London Academy of Maths

Integration Techniques

Question Paper

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1 Find

(i) $\int (x^3 + 8x - 5)\,\mathrm{d}x$, **[3]**

(ii) $\int 12\sqrt{x}\,\mathrm{d}x$. **[3]**

2 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 - 3x + \frac{5}{x^2}\right)$. **[3]**

(b) Find $\int \left(6x^2 - \frac{2}{x^3}\right) \mathrm{d}x$. **[3]**

3 It is given that $\mathrm{f}(x) = 3x - \frac{5}{x^3}$.

Find

(a) $\mathrm{f}'(x)$, **[3]**

(b) $\mathrm{f}''(x)$, **[2]**

(c) $\int \mathrm{f}(x)\,\mathrm{d}x$. **[3]**

 **

4 **(a)** Find $\int_1^4 (3\sqrt{x}+5)\,\mathrm{d}x$. **[4]**

(b) Find $\int \frac{6x^4+4}{x^2}\,\mathrm{d}x$. **[3]**

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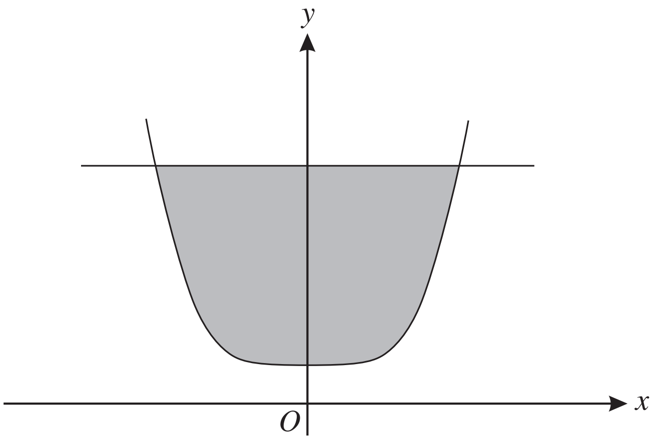


The diagram shows the curve $y = x^4 + 3$ and the line $y = 19$ which intersect at $(-2, 19)$ and $(2, 19)$. Use integration to find the exact area of the shaded region enclosed by the curve and the line. **[7]**

**

6 The gradient of a curve is given by $\frac{dy}{dx} = 5x(\sqrt{x} - 2)$ and the curve passes through the point (4, 11). Find the equation of the curve. **[6]**

 **

7 The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points (2, 5) and (p, 5).

(i) Find the equation of the curve. **[4]**

(ii) Find the value of p. **[3]**

**

8 **(i)** Find $\int \left(6x^{\frac{1}{2}} - 1\right) \mathrm{d}x$. **[3]**

(ii) Hence find the equation of the curve for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{\frac{1}{2}} - 1$ and which passes through the point (4, 17). **[3]**

 **

9 **(i)** Find the binomial expansion of $(x^2 - 5)^3$, simplifying the terms. **[4]**

(ii) Hence find $\int (x^2 - 5)^3 \, dx$. **[4]**

 **

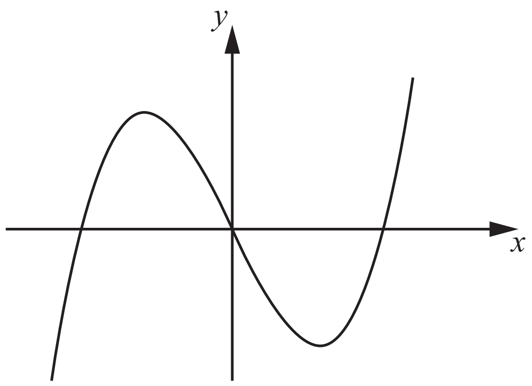
10 **(i)** Find $\int(x^2 - 2x + 5)\,\mathrm{d}x$. **[3]**

(ii) Hence find the equation of the curve for which $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 2x + 5$ and which passes through the point (3, 11). **[3]**

 **

11 **In this question you must show detailed reasoning.**

The diagram shows part of the graph of $y = x^3 - 4x$.



Determine the total area enclosed by the curve and the x-axis. **[6]**

 **

12 A curve has the following properties:

- The gradient of the curve is given by $\frac{dy}{dx} = -2x$.
- The curve passes through the point $(4, -13)$.

