −10i unless otherwise stated by the candidate) **Note:** If candidates draw the loci on separate diagrams the maximum they can score is B1 M1 A0
(c) **M1:** Formulates a correct strategy to find the shortest distance, e.g. uses right angle OXA where X is where the lines meet and proceeds at least as far as OX. **M1:** Full method to achieve the shortest distance, e.g. for $WX = OW - OX$. **A1: cao** shortest distance is 3 **Alternative 1:** **M1:** Uses a correct method to find the equation of the line from O to w, $y = -\frac{1}{\sqrt{3}}x$ and the equation of the half line $y = \sqrt{3}x - 10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$ If the incorrect gradient(s) is used with no valid method seen this is M0 **M1**: Finds the length $WX = \sqrt{\left(\text{their}\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} -\frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets. **A1: cao** shortest distance is 3 **Alternative 2:** **M1:** Uses a correct method to find the length AW and a correct method to find the angle between the horizontal and the line AW **M1**: Finds the length of $WX = \text{their}\sqrt{84} \times \sin\left(\frac{\pi}{3} - \text{their } 0.7137\right) = ...$ **A1: cao** shortest distance is 3 **Alternative 3** **M1:** Finds the vector equation of the half line, then XW. **Then either:** Sets dot product XW and the line = 0 and solves for λ. Substitutes their λ into the equation of the half line to find the point of intersection. **Or** finds the length of XW and differentiates, set = 0 and solve for λ **M1**: Finds the length $WX = \sqrt{\left(\text{their}\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} -\frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets. Or substitutes their value for λ into the length of *(d)* **A1: cao** shortest distance is 3

Question	Scheme	Marks	AOs
13(a)		M1 A1 M1 A1	1.1b 1.1b 1.1b 1.1b
		(4)	
(b)		B1ft	2.3
		(1)	

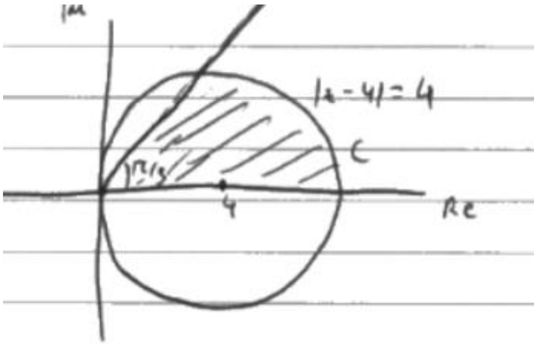
Notes

(a)
M1: Circle drawn with centre on the real axis
Look for real axis acting as a line of symmetry of the circle.
A1: Circle in the correct position with the imaginary axis as a tangent
Centre need not be labelled for either mark.
M1: Half line starting at the origin, must be in the first quadrant. Do not award if their line continues into the third quadrant
A1: Fully correct diagram that requires

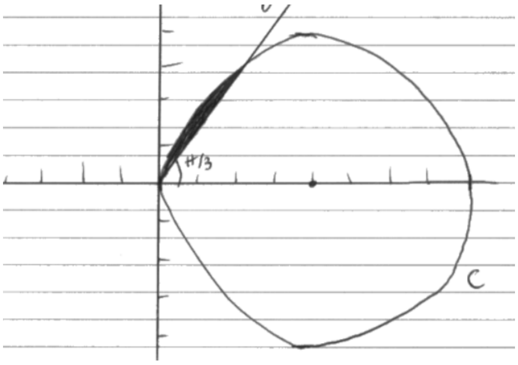
- A circle in the correct position
- A half line intersecting the circle at the origin and in the first quadrant
- The x coordinate of the intersection in the first quadrant must be to the left of the centre of the circle

(b)
B1ft: Shades the region in their circle above the real axis and below the half line
For this mark to be awarded their line must intersect the circle.

