

Integration Techniques 1

Question Paper

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Show that

$$\int_4^{4\sqrt{3}} \frac{8}{16+x^2} dx = p\pi$$

where p is a rational number to be determined.

(3)

(ii) Determine the exact value of k for which

$$\int_{\frac{3}{4}}^k \frac{2}{\sqrt{9-4x^2}} dx = \frac{\pi}{12}$$

(4)

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Question 1 continued

Lined area for writing the answer to Question 1.



2.



(5)

(6)

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Question 2 continued

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3. Evaluate $\int_1^3 \frac{1}{\sqrt{(x^2 + 4x - 5)}} dx$, giving your answer as an exact logarithm.

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Question 3 continued

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4. Determine

(i) $\int \frac{1}{\sqrt{x^2 - 3x + 5}} dx$ (3)

(ii) $\int \frac{1}{\sqrt{63 + 4x - 4x^2}} dx$ (4)



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Question 4 continued

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5. (a) Determine

$$\int \frac{1}{\sqrt{9x^2 + 16}} dx \quad (2)$$

(b) Hence determine the exact value of

$$\int_{-2}^2 \frac{1}{\sqrt{9x^2 + 16}} dx$$

Give your answer in the form $a \ln(b + c\sqrt{13})$, where a , b and c are rational numbers.

(3)

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Question 5 continued

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6. Determine

(i) $\int \frac{1}{3x^2 + 12x + 24} dx$ (4)

(ii) $\int \frac{1}{\sqrt{27 - 6x - x^2}} dx$ (4)

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Question 6 continued

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7. (a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x + 1)(x^2 + 4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x + 1)(x^2 + 4)} dx = \ln(a\sqrt{2}) + b\pi$$

where a and b are constants to be determined.

(4)

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Question 7 continued

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8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Determine

$$\int \frac{1}{\sqrt{5+4x-x^2}} dx \quad (3)$$

(ii) Use the substitution $x = 3 \sec \theta$ to determine the exact value of

$$\int_{2\sqrt{3}}^6 \frac{18}{(x^2-9)^{\frac{3}{2}}} dx$$

Give your answer in the form $A + B\sqrt{3}$ where A and B are constants to be found. (6)

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Question 8 continued

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9.

$$4x^2 + 4x + 17 \equiv (2x + p)^2 + q$$

where p and q are integers.

(a) Determine the value of p and the value of q

(2)

Given that

$$\frac{8x + 5}{\sqrt{4x^2 + 4x + 17}} \equiv \frac{1}{\sqrt{4x^2 + 4x + 17}} + \frac{Ax + B}{\sqrt{4x^2 + 4x + 17}}$$

where A and B are integers,

(b) write down the value of A and the value of B

(1)

(c) Hence use algebraic integration to show that

$$\int_{\frac{1}{3}}^1 \frac{8x + 5}{\sqrt{4x^2 + 4x + 17}} dx = k + \frac{1}{2} \ln k$$

where k is a rational number to be determined.

(5)

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Question 9 continued

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$$y = \arccos(2\sqrt{x})$$

(3)

$$\int y \, dx = x \arccos(2\sqrt{x}) + \int \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx$$

(2)

$$\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx = \frac{1}{4} \int_a^b \cos^2 \theta \, d\theta$$

(4)

$$\int_0^{\frac{1}{8}} \arccos(2\sqrt{x}) \, dx$$

(4)

Question 10 continued

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11. (a) Using the substitution $x = \frac{a}{u}$, or otherwise, find

(6)

$$\int_3^4 \frac{1}{x \sqrt{25 - x^2}} dx$$

(5)

[illegible]

Question 11 continued

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12. (a) Differentiate $x \operatorname{arcosh} 5x$ with respect to x

(2)

(b) Hence, or otherwise, show that

$$\int_{\frac{1}{4}}^{\frac{3}{5}} \operatorname{arcosh} 5x \, dx = \frac{3}{20} - \frac{2\sqrt{2}}{5} + \ln(p + q\sqrt{2})^k - \frac{1}{4} \ln r$$

where p , q , r and k are rational numbers to be determined.

(8)

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Question 12 continued

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13. Using calculus, find the exact values of

(i) $\int_1^2 \frac{1}{x^2 - 4x + 5} dx$ (3)

(ii) $\int_{\sqrt{3}}^3 \frac{\sqrt{x^2 - 3}}{x^2} dx$ (5)

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Question 13 continued

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14. (a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2 - 1}} dx = \frac{1}{2} \left[x\sqrt{x^2 - 1} + \operatorname{arcosh} x \right] + k$$

where k is an arbitrary constant.

(6)

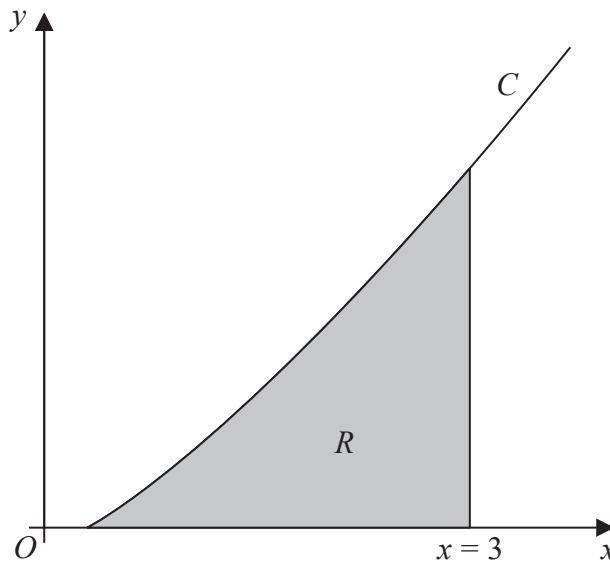


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{4}{15} x \operatorname{arcosh} x \quad x \geq 1$$

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the x -axis and the line with equation $x = 3$

- (b) Using algebraic integration and the result from part (a), show that the area of R is given by

$$\frac{1}{15} \left[17 \ln(3 + 2\sqrt{2}) - 6\sqrt{2} \right]$$

(5)



Question 14 continued

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15. Show that

$$\int_5^6 \frac{3+x}{\sqrt{x^2-9}} \, dx = 3 \ln \left(\frac{2+\sqrt{3}}{3} \right) + 3\sqrt{3} - 4.$$

(7)



Question 15 continued

Lined area for writing the answer to Question 15.

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16.


$$y = \frac{1}{10} \cosh x \arctan (\sinh x), \quad x \geqslant 0.$$

(a) Find $\int \cosh x \arctan (\sinh x) dx$.

(b) Hence show that, to 2 significant figures, the area of R is 0.34 (2)

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Question 16 continued

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17. Using the substitution $x = 4 \cosh \theta$ show that

$$\int \frac{1}{(x^2 - 16)^{\frac{3}{2}}} dx = \frac{ax}{\sqrt{x^2 - 16}} + c \quad |x| > 4$$

where a is a constant to be determined and c is an arbitrary constant.

(6)

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Question 17 continued

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A graph in the xy -plane. The horizontal axis is the x -axis, and the vertical axis is the y -axis. The origin is labeled O . A curve, labeled C , starts at the origin and increases, concave down. A vertical dashed line is drawn at $x = 4$. The region R is the area bounded by the x -axis, the curve C , and the vertical line $x = 4$. This region is shaded with diagonal lines.

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Question 18 continued

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