Determine the coordinates of the points where the curve meets the line $y = 2x$. **[7]**

 **

13 The gradient of a curve is given by $\frac{dy}{dx} = x^2 - 3x$. The curve passes through the point (6, 20).

(a) Determine the equation of the curve. **[4]**

(b) Hence determine $\int_1^p y\,dx$ in terms of the constant p. **[4]**

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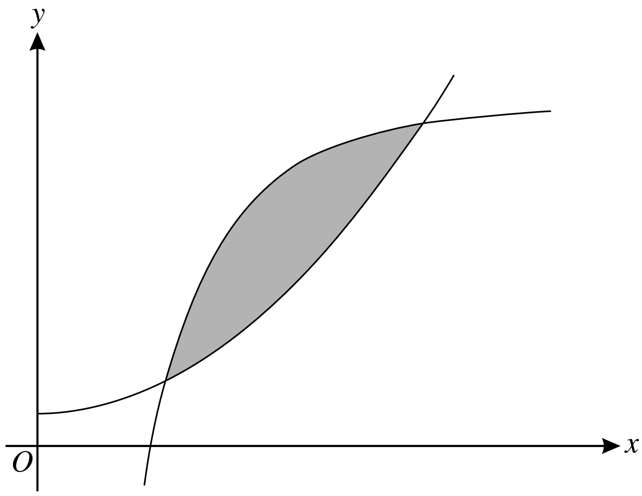

The diagram shows parts of the curves $y = x^2 + 1$ and $y = 11 - \frac{9}{x^2}$, which intersect at $(1, 2)$ and $(3, 10)$. Use integration to find the exact area of the shaded region enclosed between the two curves. **[7]**

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15 The function f is defined by $f(x) = (x+a)(x+3a)(x-b)$ where a and b are positive integers.

(a) On the axes in the Printed Answer Booklet, sketch the curve $y = f(x)$. **[2]**

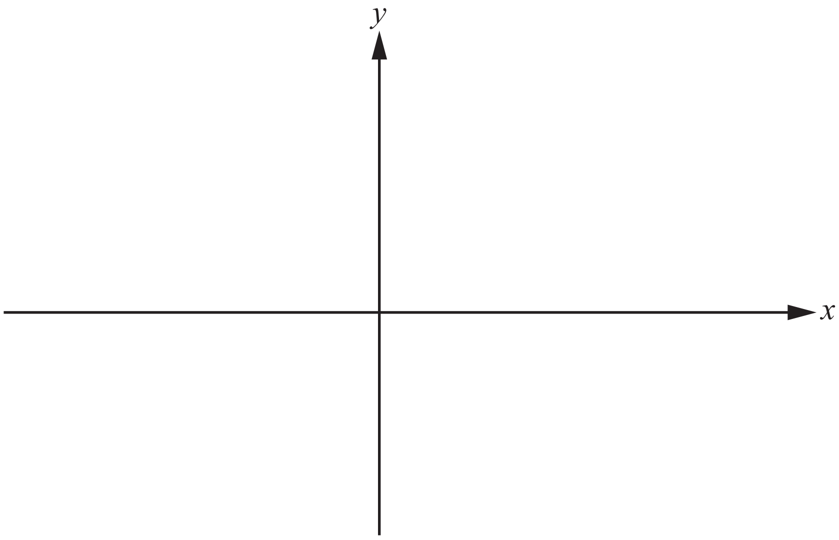
(b) On your sketch show, in terms of a and b, the coordinates of the points where the curve meets the axes. **[2]**

It is now given that $a = 1$ and $b = 4$.