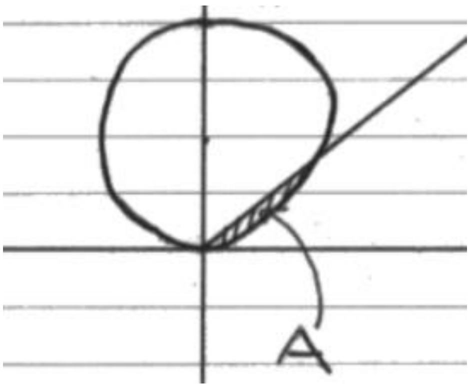
Q5 (a) and (b) examples



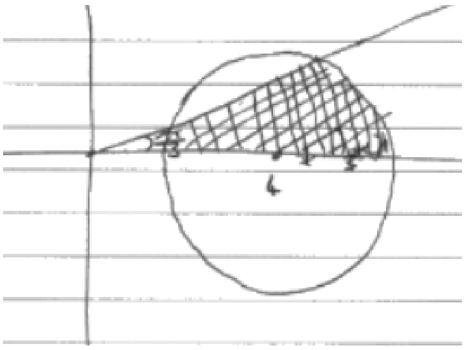
a) M1A1M1A1 b) B1



a) M1A1M1A1 b) B0



a) M0A0M1A0 b) B1



a) M1A0M1A0 b) B1

(c)	$(x-4)^2+y^2=16,\quad y=\sqrt{3}x \Rightarrow x^2-8x+16+3x^2=16 \Rightarrow x=...$ $x=2,\ \ y=2\sqrt{3}$	M1 A1	3.1a 1.1b
	$\frac{1}{2}\pi\times4^2-\left(\frac{1}{2}\times4^2\times\frac{\pi}{3}-\frac{1}{2}\times4^2\times\sin\frac{\pi}{3}\right)$ or $\frac{1}{2}\times4^2\times\frac{2\pi}{3}+\frac{1}{2}\times4\times2\sqrt{3}$ or $\frac{1}{2}\times4^2\times\frac{2\pi}{3}+\frac{1}{2}\times4\times4\times\frac{\sqrt{3}}{2}$	dM1	3.1a
	$=\frac{16}{3}\pi+4\sqrt{3}$	A1	1.1b
		(4)	
	Alternative 1 for part (c)		
	$x=4\cos\frac{\pi}{3},\ \ y=4\sin\frac{\pi}{3}$	M1 A1	3.1a 1.1b
	$\frac{1}{2}\pi\times4^2-\left(\frac{1}{2}\times4^2\times\frac{\pi}{3}-\frac{1}{2}\times4^2\times\sin\frac{\pi}{3}\right)$ or $\frac{1}{2}\times4^2\times\frac{2\pi}{3}+\frac{1}{2}\times4\times2\sqrt{3}$ or $\frac{1}{2}\times4^2\times\frac{2\pi}{3}+\frac{1}{2}\times4\times4\times\frac{\sqrt{3}}{2}$	dM1	3.1a
	$=\frac{16}{3}\pi+4\sqrt{3}$	A1	1.1b
		(4)	
	Alternative 2 for part (c)		
	Deduces $\frac{\pi}{3}$ means there is an equilateral triangle of length 4 4 and $\sin\frac{\pi}{3}$ oe are seen or used in their workings	M1 A1	3.1a 1.1b
	$\frac{1}{2}\pi\times4^2-\left(\frac{1}{2}\times4^2\times\frac{\pi}{3}-\frac{1}{2}\times4^2\times\sin\frac{\pi}{3}\right)$ or $\frac{1}{2}\times4^2\times\frac{2\pi}{3}+\frac{1}{2}\times4\times2\sqrt{3}$ or	dM1	3.1a

	$\frac{1}{2}\times4^2\times\frac{2\pi}{3}+\frac{1}{2}\times4\times4\times\frac{\sqrt{3}}{2}$		
	$=\frac{16}{3}\pi+4\sqrt{3}$	A1	1.1b
		(4)	
	Alternative 3 for part (c) Polar coordinates		
	$(x-4)^2+y^2=16, \Rightarrow r^2=8r\cos\theta \Rightarrow r=8\cos\theta$ Area = $\int\frac{1}{2}r^2\ \mathrm{d}\theta = \int\frac{1}{2}.64\cos^2\theta\ \mathrm{d}\theta=\int\ (16+16\cos2\theta)\,\mathrm{d}\theta$	M1 A1	3.1a 1.1b
	$=\ [16\theta+8\sin2\theta]_0^{\frac{\pi}{3}}=...$	dM1	3.1a
	$=\frac{16}{3}\pi+4\sqrt{3}$	A1	1.1b
		(4)	
			(9 marks)

Notes

(c)
This is not for finding their shaded area of their diagram in part (a) but is for a correct process for finding the correct required area.

