London Academy of Maths

Trigonometric Equations

Question Paper

 **

1 Solve each of the following equations, for $0° \leqslant x \leqslant 360°$.

(i) $\sin\frac{1}{2}x = 0.8$ **[3]**

(ii) $\sin x = 3\cos x$ **[3]**

2 **(i)** Show that the equation

$$2\sin^2 x = 5\cos x - 1$$

can be expressed in the form

$$2\cos^2 x + 5\cos x - 3 = 0.$$ **[2]**

(ii) Hence solve the equation

$$2\sin^2 x = 5\cos x - 1,$$

giving all values of x between 0° and 360°. **[4]**

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3 **(a)** **(i)** Show that the equation

$$2\sin x \tan x - 5 = \cos x$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ [3]

(ii) Hence solve the equation

$$2\sin x \tan x - 5 = \cos x,$$

giving all values of x, in radians, for $0 \leqslant x \leqslant 2\pi$. [4]

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4 **(i)** Solve the equation $\sin^2\theta = 0.25$ for $0° \leqslant \theta < 360°$. **[3]**

(ii) **In this question you must show detailed reasoning.**

Solve the equation $\tan 3\phi = \sqrt{3}$ for $0° \leqslant \phi < 90°$. **[3]**

5 **(i)** The polynomial $f(x)$ is defined by

$$f(x) = x^3 - x^2 - 3x + 3.$$

Show that $x = 1$ is a root of the equation $f(x) = 0$, and hence find the other two roots. **[6]**

(ii) Hence solve the equation

$$\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$$

for $0 \leqslant x \leqslant 2\pi$. Give each solution for x in an exact form. **[6]**

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6 Solve each of the following equations for $0^\circ \leqslant x \leqslant 180^\circ$.

(i) $3\tan 2x = 1$ **[3]**

(ii) $3\cos^2 x + 2\sin x - 3 = 0$ **[5]**

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7 **(i)** Show that the equation $2\sin x = \dfrac{4\cos x - 1}{\tan x}$ can be expressed in the form

$$6\cos^2 x - \cos x - 2 = 0.$$ **[3]**

(ii) Hence solve the equation $2\sin x = \dfrac{4\cos x - 1}{\tan x}$, giving all values of x between 0° and 360°. **[4]**

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8 Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

9 **(i)** Show that $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. **[2]**

(ii) Hence solve the equation

$$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x,$$

for $0° \leqslant x \leqslant 360°$. **[6]**

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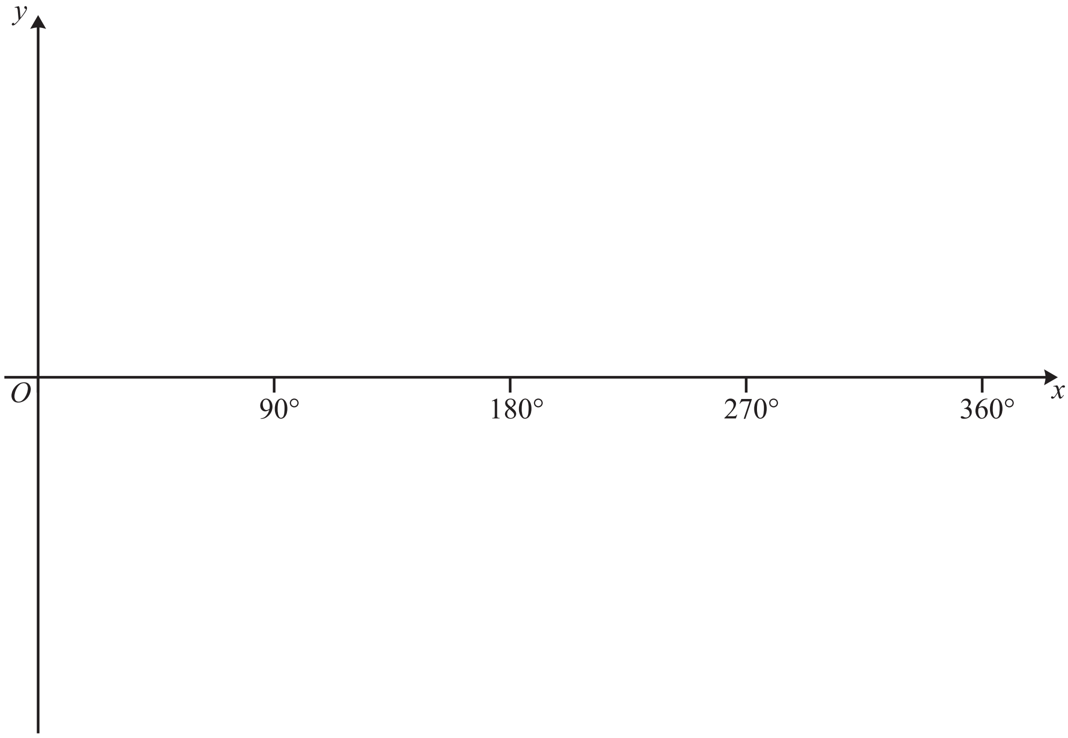
10 **(i)** Sketch the graphs of $y = 2\cos x$ and $y = 3\tan x$ for $0° \leqslant x \leqslant 360°$ on the axes provided. **[2]**

