London Academy of Maths

Integration Techniques 1

Question Paper

1 It is given that $f(x) = 3x - \frac{5}{x^3}$.

Find

(a) $f'(x)$, **[3]**

(b) $f''(x)$, **[2]**

(c) $\int f(x)\,dx$. **[3]**

2 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 - 3x + \frac{5}{x^2}\right)$. **[3]**

(b) Find $\int\left(6x^2 - \frac{2}{x^3}\right)\mathrm{d}x$. **[3]**

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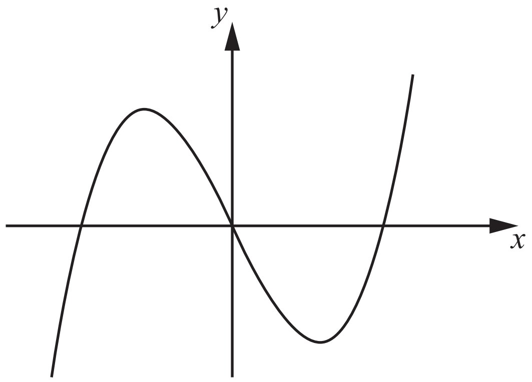
3 In this question you must show detailed reasoning.

The diagram shows part of the graph of $y = x^3 - 4x$.



Determine the total area enclosed by the curve and the x-axis. **[6]**

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4 A curve has the following properties:

- The gradient of the curve is given by $\frac{dy}{dx} = -2x$.
- The curve passes through the point $(4, -13)$.

Determine the coordinates of the points where the curve meets the line $y = 2x$. **[7]**

5 The gradient of a curve is given by $\frac{dy}{dx} = x^2 - 3x$. The curve passes through the point (6, 20).

(a) Determine the equation of the curve. **[4]**

(b) Hence determine $\int_1^p y \, dx$ in terms of the constant p. **[4]**

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6 The function f is defined by $f(x) = (x+a)(x+3a)(x-b)$ where a and b are positive integers.

(a) On the axes in the Printed Answer Booklet, sketch the curve $y = f(x)$. **[2]**

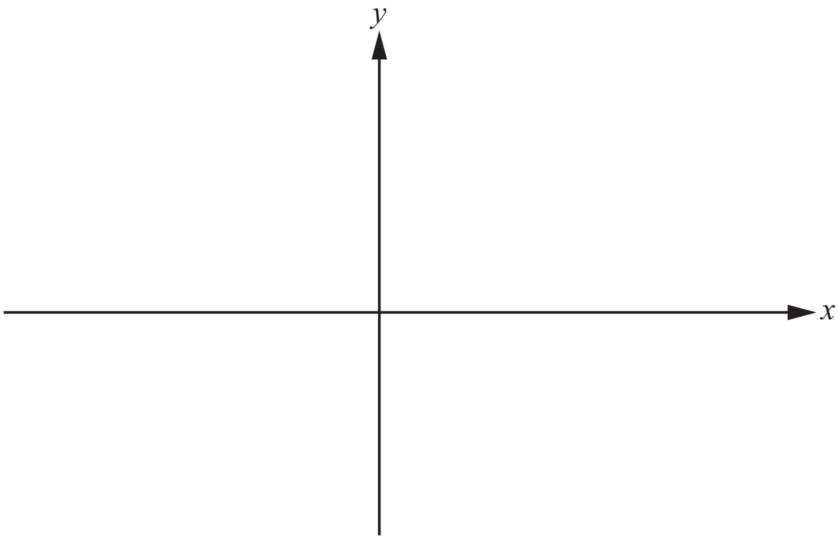
(b) On your sketch show, in terms of a and b, the coordinates of the points where the curve meets the axes. **[2]**

It is now given that $a = 1$ and $b = 4$.