M1: Correct strategy for identifying both coordinates of the point of intersection.
Award for substituting a line of the form $y=kx$ into the equation of a circle $(x-4)^2+y^2=16$ then proceeds to find a value for x from a quadratic, where $x\neq0$ and then find their y coordinate.
A1: Correct coordinates, either written separately, or as $(2,2\sqrt{3})$ or $2+\ 2\sqrt{3}\,\mathrm{i}$
dM1: Fully correct strategy for the area, must be consistent with a half line making an angle of $\frac{\pi}{3}$ with the real axis.
Can be found by subtracting the area of a segment from the area of a semicircle or by adding the area of a sector to the area of a triangle.
The candidate may do combinations of semicircles, triangles and sectors so look carefully for the method.
A1: Correct answer in the required form.

Alternative 1

M1: Deduces that since the half line makes an angle of $\frac{\pi}{3}$ with the real axis, the horizontal and vertical distances from the origin to the point of intersection are $4\cos\frac{\pi}{3}$ and $4\sin\frac{\pi}{3}$.

A1: $4\cos\frac{\pi}{3}$ and $4\sin\frac{\pi}{3}$ oe are seen or used in their workings
dM1: Fully correct strategy for the area, must be consistent with a half line making an angle of $\frac{\pi}{3}$ with the real axis.
Can be found by subtracting the area of a segment from the area of a semicircle or by adding the area of a sector to the area of a triangle.

Question	Scheme	Marks	AOs
14(a)	Centre of circle C is (1, –1)	B1	1.1b
	$r=\sqrt{(5-1)^2+(-4+1)^2}=5$ or $r=\sqrt{(-3-1)^2+(2+1)^2}=5$ or $r=\frac{1}{2}\sqrt{(-3-5)^2+(2+4)^2}=5$	M1	3.1a
	$\lvert z-1+\mathrm{i}\rvert=5$ or $\lvert z-(1-\mathrm{i})\rvert=5$	A1	2.5
		(3)	
(b)	$(x-1)^2+(y+1)^2=25,\quad (x-2)^2+(y-3)^2=4$ $x^2-2x+1+y^2+2y+1=25$ $x^2-4x+4+y^2-6y+9=4$ $\Rightarrow 2x+8y=32$	M1	3.1a
	$(16-4y)^2-4(16-4y)+4+y^2-6y+9=4$ or $x^2-4x+4+\left(\frac{16-x}{4}\right)^2-6\left(\frac{16-x}{4}\right)+9=4$	M1	1.1b
	$17y^2-118y+201=0$ or $17x^2-72x+16=0$	A1	1.1b
	$17y^2-118y+201=0\Rightarrow(17y-67)(y-3)=0\Rightarrow y=\frac{67}{17},3$ or $17x^2-72x+16=0\Rightarrow(17x-4)(x-4)=0\Rightarrow x=\frac{4}{17},4$	M1	1.1b
	$y=\frac{67}{17},3\Rightarrow x=\frac{4}{17},4$ or $x=\frac{4}{17},4\Rightarrow y=\frac{67}{17},3$	M1	2.1
	$4+3\mathrm{i},\ \frac{4}{17}+\frac{67}{17}\mathrm{i}$	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)
B1: Correct coordinates of centre
M1: Fully correct strategy for identifying the radius. If the diameter is calculated this must be halved to achieve this mark.
A1: Correct equation using the required notation
(b)
M1: Begins the process of finding z_1 and z_2 by using the Cartesian equations to obtain the equation of the line of intersection
M1: Substitutes back into the equation of one of the circles to obtain an equation in one variable
A1: Correct 3 term quadratic
M1: Solves their 3TQ
M1: Substitutes to find values of the other variable to complete the process of finding z_1 and z_2
A1: Correct complex numbers