(c) Find the total area enclosed between the curve $y = f(x)$ and the x-axis. **[4]**

15(a)

15(b)



15(c)

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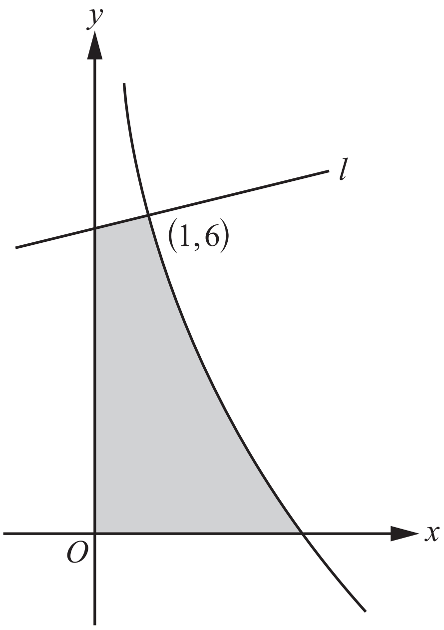

The diagram shows the curve $y = 1 - x + \frac{6}{\sqrt{x}}$ and the line l, which is the normal to the curve at the point (1, 6).

(a) Determine the equation of l in the form

$ax + by = c$

where a, b and c are integers whose values are to be stated. **[5]**

(b) Verify that the curve intersects the x-axis at the point where $x = 4$. **[1]**

(c) **In this question you must show detailed reasoning.**

Determine the exact area of the shaded region enclosed between l, the curve, the x-axis and the y-axis. **[5]**

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17 In this question you must show detailed reasoning.

Given that $\int_4^a \left(\frac{4}{\sqrt{x}} + 3\right) \mathrm{d}x = 7$, find the value of a. **[7]**

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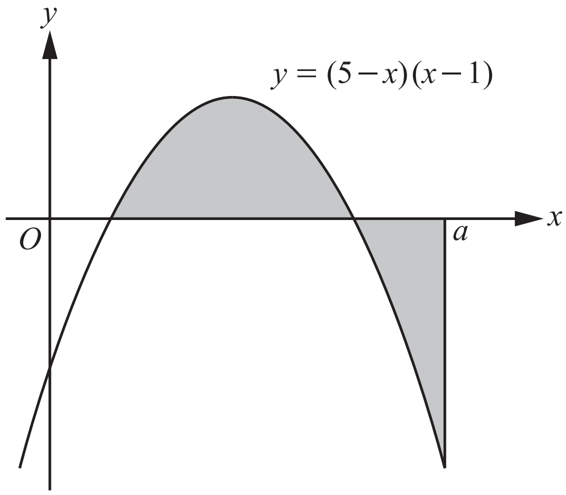

The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a. **[8]**

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19 **(a)** Find $\int \frac{x^3 + 3x^{\frac{1}{2}}}{x}\,\mathrm{d}x$. **[4]**

(b) **(i)** Find, in terms of a, the value of $\int_2^a 6x^{-4}\,\mathrm{d}x$, where a is a constant greater than 2. **[3]**

(ii) Deduce the value of $\int_2^\infty 6x^{-4}\,\mathrm{d}x$. **[1]**

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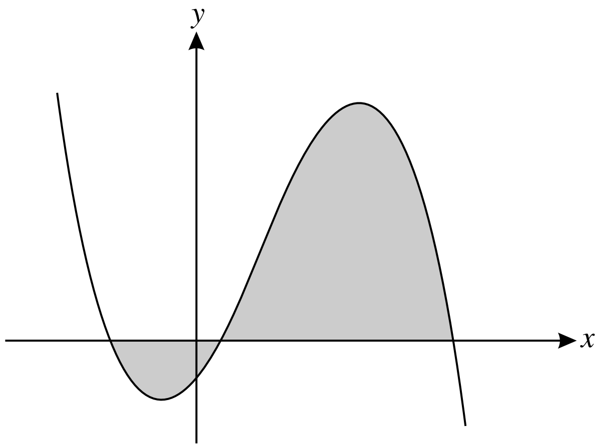

The diagram shows the curve $y = \text{f}(x)$, where $\text{f}(x) = -4x^3 + 9x^2 + 10x - 3$.

(i) Verify that the curve crosses the x-axis at $(3, 0)$ and hence state a factor of $\text{f}(x)$. **[2]**

(ii) Express $\text{f}(x)$ as the product of a linear factor and a quadratic factor. **[3]**

(iii) Hence find the other two points of intersection of the curve with the x-axis. **[2]**

(iv) The region enclosed by the curve and the x-axis is shaded in the diagram. Use integration to find the total area of this region. **[5]**

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21 (a) Find $\int x^3(x^2 - x + 5)\,dx$. [4]

(b) **(i)** Find $\int 18x^{-4}\,dx$. [2]

(ii) Hence evaluate $\int_2^{\infty} 18x^{-4}\,dx$. [2]

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22 A curve has an equation which satisfies $\frac{dy}{dx} = kx(2x - 1)$ for all values of x. The point P (2, 7) lies on the curve and the gradient of the curve at P is 9.

