

# SUVAT Equations (with vectors)

Worked Solutions

1. A particle,  $P$ , moves with constant acceleration  $(2\mathbf{i} - 3\mathbf{j})\text{m s}^{-2}$

At time  $t = 0$ , the particle is at the point  $A$  and is moving with velocity  $(-\mathbf{i} + 4\mathbf{j})\text{m s}^{-1}$

At time  $t = T$  seconds,  $P$  is moving in the direction of vector  $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of  $T$ .

(4)

At time  $t = 4$  seconds,  $P$  is at the point  $B$ .

(b) Find the distance  $AB$ .

(4)

a)  $\underline{s} = ?$

$$\underline{u} = -\underline{i} + 4\underline{j}$$

$$\underline{v} = \underline{u} + \underline{a}t$$

$$\underline{v} = k(3\underline{i} - 4\underline{j})$$

$$\underline{a} = 2\underline{i} - 3\underline{j}$$

$$\begin{pmatrix} 3k \\ -4k \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} T$$

$$t = T$$

$$\textcircled{1} \quad 3k = -1 + 2T$$

$$\textcircled{2} \quad -4k = 4 - 3T$$

$$\textcircled{1} \times 4 \quad 12k = -4 + 8T \quad \textcircled{3}$$

$$\textcircled{2} \times 3 \quad -12k = 12 - 9T \quad \textcircled{4}$$

$$\textcircled{3} + \textcircled{4} \quad 0 = 8 - T \quad \Rightarrow T = 8 \text{ s}$$

b)  $\underline{s} = ?$

$$\underline{u} = -\underline{i} + 4\underline{j}$$

$$\underline{s} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} 4 + \frac{1}{2} \begin{pmatrix} 2 \\ -3 \end{pmatrix} 4^2$$

$$\underline{v} = ?$$

$$\underline{a} = 2\underline{i} - 3\underline{j}$$

$$= \begin{pmatrix} -4 \\ 16 \end{pmatrix} + \begin{pmatrix} 16 \\ -24 \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$$

$$t = 4$$

$$d_{AB} = \left| \begin{pmatrix} 12 \\ -8 \end{pmatrix} \right| = \sqrt{12^2 + 8^2}$$

$$= 14.4 \text{ m to 3 s.f.}$$



2. A particle  $P$  moves with constant acceleration  $(2\mathbf{i} - 3\mathbf{j})\text{ms}^{-2}$

At time  $t = 0$ ,  $P$  is moving with velocity  $4\mathbf{i}\text{ms}^{-1}$

(a) Find the velocity of  $P$  at time  $t = 2$  seconds.

(2)

At time  $t = 0$ , the position vector of  $P$  relative to a fixed origin  $O$  is  $(\mathbf{i} + \mathbf{j})\text{m}$ .

(b) Find the position vector of  $P$  relative to  $O$  at time  $t = 3$  seconds.

(2)

a)  $\underline{s} = /$   
 $\underline{u} = 4\mathbf{i}$   
 $\underline{v} =$   
 $\underline{a} = 2\mathbf{i} - 3\mathbf{j}$   
 $\underline{t} = 2$

$$v = u + at$$

$$= \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} 2$$

$$= \begin{pmatrix} 8 \\ -6 \end{pmatrix} = \boxed{8\mathbf{i} - 6\mathbf{j}} \text{ms}^{-1}$$

b)  $\underline{s} =$   
 $\underline{u} = 4\mathbf{i}$   
 $\underline{v} =$   
 $\underline{a} = 2\mathbf{i} - 3\mathbf{j}$   
 $\underline{t} = 3$

$$s = \begin{pmatrix} 4 \\ 0 \end{pmatrix} 3 + \frac{1}{2} \begin{pmatrix} 2 \\ -3 \end{pmatrix} 3^2$$

$$= \begin{pmatrix} 12 \\ 0 \end{pmatrix} + \begin{pmatrix} 9 \\ -\frac{27}{2} \end{pmatrix} = \begin{pmatrix} 21 \\ -\frac{27}{2} \end{pmatrix}$$

$$\underline{r}(3) = \underline{r}_0 + \underline{s}$$

$$= \mathbf{i} + \mathbf{j} + 21\mathbf{i} - \frac{27}{2}\mathbf{j}$$

$$= \boxed{22\mathbf{i} - \frac{25}{2}\mathbf{j}}$$

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3. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors due east and due north respectively]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time  $t = 0$ , the boat is at the fixed point  $O$  and is moving due north with speed  $0.6 \text{ m s}^{-1}$ .

Relative to  $O$ , the position vector of the boat at time  $t$  seconds is  $\mathbf{r}$  metres.

At time  $t = 15$ , the velocity of the boat is  $(10.5\mathbf{i} - 0.9\mathbf{j}) \text{ m s}^{-1}$ .

The acceleration of the boat is constant.