Question	Scheme	Marks	AOs
15(a)	Examples: $\begin{pmatrix}\cos 120 & -\sin 120\\ \sin 120 & \cos 120\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix} = \ldots$ **or** $(6+2\mathrm{i})\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\mathrm{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\mathrm{e}^{\mathrm{i}\arctan\left(\frac{2}{6}\right)}\mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{3}\right)}$	M1	3.1a
	$\left(-3-\sqrt{3}\right)$ **or** $\left(3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
	$\left(-3-\sqrt{3}\right)+\left(3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
	Examples: $\begin{pmatrix}\cos 240 & -\sin 240\\ \sin 240 & \cos 240\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix} = \ldots$ **or** $(6+2\mathrm{i})\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\mathrm{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{4\pi}{3}\right)+\mathrm{i}\sin\left(\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)+\mathrm{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\mathrm{e}^{\mathrm{i}\arctan\left(\frac{2}{6}\right)}\mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{3}\right)}$	M1	3.1a
	$\left(-3+\sqrt{3}\right)$ **or** $\left(-3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
	$\left(-3+\sqrt{3}\right)+\left(-3\sqrt{3}-1\right)\mathrm{i}$	A1	1.1b
		(6)	
(b) Way 1	Area $ABC = 3\times\frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$ or Area $AOB = \frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$	M1	2.1
	Area $DEF = \frac{1}{4}ABC$ or $\frac{3}{4}AOB$	**d**M1	3.1a
	$=\frac{3}{8}\times 40\times\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$	A1	1.1b
		(3)	

(b) Way 2	$D\left(\frac{3-\sqrt{3}}{2},\frac{3\sqrt{3}+1}{2}\right)$ $OD=\sqrt{\left(\frac{3-\sqrt{3}}{2}\right)^2+\left(\frac{3\sqrt{3}+1}{2}\right)^2}=\sqrt{10}$ Area $DOF=\frac{1}{2}\sqrt{10}\sqrt{10}\sin 120°$	M1	2.1
	Area $DEF = 3DOF$	**d**M1	3.1a
	$=3\times\frac{1}{2}\times\sqrt{10}\sqrt{10}\times\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 3	$AB=\sqrt{\left(9+\sqrt{3}\right)^2+\left(3-3\sqrt{3}\right)^2}=\sqrt{120}$ Area $ABC=\frac{1}{2}\sqrt{120}\sqrt{120}\sin 60°\left(=30\sqrt{3}\right)$	M1	2.1
	Area $DEF=\frac{1}{4}ABC$	**d**M1	3.1a
	$=\frac{1}{4}\times 30\sqrt{3}=\frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 4	$D\left(\frac{3-\sqrt{3}}{2},\frac{3\sqrt{3}+1}{2}\right)$, $E(-3,-1)$, $F\left(\frac{3+\sqrt{3}}{2},\frac{-3\sqrt{3}+1}{2}\right)$ $DE=\sqrt{\left(\frac{3-\sqrt{3}}{2}+3\right)^2+\left(\frac{3\sqrt{3}+1}{2}+1\right)^2}\left(=\sqrt{30}\right)$ Area $DEF=\frac{1}{2}\sqrt{30}\sqrt{30}\sin 60°$	M1 **d**M1	2.1 3.1a
	$=\frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 5	Area $ABC=\frac{1}{2}\begin{vmatrix}6 & -3-\sqrt{3} & \sqrt{3}-3 & 6\\ 2 & 3\sqrt{3}-1 & -3\sqrt{3}-1 & 2\end{vmatrix}=30\sqrt{3}$	M1	2.1
	Area $DEF=\frac{1}{4}ABC$	**d**M1	3.1a
	$=\frac{1}{4}\times 30\sqrt{3}=\frac{15\sqrt{3}}{2}$	A1	1.1b

(9 marks)

Notes

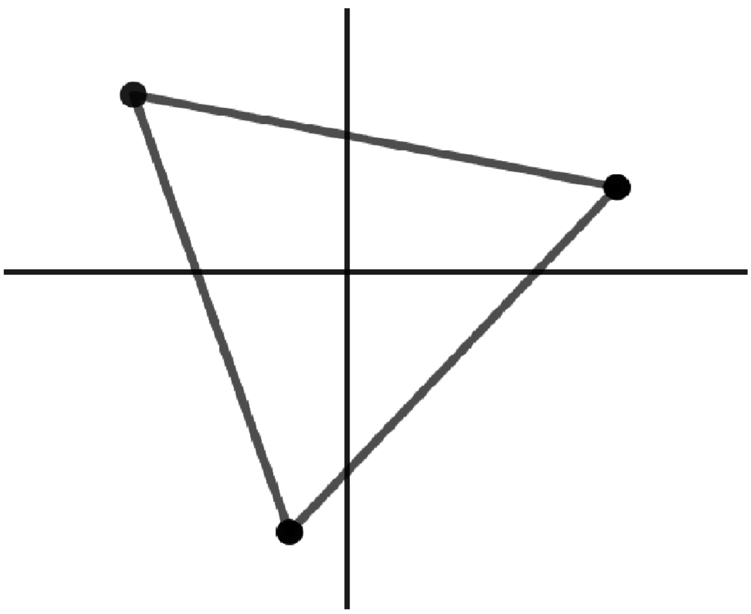
(a)
M1: Identifies a suitable method to rotate the given point by 120° (or equivalent) about the origin. May see equivalent work with modulus/argument or exponential form e.g. an attempt to multiply by $\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3}$ or $\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$
A1: Correct real part or correct imaginary part
A1: Completely correct complex number
M1: Identifies a suitable method to rotate the given point by 240° (or equivalent e.g. rotate their *B* by 120°) about the origin