(i) Find the value of the constant k. **[2]**

(ii) Find the equation of the curve. **[5]**

 ***

23 **(a)** Find $\int (x^2 + 2)(2x - 3)\,dx$. **[3]**

(b) **(i)** Find, in terms of a, the value of $\int_1^a (6x^{-2} - 4x^{-3})\,dx$, where a is a constant greater than 1. **[4]**

(ii) Deduce the value of $\int_1^{\infty} (6x^{-2} - 4x^{-3})\,dx$. **[1]**

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24 **(a)** Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. **[4]**

(b) Find $\int \left(2 - 6\sqrt{y}\right) \mathrm{d}y$. **[3]**

(c) Evaluate $\int_1^{\infty} \frac{8}{x^3} \, \mathrm{d}x$. **[4]**

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25 **(i)** Find the binomial expansion of $\left(x^3+\dfrac{2}{x^2}\right)^4$, simplifying the terms. **[5]**

(ii) Hence find $\displaystyle\int\left(x^3+\frac{2}{x^2}\right)^4 \mathrm{d}x$. **[4]**

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26 A curve has an equation which satisfies $\frac{d^2y}{dx^2} = 3x^{-\frac{1}{2}}$ for all positive values of x. The point $P(4, 1)$ lies on the curve, and the gradient of the curve at P is 5. Find the equation of the curve. **[7]**

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27 The positive constant a is such that $\int_a^{2a} \frac{2x^3 - 5x^2 + 4}{x^2} \, dx = 0$.

(i) Show that $3a^3 - 5a^2 + 2 = 0$. **[6]**

(ii) Show that $a = 1$ is a root of $3a^3 - 5a^2 + 2 = 0$, and hence find the other possible value of a, giving your answer in simplified surd form. **[6]**

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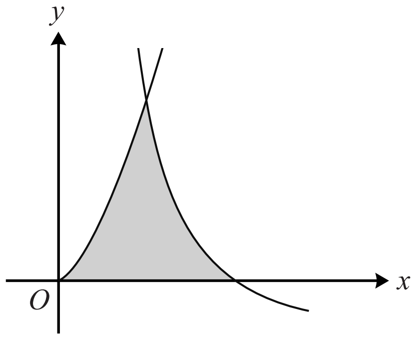
28 **(a)** Find $\int (x^2 + 4)(x - 6)\,dx$. **[3]**

(b)



The diagram shows the curve $y = 6x^{\frac{3}{2}}$ and part of the curve $y = \frac{8}{x^2} - 2$, which intersect at the point (1, 6). Use integration to find the area of the shaded region enclosed by the two curves and the x-axis. **[8]**

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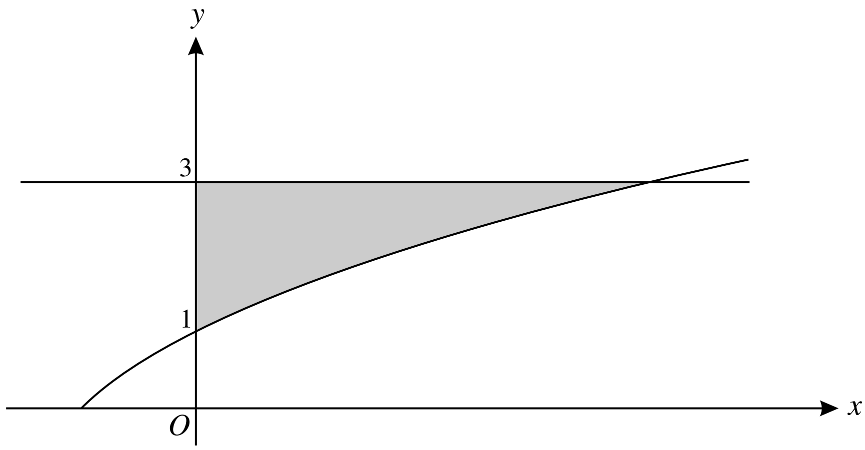

The diagram shows the curve $y = -1 + \sqrt{x+4}$ and the line $y = 3$.