- (a) Show that the acceleration of the boat is  $(0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$ . (2)
- (b) Find  $\mathbf{r}$  in terms of  $t$ . (2)
- (c) Find the value of  $t$  when the boat is north-east of  $O$ . (3)
- (d) Find the value of  $t$  when the boat is moving in a north-east direction. (3)

a)

$$\underline{u} = 0.6\mathbf{j}$$

$$\underline{v} = 10.5\mathbf{i} - 0.9\mathbf{j}$$

$$\underline{a} = ?$$

$$t = 15$$

$$\underline{v} = \underline{u} + \underline{a}t$$

$$\begin{pmatrix} 10.5 \\ -0.9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.6 \end{pmatrix} + 15\underline{a}$$

$$\begin{pmatrix} 10.5 \\ -1.5 \end{pmatrix} = 15\underline{a}$$

$$\Rightarrow \underline{a} = 0.7\mathbf{i} - 0.1\mathbf{j}$$

b)

$$\underline{s} = \underline{r}$$

$$\underline{u} = 0.6\mathbf{j}$$

$$\underline{a} = 0.7\mathbf{i} - 0.1\mathbf{j}$$

$$t = t$$

$$\underline{r} = 0.6\mathbf{j}t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})t^2$$

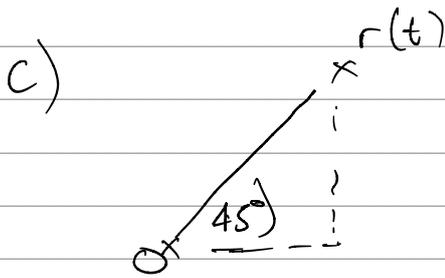
$$= 0.35t^2\mathbf{i} + (0.6t - 0.05t^2)\mathbf{j}$$

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Question 3 continued



$r(t)$  is North east of  $O$  when

$$r_x(t) = r_y(t)$$

$$\text{i.e. } 0.35t^2 = 0.6t - 0 - 0.05t^2$$

$$0.4t^2 - 0.6t = 0$$

$$t(0.4t - 0.6) = 0$$

$$t = 0, \quad t = 1.5$$

ignore since this is when the boat is at the origin.

d)

$$\underline{s} = /$$

$$\underline{u} = 0.6\mathbf{j}$$

$$\underline{v} = k\mathbf{i} + k\mathbf{j}$$

$$\underline{a} = 0.7\mathbf{i} - 0.1\mathbf{j}$$

$$t = ?$$

$$\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 0.6 \end{pmatrix} + \begin{pmatrix} 0.7 \\ -0.1 \end{pmatrix} t$$

$$\left. \begin{array}{l} \textcircled{1} \quad k = 0.7t \\ \textcircled{2} \quad k = 0.6 - 0.1t \end{array} \right\} \Rightarrow \begin{array}{l} 0.7t = 0.6 - 0.1t \\ t = 0.75 \text{ s} \end{array}$$

4. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point  $O$ .]

A particle  $P$  moves with constant acceleration.

At time  $t = 0$ , the particle is at  $O$  and is moving with velocity  $(2\mathbf{i} - 3\mathbf{j})\text{ms}^{-1}$

At time  $t = 2$  seconds,  $P$  is at the point  $A$  with position vector  $(7\mathbf{i} - 10\mathbf{j})\text{m}$ .

- (a) Show that the magnitude of the acceleration of  $P$  is  $2.5\text{ms}^{-2}$  (4)

At the instant when  $P$  leaves the point  $A$ , the acceleration of  $P$  changes so that  $P$  now moves with constant acceleration  $(4\mathbf{i} + 8.8\mathbf{j})\text{ms}^{-2}$

At the instant when  $P$  reaches the point  $B$ , the direction of motion of  $P$  is north east.

- (b) Find the time it takes for  $P$  to travel from  $A$  to  $B$ . (4)

a)  $\underline{s} = 7\mathbf{i} - 10\mathbf{j}$  (since particle starts at  $O$ )

$\underline{u} = 2\mathbf{i} - 3\mathbf{j}$

$\underline{v} = /$

$\underline{a} = ?$

$t = 2$

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\begin{pmatrix} 7 \\ -10 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} 2 + \frac{1}{2}\underline{a}(4)$

$\begin{pmatrix} 7 \\ -10 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} + 2\underline{a}$

$\begin{pmatrix} 3 \\ -4 \end{pmatrix} = 2\underline{a} \quad \underline{a} = \begin{pmatrix} 3/2 \\ -2 \end{pmatrix}$