(ii) Show that the equation $2\cos x = 3\tan x$ can be expressed in the form $2\sin^2 x + 3\sin x - 2 = 0$. **[3]**

(iii) Hence solve the equation $2\cos x = 3\tan x$, giving all values of x between $0°$ and $360°$. **[4]**

10 (i)



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11 **(i)** Show that the equation

$$2\sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ **[3]**

(ii) Hence solve the equation

$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of θ between 0° and 180°, correct to 1 decimal place. **[5]**

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12 Solve the equation

$$4\cos^2 x + 7\sin x - 7 = 0,$$

giving all values of x between 0° and 360°. **[6]**

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13 **In this question you must show detailed reasoning.**

Solve the following equations, for $0° \leqslant x \leqslant 360°$.

(a) $2\tan x + 1 = 4$ **[3]**

(b) $5\sin x - 1 = 2\cos^2 x$ **[5]**

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14 **(a)** Prove that $\cos x + \sin x \tan x \equiv \dfrac{1}{\cos x}$ (where $x \neq \frac{1}{2}n\pi$ for any odd integer n). **[3]**

(b) Solve the equation $2\sin^2 x = \cos^2 x$ for $0° \leqslant x \leqslant 180°$. **[2]**

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15 **(i)** Show that the equation

$$\sin x - \cos x = \frac{6\cos x}{\tan x}$$

can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0.$$ **[2]**

(ii) Hence solve the equation $\sin x - \cos x = \dfrac{6\cos x}{\tan x}$ for $0° \leqslant x \leqslant 360°$. **[4]**

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16 **In this question you must show detailed reasoning.**

Solve the equation $2\cos^2 x = 2 - \sin x$ for $0^\circ \le x \le 180^\circ$. **[5]**

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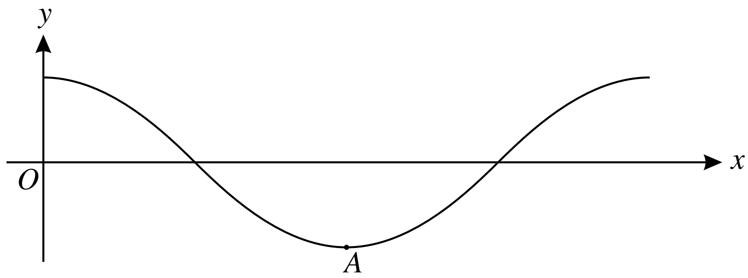
17 **(a)**



The diagram shows part of the curve $y = \cos 2x$, where x is in radians. The point A is the minimum point of this part of the curve.

(i) State the period of $y = \cos 2x$. **[1]**

(ii) State the coordinates of A. **[2]**

(iii) Solve the inequality $\cos 2x \leqslant 0.5$ for $0 \leqslant x \leqslant \pi$, giving your answers exactly. **[4]**

(b) Solve the equation $\cos 2x = \sqrt{3} \sin 2x$ for $0 \leqslant x \leqslant \pi$, giving your answers exactly. **[4]**

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18 The cubic polynomial $f(x)$ is defined by $f(x) = 4x^3 - 7x - 3$.