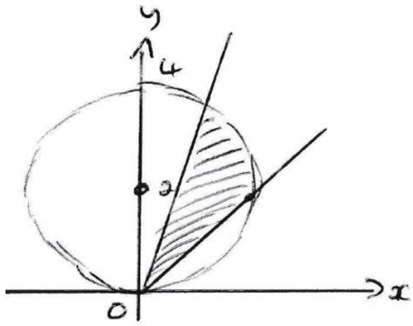
May see equivalent work with modulus/argument or exponential form e.g. an attempt to multiply 6 + 2i by $\cos\frac{4\pi}{3}+\mathrm{i}\sin\frac{4\pi}{3}$ or $\mathrm{e}^{\frac{4\pi}{3}\mathrm{i}}$ or their *B* by $\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3}$ or $\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$
A1: Correct real part or correct imaginary part
A1: Completely correct complex number
(b)

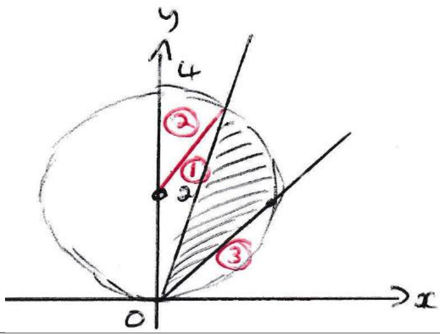
In general, the marks in (b) should be awarded as follows:
M1: Attempts to find the area of a relevant triangle
dM1: completes the problem by multiplying by an appropriate factor to find the area of *DEF*
Dependent on the first method mark
A1: Correct exact area
In some cases it may not be possible to distinguish the 2 method marks. In such cases they can be awarded together for a direct method that finds the area of *DEF*

Examples:

Way 1
M1: A correct strategy for the area of a relevant triangle such as *ABC* or *AOB*
dM1: Completes the problem by linking the area of *DEF* correctly with *ABC* or with *AOB*
A1: Correct value
Way 2
M1: A correct strategy for the area of a relevant triangle such as *DOF*
dM1: Completes the problem by linking the area of *DEF* correctly with *DOF*
A1: Correct value
Way 3
M1: A correct strategy for the area of a relevant triangle such as *ABC*
dM1: Completes the problem by linking the area of *DEF* correctly with *ABC*
A1: Correct value
Way 4
M1**d**M1: A correct strategy for the area of *DEF*. Finds 2 midpoints and attempts one side of *DEF* and uses a correct triangle area formula. By implication this scores both M marks.
A1: Correct value
Way 5
M1: A correct strategy for the area of *ABC* using the "shoelace" method.
dM1: Completes the problem by linking the area of *DEF* correctly with *ABC*
A1: Correct value
Note the marks in (b) can be scored using inexact answers from (a) and the A1 scored if an exact area is obtained.