(i) Show that $y = -1 + \sqrt{x+4}$ can be rearranged as $x = y^2 + 2y - 3$. **[2]**

(ii) Hence find by integration the exact area of the shaded region enclosed between the curve, the y-axis and the line $y = 3$. **[5]**

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30 The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve. **[8]**

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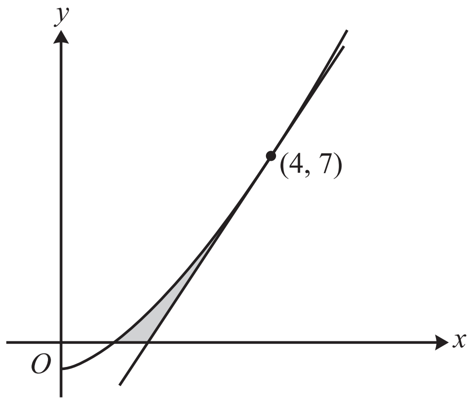

The diagram shows the curve $y = x^{\frac{3}{2}} - 1$, which crosses the x-axis at (1, 0), and the tangent to the curve at the point (4, 7).

(i) Show that $\int_1^4 (x^{\frac{3}{2}} - 1)\,\mathrm{d}x = 9\frac{2}{5}$. **[4]**

(ii) Hence find the exact area of the shaded region enclosed by the curve, the tangent and the x-axis. **[5]**

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32 The cubic polynomial $f(x)$ is defined by $f(x) = x^3 - 19x + 30$.

(i) Given that $x = 2$ is a root of the equation $f(x) = 0$, express $f(x)$ as the product of 3 linear factors. **[4]**

(ii) Use integration to find the exact value of $\int_{-5}^{3} f(x)\,dx$. **[4]**

(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area enclosed by the curve $y = f(x)$ and the x-axis for $-5 \leqslant x \leqslant 3$. **[2]**

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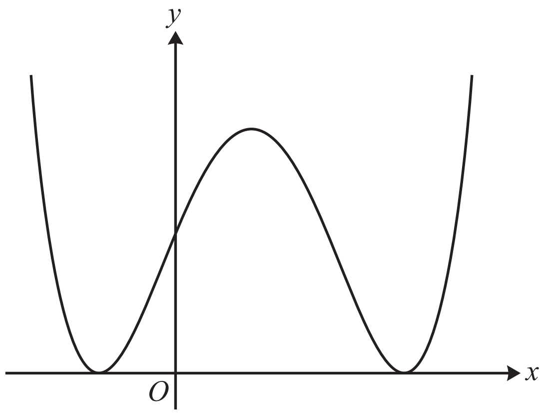
33 The cubic polynomial f(x) is defined by $f(x) = x^3 - 3x^2 - x + 3$.

(i) Find the quotient and remainder when f(x) is divided by $(x + 1)$. **[3]**

(ii) Hence find the three roots of the equation $f(x) = 0$. **[3]**



The diagram shows the curve C with equation $y = x^4 - 4x^3 - 2x^2 + 12x + 9$.

(iii) Show that the x-coordinates of the stationary points on C are given by $x^3 - 3x^2 - x + 3 = 0$. **[2]**

(iv) Use integration to find the exact area of the region enclosed by C and the x-axis. **[4]**