(i) Find the remainder when $f(x)$ is divided by $(x - 2)$. **[2]**

(ii) Show that $(2x + 1)$ is a factor of $f(x)$ and hence factorise $f(x)$ completely. **[6]**

(iii) Solve the equation

$$4\cos^3\theta - 7\cos\theta - 3 = 0$$

for $0 \leqslant \theta \leqslant 2\pi$. Give each solution for θ in an exact form. **[4]**

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19 **(i)** Sketch the graph of $y = \tan\left(\frac{1}{2}x\right)$ for $-2\pi \leqslant x \leqslant 2\pi$ on the axes provided.

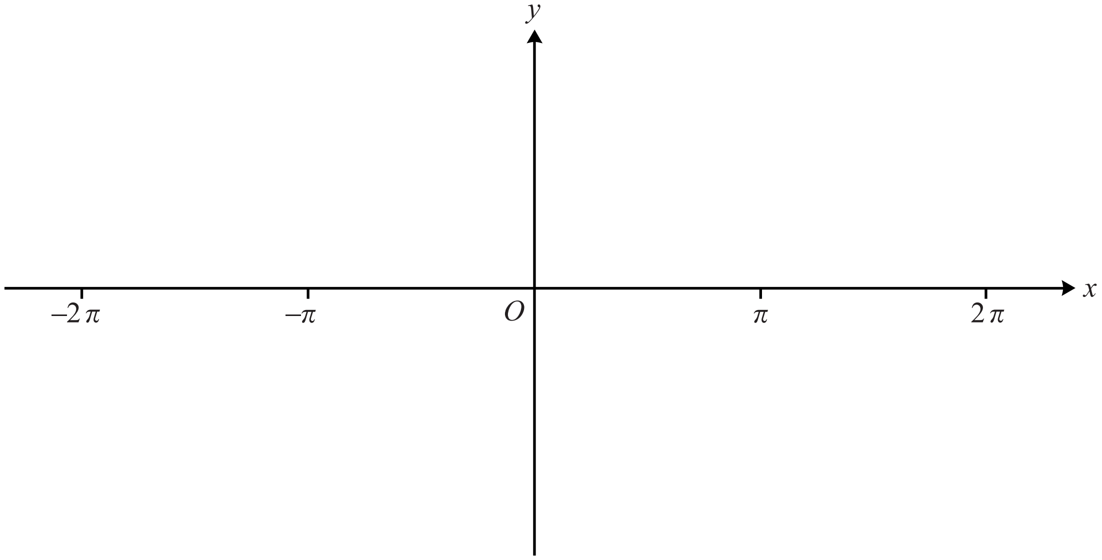
On the same axes, sketch the graph of $y = 3\cos\left(\frac{1}{2}x\right)$ for $-2\pi \leqslant x \leqslant 2\pi$, indicating the point of intersection with the y-axis. **[3]**

(ii) Show that the equation $\tan\left(\frac{1}{2}x\right) = 3\cos\left(\frac{1}{2}x\right)$ can be expressed in the form

$$3\sin^2\left(\tfrac{1}{2}x\right) + \sin\left(\tfrac{1}{2}x\right) - 3 = 0.$$

Hence solve the equation $\tan\left(\frac{1}{2}x\right) = 3\cos\left(\frac{1}{2}x\right)$ for $-2\pi \leqslant x \leqslant 2\pi$. **[6]**

19 (i)



19 (ii)

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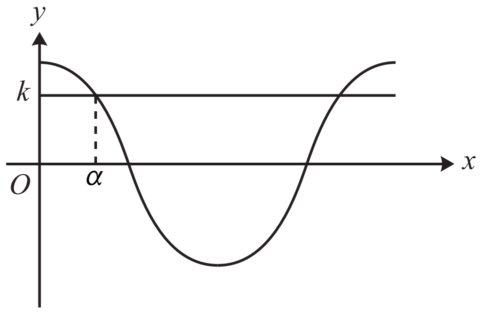


The diagram shows part of the curve $y = 2\cos\frac{1}{3}x$, where x is in radians, and the line $y = k$.