(c) Find the total area enclosed between the curve $y = f(x)$ and the x-axis. **[4]**

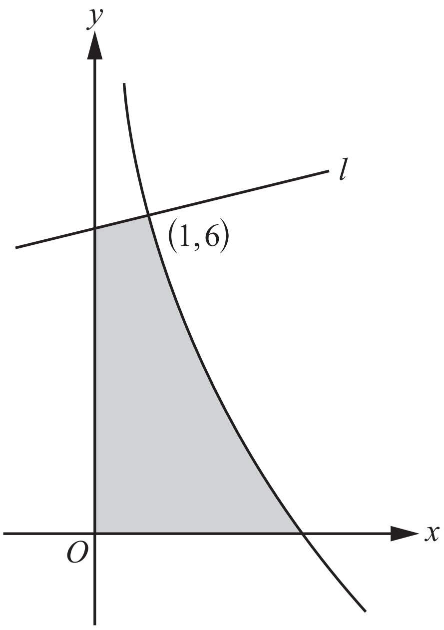
6(a)

6(b)



6(c)

7



The diagram shows the curve $y = 1 - x + \frac{6}{\sqrt{x}}$ and the line l, which is the normal to the curve at the point (1, 6).

(a) Determine the equation of l in the form

$$ax + by = c$$

where a, b and c are integers whose values are to be stated. **[5]**

(b) Verify that the curve intersects the x-axis at the point where $x = 4$. **[1]**

(c) **In this question you must show detailed reasoning.**

Determine the exact area of the shaded region enclosed between l, the curve, the x-axis and the y-axis. **[5]**

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8 **In this question you must show detailed reasoning.**

Given that $\int_4^a \left(\frac{4}{\sqrt{x}} + 3\right) dx = 7$, find the value of a. **[7]**

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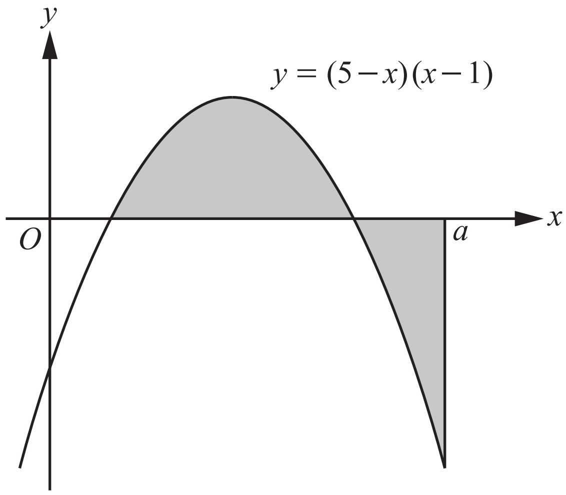

The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a. **[8]**

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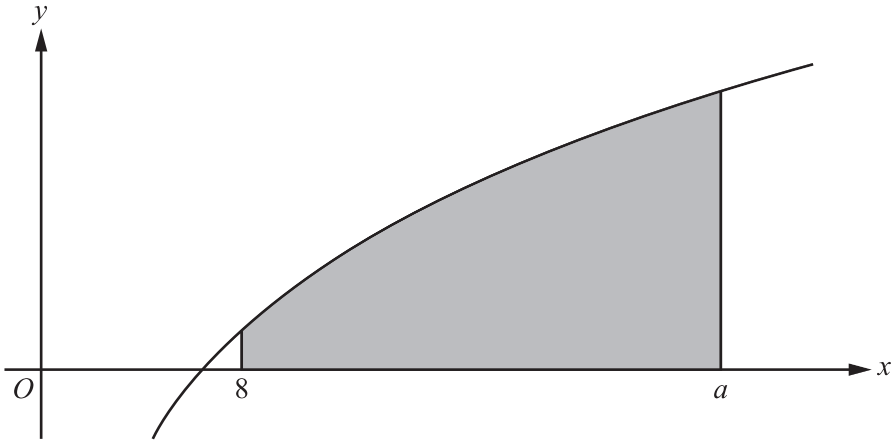
10 **In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \dfrac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x-axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a. **[9]**

