

Parametric Equations

Question Paper

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Question 1 continued

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2. A curve C has parametric equations

$$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t} \quad t \neq 0$$

(b) Show that a cartesian equation of C can be expressed in the form

$$y = \frac{ax + b}{x + 5} \quad x \neq k$$

where a , b and k are integers to be found.

(4)

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Question 2 continued

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3. A curve C has parametric equations

$$x = \frac{t}{t-3} \quad y = \frac{1}{t} + 2 \quad t \in \mathbb{R} \quad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax-1}{bx}$$

where a and b are constants to be found.

(3)

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Question 3 continued

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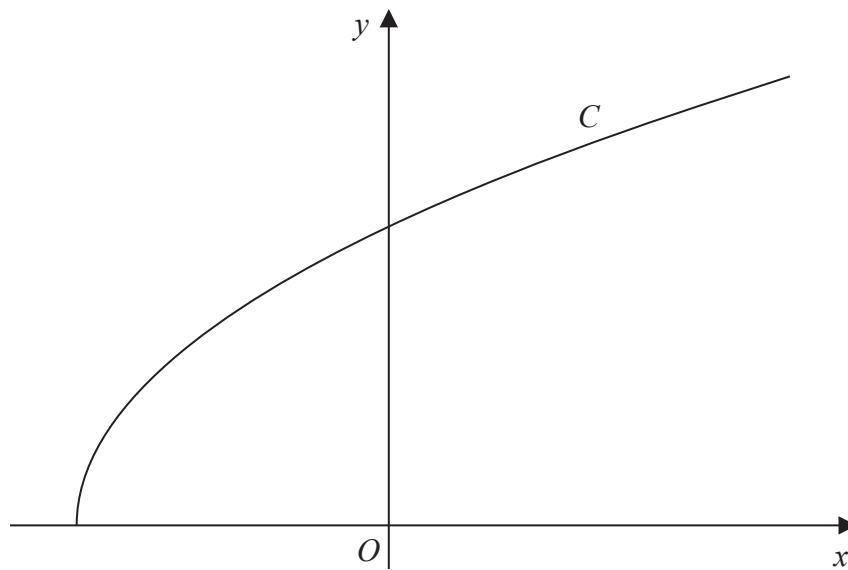


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 - 8\cos 2t \quad y = 9\sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

(b) Show that all points on C satisfy

$$y = \frac{9}{4}\sqrt{x+7} \quad (3)$$

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \frac{9}{4}\sqrt{x+7} \quad a \leq x \leq b$$

and a and b are constants.

(c) Find the value of a and the value of b . (2)

(d) State the range of f . (1)

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Question 4 continued

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- (c) Hence, or otherwise, find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$
- (3)**

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Question 5 continued

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6. A curve C has parametric equations

$$x = 6 \cos 2t, \quad y = 2 \sin t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

The cartesian equation for the curve C can be written in the form

$$x = f(y), \quad -k < y < k$$

where $f(y)$ is a polynomial in y and k is a constant.

(c) Find $f(y)$.

(3)

(d) State the value of k .

(1)

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Question 6 continued

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- $$x = \frac{1}{t} + 2 \quad y = \frac{1-2t}{3+t} \quad t > 0$$

- $$g(x) = \frac{ax + b}{cx + d} \quad x > k$$

(5)

- (2)

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Question 7 continued

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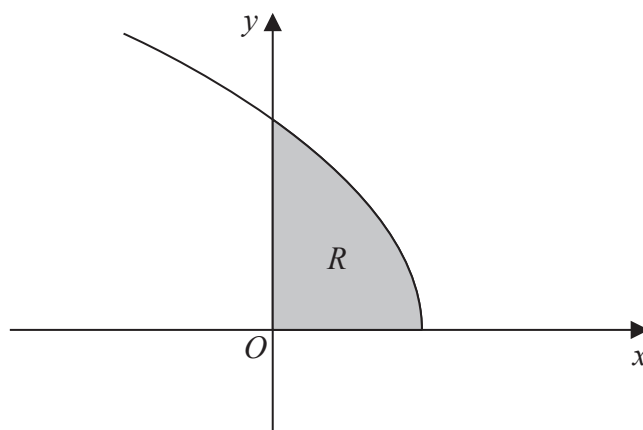


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = 4 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the y -axis.

- (b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \sqrt{ax + b} \quad -2 \leq x \leq 2$$

and a and b are the constants found in part (b).

- (c) State the range of f .

(1)



Question 8 continued

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Figure 1 shows a sketch of the curve C with parametric equations

The curve C has equation $y = f(x)$, where f is a quadratic function.

- (c) Find the range of f .

(2)

Question 9 continued

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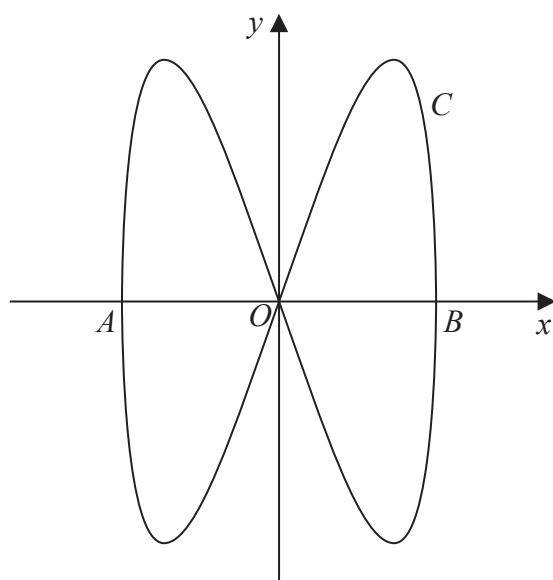


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = 3 \cos t, \quad y = 9 \sin 2t, \quad 0 \leq t \leq 2\pi$$

The curve C meets the x -axis at the origin and at the points A and B , as shown in Figure 3.