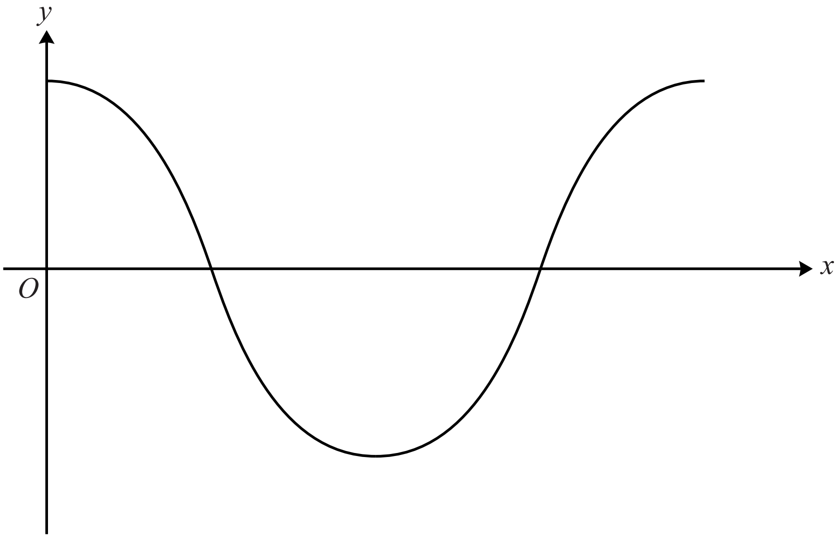
(i) The smallest positive solution of the equation $2\cos\frac{1}{3}x = k$ is denoted by α. State, in terms of α,

(a) the next smallest positive solution of the equation $2\cos\frac{1}{3}x = k$, **[1]**

(b) the smallest positive solution of the equation $2\cos\frac{1}{3}x = -k$. **[2]**

(ii) The curve $y = 2\cos\frac{1}{3}x$ is shown in the Printed Answer Book. On the diagram, and for the same values of x, sketch the curve of $y = \sin\frac{1}{3}x$. **[2]**

(iii) Calculate the x-coordinates of the points of intersection of the curves in part **(ii)**. Give your answers in radians correct to 3 significant figures. **[4]**

20 (ii)	

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21 The cubic polynomial $f(x)$ is defined by $f(x) = 4x^3 + 9x - 5$.

(i) Show that $(2x-1)$ is a factor of $f(x)$ and hence express $f(x)$ as the product of a linear factor and a quadratic factor. **[4]**

(ii) (a) Show that the equation

$$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13\tan 2\theta$$

can be expressed in the form

$$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0.$$ **[4]**

(b) Hence solve the equation

$$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13\tan 2\theta$$

for $0 \leqslant \theta \leqslant 2\pi$. Give each answer in an exact form. **[4]**

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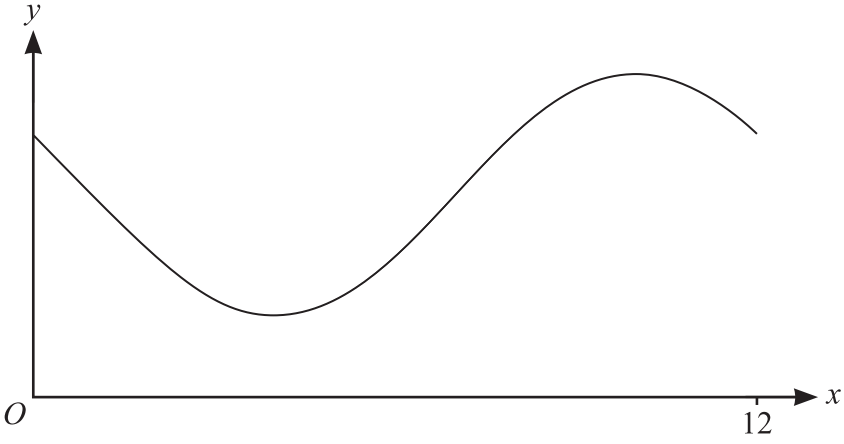

The diagram shows the curve with equation $y = 5 + 3\cos(30x + 60)°$, for $0 \leqslant x \leqslant 12$.

(a) **(i)** State the greatest value of y for points on the curve. **[1]**

(ii) Determine the value of x for which this greatest value of y occurs. **[3]**

There are two points on the curve for which the value of y is 7.