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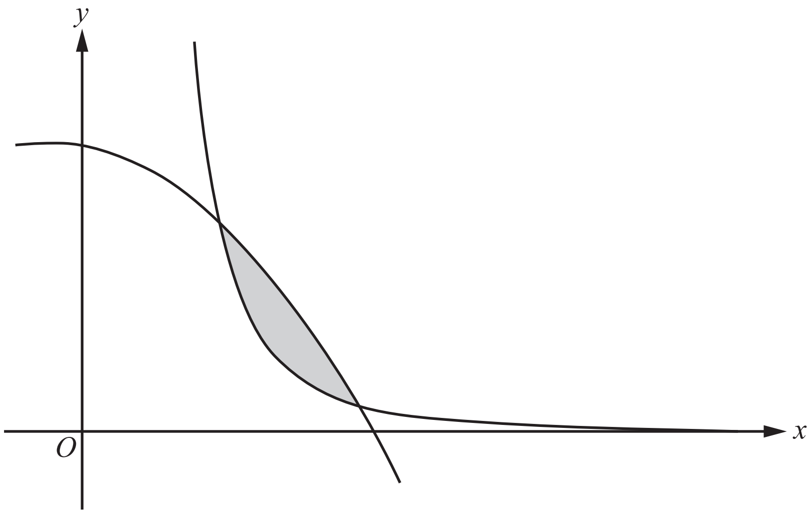

The diagram shows parts of the curves $y = 11 - x - 2x^2$ and $y = \dfrac{8}{x^3}$. The curves intersect at (1, 8) and (2, 1). Use integration to find the exact area of the shaded region enclosed between the two curves. **[7]**

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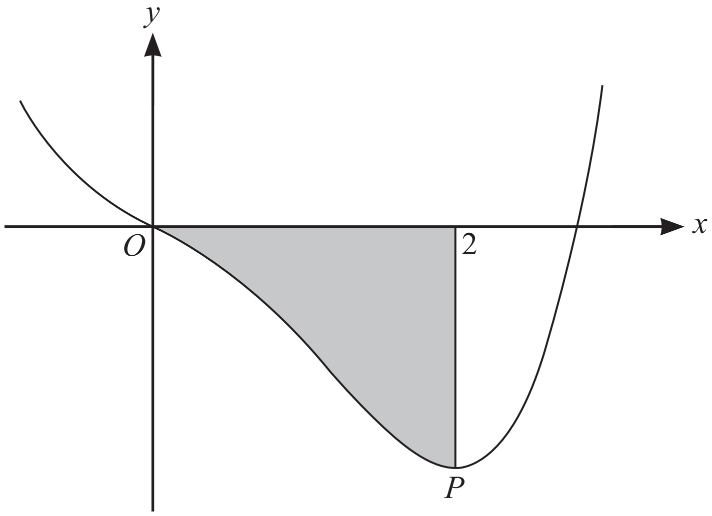

The diagram shows the curve with equation $y = 5x^4 + ax^3 + bx$, where a and b are integers. The curve has a minimum at the point P where $x = 2$.

The shaded region is enclosed by the curve, the x-axis and the line $x = 2$.

Given that the area of the shaded region is 48 units^2, determine the y-coordinate of P. **[7]**

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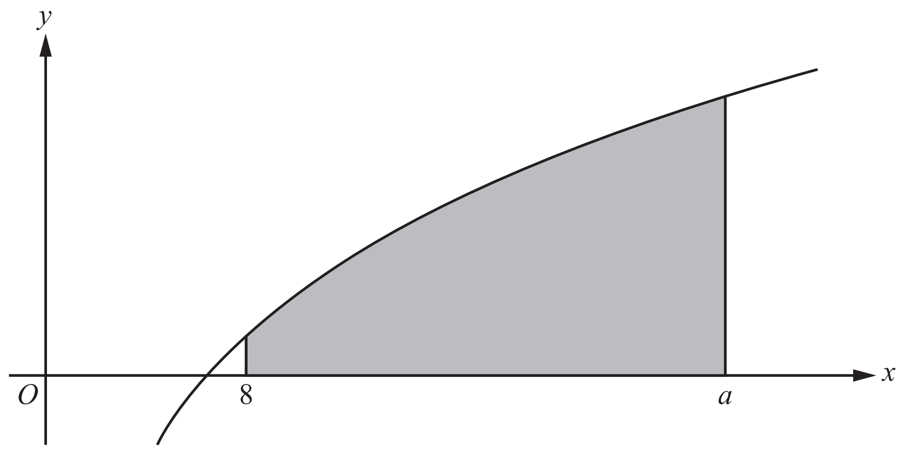
36 In this question you must show detailed reasoning.