Question	Scheme	Marks	AOs
16(i)	$\lvert z\rvert = \sqrt{6^2+6^2}$ $= 6\sqrt{2}$ $\arg(z) = \tan^{-1}\left(\frac{6}{6}\right) = \frac{\pi}{4}$ $\therefore\ z = 6\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$ similar to Q15, each point is rotated anticlockwise $\frac{2\pi}{5}$ rad so, $\begin{pmatrix}\cos(\frac{2}{5}\pi) & -\sin(\frac{2}{5}\pi)\\ \sin(\frac{2}{5}\pi) & \cos(\frac{2}{5}\pi)\end{pmatrix}$ is the matrix $\therefore\ 6\sqrt{2}\left(\cos\left(\frac{13\pi}{20}\right) + i\sin\left(\frac{13\pi}{20}\right)\right)$ $6\sqrt{2}\left(\cos\left(\frac{21\pi}{20}\right) + i\sin\left(\frac{21\pi}{20}\right)\right)$ $6\sqrt{2}\left(\cos\left(\frac{29\pi}{20}\right) + i\sin\left(\frac{29\pi}{20}\right)\right)$ $6\sqrt{2}\left(\cos\left(\frac{37\pi}{20}\right) + i\sin\left(\frac{37\pi}{20}\right)\right)$ are the other vertices, (by adding multiples of $2\pi/5$ to the argument)		
		(5)	
(ii)(a)	Circle centre (0, 2) and radius 2 or  with the point on the origin	B1	1.1b
	Fully correct 	B1	1.1b
		(2)	
(ii)(b)	$\text{area} = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (4\sin\theta)^2\,\mathrm{d}\theta$ or $\text{area} = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\alpha\sin\theta)^2\,\mathrm{d}\theta$	M1	3.1a
	Uses $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and integrates to the form $A\theta + B\sin 2\theta$ $\text{area} = 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2\theta\,\mathrm{d}\theta = 4\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1 - \cos 2\theta\,\mathrm{d}\theta = 4\theta - 2\sin 2\theta$	M1	3.1a
	Uses the limits of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ and subtracts the correct way around $\left[4\left(\frac{\pi}{3}\right) - 2\sin\left(\frac{2\pi}{3}\right)\right] - \left[4\left(\frac{\pi}{4}\right) - 2\sin\left(\frac{2\pi}{4}\right)\right]$	M1	1.1b

	Area = $\frac{\pi}{3}-\sqrt{3}+2$	A1	1.1b
		(4)	
	<u>Alternative</u> 		
	Finds either the areas 1 or 2 Area 1 $=\frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\left\{=\sqrt{3}\right\}$ Area 2 $=\frac{1}{2}\times 2^2\times\frac{\pi}{3}\left\{=\frac{2\pi}{3}\right\}$	M1	1.1b
	A complete method to find area 3 Area 3 $=\frac{1}{4}\pi\times 2^2-\frac{1}{2}\times 2^2\left\{=\pi-2\right\}$	M1	3.1a
	A complete method to find the required area Shaded area $=$ Area of semi circle $-$ area 1 $-$ area 2 $-$ area 3 $=\left[\frac{1}{2}\pi\times 2^2\right]-\left[\frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\right]-\left[\frac{1}{2}\times 2^2\times\frac{\pi}{3}\right]-\left[\frac{1}{4}\pi\times 2^2-\frac{1}{2}\times 2^2\right]$ $=2\pi-\sqrt{3}-\frac{2\pi}{3}-(\pi-2)$ Or Shaded area $=$ Area of sector $-$ area 1 $-$ area 3 $=\left[\frac{1}{2}\times 4\times\left(\frac{2\pi}{3}\right)\right]-\left[\frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\right]-\left[\frac{1}{4}\pi\times 2^2-\frac{1}{2}\times 2^2\right]$ $=\frac{4\pi}{3}-\sqrt{3}-(\pi-2)$	M1	3.1a
	Area = $\frac{\pi}{3}-\sqrt{3}+2$	A1	1.1b
		(4)	
(11 marks)			

Notes:

(i)
M1: Finds the modulus and argument of z
A1: Correct modulus and argument of z

M1: Uses a correct method to find to all the other 4 vertices of the pentagon. Must be doing the equivalent of adding/ subtracting multiples of   to the argument.

A1ft: All 4 vertices following through on their modulus and argument. Does not need to be simplified for this mark.
A1: All 4 vertices correct in the required form

(ii)(a)
B1: Circle centre (0, 2) and radius 2 or  with the vertex on the origin.
B1: Fully correct region shaded.

(ii) (b)
M1: Writes the required area using polar coordinates
M1: Uses $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and integrates to the form $A\theta + B\sin 2\theta$

M1: Uses the limits of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ and subtracts the correct way around. Must be some attempt at $\text{area} = \frac{1}{2}\int \alpha\sin\theta^{\;2}\, d\theta$ and integration.

A1: Correct exact area = $\frac{\pi}{3} - \sqrt{3} + 2$

Alternative
M1: Finds either area 1 or area 2
M1: A complete method to find the area 3
M1: A complete method to find the required area = Area of semi circle – area 1 – area 2 – area 3 or = Area of sector – area 1 – area 3
A1: Correct exact area = $\frac{\pi}{3} - \sqrt{3} + 2$