(b) Determine the values of x at these two points. **[5]**

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23 **In this question you must show detailed reasoning.**

The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 4x^3 + 4x^2 + 7x - 5$.

(a) Show that $(2x-1)$ is a factor of $\mathrm{f}(x)$. **[2]**

(b) Hence solve the equation $4\sin^3\theta + 4\sin^2\theta + 7\sin\theta - 5 = 0$ for $0^\circ \le \theta \le 360^\circ$. **[7]**

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24 In this question you must show detailed reasoning.

(a) The polynomial $f(x)$ is defined by $f(x) = 2x^3 + 3x^2 - 8x + 3$.

(i) Show that $f(1) = 0$. **[1]**

(ii) Solve the equation $f(x) = 0$. **[4]**

(b) Hence solve the equation $2\sin^3\theta + 3\sin^2\theta - 8\sin\theta + 3 = 0$ for $0° \leqslant \theta < 360°$. **[5]**

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25 **(a)** Show that the equation $2\cos x \tan^2 x = 3(1+\cos x)$ can be expressed in the form

$5\cos^2 x + 3\cos x - 2 = 0.$ **[3]**

(b) **In this question you must show detailed reasoning.**

Hence solve the equation

$2\cos 3\theta \tan^2 3\theta = 3(1+\cos 3\theta)$,

giving all values of θ between 0° and 120°, correct to **1** decimal place where appropriate. **[6]**

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26 **In this question you must show detailed reasoning.**

(a) Solve the equation $4\sin^2\theta = \tan^2\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

(b) Prove that $\dfrac{\sin^2\theta - 1 + \cos\theta}{1 - \cos\theta} \equiv \cos\theta \quad (\cos\theta \neq 1)$. **[3]**

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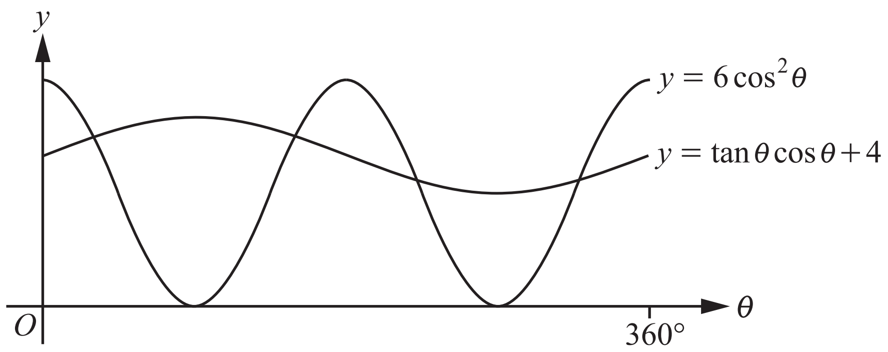
27 In this question you must show detailed reasoning.

(a) Show that the equation $6\cos^2\theta = \tan\theta\cos\theta + 4$

can be expressed in the form $6\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

(b)



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = \tan\theta\cos\theta + 4$, where θ is in degrees.

Solve the inequality $6\cos^2\theta > \tan\theta\cos\theta + 4$ for $0° < \theta < 360°$. **[5]**

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28 A Ferris wheel at a fairground rotates in a vertical plane. The height above the ground of a seat on the wheel is h metres at time t seconds after the seat is at its lowest point.

The height is given by the equation $h = 15 - 14\cos(kt)^\circ$, where k is a positive constant.

(a) **(i)** Write down the greatest height of a seat above the ground. **[1]**

(ii) Write down the least height of a seat above the ground. **[1]**

(b) Given that a seat first returns to its lowest point after 150 seconds, calculate the value of k. **[2]**

(c) Determine for how long a seat is 20 metres or more above the ground during one complete revolution. Give your answer correct to the nearest tenth of a second. **[4]**

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29 A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

(i) State the period of $y = \sin(ax)$, giving your answer in an exact form in terms of a. **[1]**

(ii) Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, where k is a positive constant, find the values of a and k. **[3]**

(iii) Given instead that $\sin(ax) = \sqrt{3}\cos(ax)$, find the two smallest positive solutions for x, giving your answers in an exact form in terms of a. **[4]**