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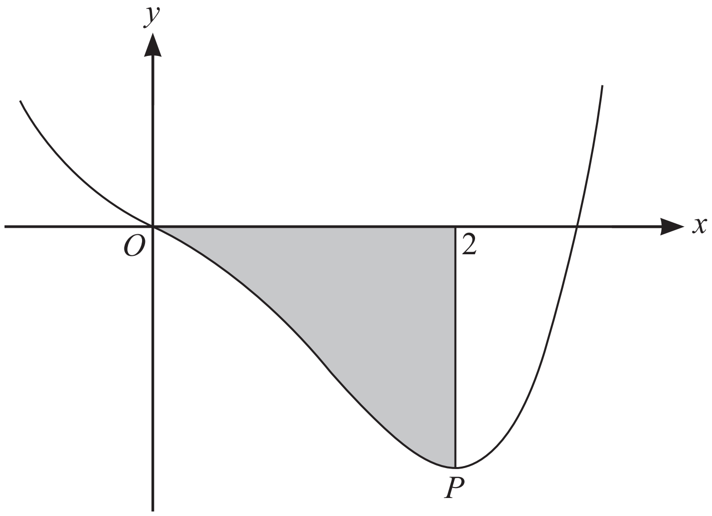

The diagram shows the curve with equation $y = 5x^4 + ax^3 + bx$, where a and b are integers. The curve has a minimum at the point P where $x = 2$.

The shaded region is enclosed by the curve, the x-axis and the line $x = 2$.

Given that the area of the shaded region is 48 units^2, determine the y-coordinate of P. **[7]**

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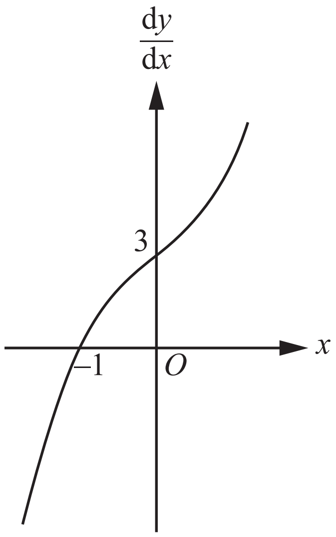
12 A curve C has an equation which satisfies $\frac{d^2y}{dx^2} = 3x^2 + 2$, for all values of x.

(a) It is given that C has a single stationary point. Determine the nature of this stationary point. **[1]**

The diagram shows the graph of the **gradient function** for C.



(b) Given that C passes through the point $(-1, \frac{1}{4})$, find the equation of C in the form $y = f(x)$. **[5]**

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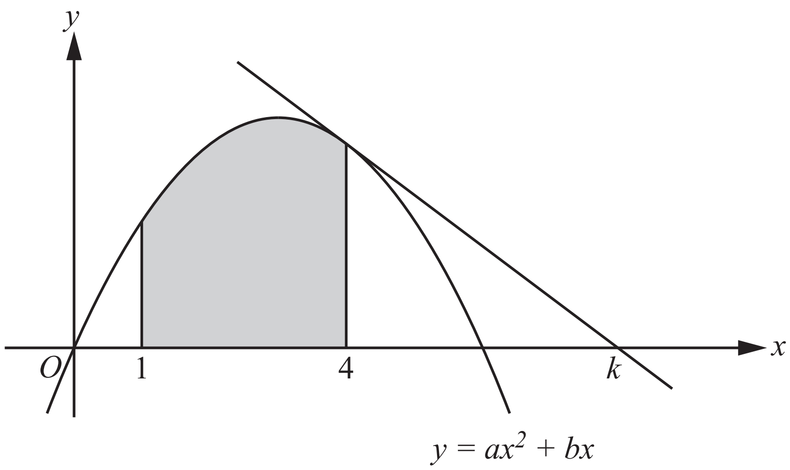
13 **(a)** The quadratic polynomial $ax^2 + bx$, where a and b are constants, is denoted by $f(x)$.

Use differentiation from first principles to determine, in terms of a, b and x, an expression for $f'(x)$. **[4]**

(b)



The diagram shows the quadratic curve $y = ax^2 + bx$, where a and b are constants. The shaded region is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

The tangent to the curve at $x = 4$ intersects the x-axis at the point with coordinates $(k, 0)$.

Given that the area of the shaded region is 9 units^2, and the gradient of this tangent is $-\frac{3}{4}$, determine the value of k. **[7]**