(a) Write down the coordinates of A and B . (2)

(b) Find the values of t at which the curve passes through the origin. (2)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y^2 = ax^2(b - x^2)$$

where a and b are integers to be determined. (4)

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Question 10 continued

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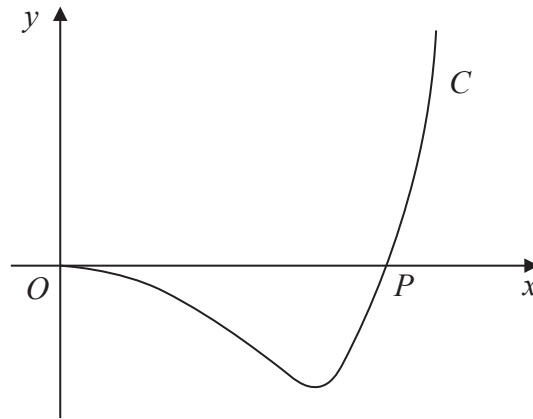


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = \frac{20t}{2t+1} \quad y = t(t-4), \quad t > 0$$

The curve cuts the x -axis at the point P .

(a) Find the x coordinate of P .

(2)

(c) (i) Make t the subject of the formula

$$x = \frac{20t}{2t+1}$$

(ii) Hence find a cartesian equation of the curve C . Write your answer in the form

$$y = f(x), \quad 0 < x < k$$

where $f(x)$ is a single fraction and k is a constant to be found.

(6)

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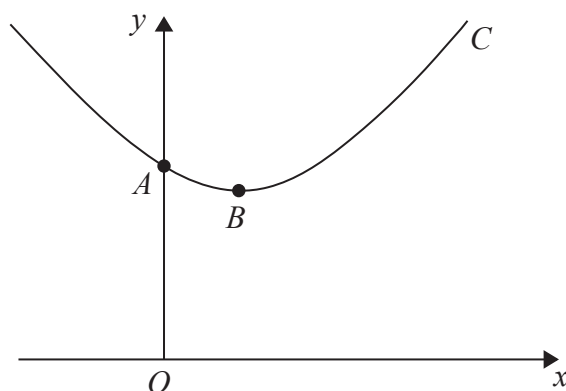


Figure 4

The curve C shown in Figure 4 has parametric equations

$$x = 1 + \sqrt{3} \tan \theta, \quad y = 5 \sec \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

The curve C crosses the y -axis at A and has a minimum turning point at B , as shown in Figure 4.

(a) Find the exact coordinates of A . (3)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y = k\sqrt{x^2 - 2x + 4}$$

where k is a simplified surd to be found. (3)

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Question 12 continued

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13. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \quad y = \frac{6}{2t+1} \quad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

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Question 13 continued

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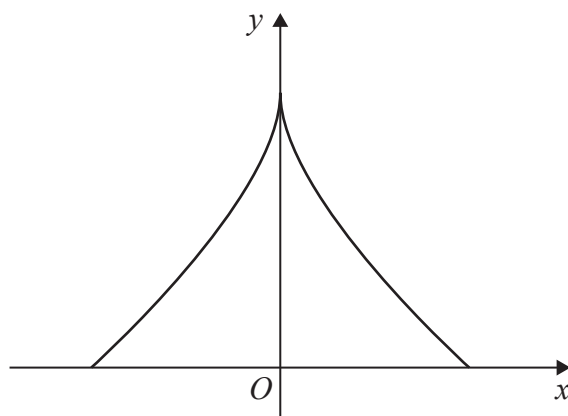


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3 \sin^3 \theta \quad y = 1 + \cos 2\theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \quad -q \leq x \leq q$$

where q is a constant to be found.

(3)

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Question 14 continued

Handwriting practice area with 30 horizontal lines.

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15. The curve C has parametric equations

$$x = \frac{t^4}{2t+1} \quad y = \frac{t^3}{2t+1} \quad t > 0$$

(a) Write down $\frac{x}{y}$ in terms of t , giving your answer in simplest form.

(1)

(b) Hence show that all points on C satisfy the equation

$$x^3 - 2xy^3 - y^4 = 0$$

(3)

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Question 15 continued

Handwriting practice area with 30 horizontal lines.

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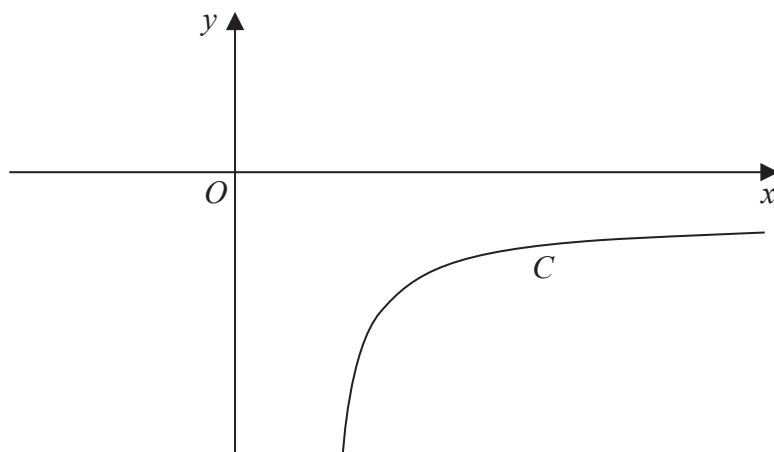


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \quad y = \sqrt{3} \tan\left(t + \frac{\pi}{3}\right) \quad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found.

(5)

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Question 17 continued

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