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x-axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a. **[9]**

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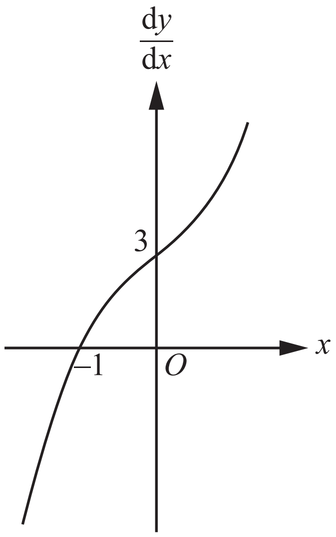
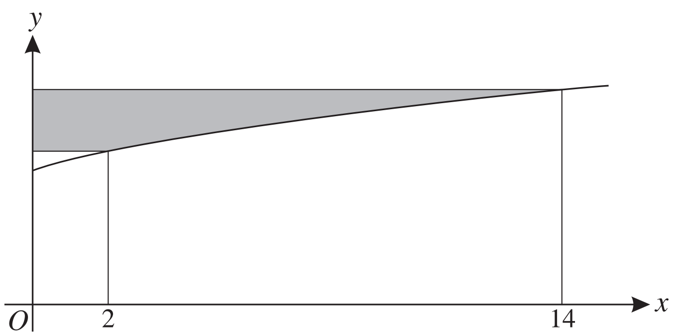
37 A curve C has an equation which satisfies $\dfrac{d^2y}{dx^2} = 3x^2 + 2$, for all values of x.

(a) It is given that C has a single stationary point. Determine the nature of this stationary point. **[1]**

The diagram shows the graph of the **gradient function** for C.



(b) Given that C passes through the point $(-1, \frac{1}{4})$, find the equation of C in the form $y = f(x)$. **[5]**



The diagram shows the curve $y = 3 + \sqrt{x + 2}$.

The shaded region is bounded by the curve, the y-axis, and two lines parallel to the x-axis which meet the curve where $x = 2$ and $x = 14$.

(i) Show that the area of the shaded region is given by

$$\int_5^7 (y^2 - 6y + 7)\,\mathrm{d}y. \qquad [3]$$

(ii) Hence find the exact area of the shaded region. [4]

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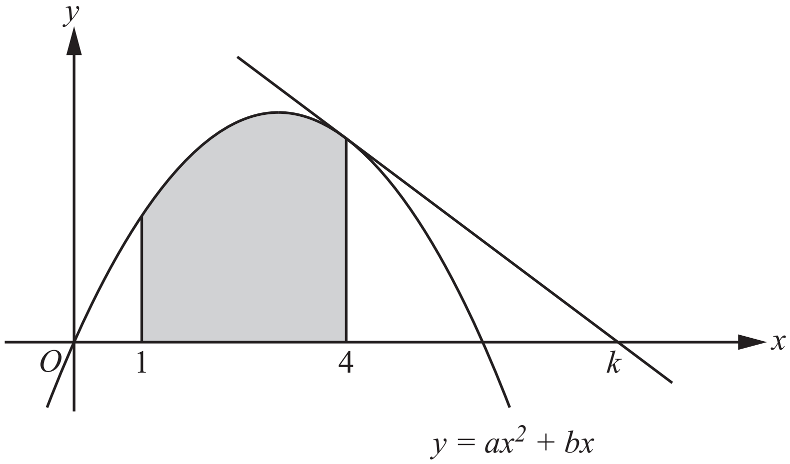
39 A curve passes through the point (1, 8) and has an equation which satisfies $\frac{dy}{dx} = 2x + \frac{a}{x^3} + 3$ for all non-zero values of x. The area enclosed by the curve, the x-axis, the line $x = 1$ and the line $x = 3$ is 30 square units. Find the value of the positive constant a. **[9]**

40 **(a)** The quadratic polynomial $ax^2 + bx$, where a and b are constants, is denoted by $f(x)$.

Use differentiation from first principles to determine, in terms of a, b and x, an expression for $f'(x)$. **[4]**

(b)



The diagram shows the quadratic curve $y = ax^2 + bx$, where a and b are constants. The shaded region is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

The tangent to the curve at $x = 4$ intersects the x-axis at the point with coordinates $(k, 0)$.

Given that the area of the shaded region is 9 units^2, and the gradient of this tangent is $-\frac{3}{4}$, determine the value of k. **[7]**