$|\underline{a}| = \sqrt{(3/2)^2 + (-2)^2}$

$= \boxed{2.5\text{ms}^{-2}}$

b) from part (a),  $\underline{v} = \underline{u} + \underline{a}t$

$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -2 \end{pmatrix} 2$

$= \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

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## Question 4 continued

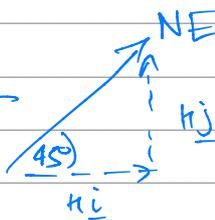
$$S = /$$

$$\underline{u} = 5\underline{i} - 7\underline{j}$$

$$\underline{v} = k\underline{i} + k\underline{j}$$

$$\underline{a} = 4\underline{i} + 8.8\underline{j}$$

$$t = ?$$



u in part (b) is equal to v of part (a)

$$\underline{v} = \underline{u} + \underline{a}t$$

$$\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \end{pmatrix} + \begin{pmatrix} 4 \\ 8.8 \end{pmatrix} t$$

$$\begin{cases} \textcircled{1} & k = 5 + 4t \\ \textcircled{2} & k = -7 + 8.8t \end{cases} \Rightarrow \begin{cases} 5 + 4t = -7 + 8.8t \\ \boxed{t = 2.5 \text{ s}} \end{cases}$$

5. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors and position vectors are given relative to a fixed origin  $O$ ]

A particle  $P$  is moving on a smooth horizontal plane.

The particle has constant acceleration  $(2.4\mathbf{i} + \mathbf{j})\text{ms}^{-2}$

At time  $t = 0$ ,  $P$  passes through the point  $A$ .

At time  $t = 5$  s,  $P$  passes through the point  $B$ .

The velocity of  $P$  as it passes through  $A$  is  $(-16\mathbf{i} - 3\mathbf{j})\text{ms}^{-1}$

(a) Find the speed of  $P$  as it passes through  $B$ . (4)

The position vector of  $A$  is  $(44\mathbf{i} - 10\mathbf{j})\text{m}$ .

At time  $t = T$  seconds, where  $T > 5$ ,  $P$  passes through the point  $C$ .

The position vector of  $C$  is  $(4\mathbf{i} + c\mathbf{j})\text{m}$ .

(b) Find the value of  $T$ . (3)

(c) Find the value of  $c$ . (3)

2)  $\underline{s}$   
 $\underline{u} = -16\mathbf{i} - 3\mathbf{j}$   
 $\underline{v} = ?$   
 $\underline{a} = 2.4\mathbf{i} + \mathbf{j}$   
 $\underline{t} = 5$

$$\underline{v} = \underline{u} + \underline{a}t$$

$$\underline{v} = \begin{pmatrix} -16 \\ -3 \end{pmatrix} + \begin{pmatrix} 2.4 \\ 1 \end{pmatrix} 5$$

$$= \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\text{speed} = |\underline{v}| = \sqrt{4^2 + 2^2} = \sqrt{20} \text{ms}^{-1} = 4.47 \text{ms}^{-1} \text{ to 3sf}$$

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## Question 5 continued

$$b) \quad \underline{s} = \underline{r}_c - \underline{r}_A$$

$$= \begin{pmatrix} 4 \\ c \end{pmatrix} - \begin{pmatrix} 14 \\ -10 \end{pmatrix} = \begin{pmatrix} -40 \\ c+10 \end{pmatrix}$$

$$\underline{s} = -40\hat{i} + (c+10)\hat{j}$$

$$\underline{u} = -16\hat{i} - 3\hat{j}$$

$$v = 1$$

$$a = 2.4\hat{i} + \hat{j}$$

$$t = T$$

$$\begin{pmatrix} -40 \\ c+10 \end{pmatrix} = \begin{pmatrix} -16 \\ -3 \end{pmatrix} T + \frac{1}{2} \begin{pmatrix} 2.4 \\ 1 \end{pmatrix} T^2 \quad \Rightarrow \quad -40 = -16T + 1.2T^2$$

$$1.2T^2 - 16T + 40 = 0$$

$$T = 10s, \quad \cancel{10/3} s$$

$$T > 5$$

$$c) \quad \text{from (b),} \quad c+10 = -3T + \frac{1}{2}T^2$$

$$T=10 \Rightarrow c+10 = -30 + 50$$

$$c+10 = 20$$

$$\boxed{c = 10}$$

6. A particle  $P$  moves with acceleration  $(4\mathbf{i} - 5\mathbf{j})\text{ms}^{-2}$

At time  $t = 0$ ,  $P$  is moving with velocity  $(-2\mathbf{i} + 2\mathbf{j})\text{ms}^{-1}$

(a) Find the velocity of  $P$  at time  $t = 2$  seconds.

(2)

At time  $t = 0$ ,  $P$  passes through the origin  $O$ .

At time  $t = T$  seconds, where  $T > 0$ , the particle  $P$  passes through the point  $A$ .

The position vector of  $A$  is  $(\lambda\mathbf{i} - 4.5\mathbf{j})\text{m}$  relative to  $O$ , where  $\lambda$  is a constant.

(b) Find the value of  $T$ .

(4)

(c) Hence find the value of  $\lambda$

(2)

a)  $\underline{s} = /$

$$\underline{u} = -2\mathbf{i} + 2\mathbf{j}$$

$$\underline{v} = \underline{u} + \underline{a}t$$

$$\underline{v} = ?$$

$$\underline{a} = 4\mathbf{i} - 5\mathbf{j}$$

$$= \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix} 2$$

$$t = 2$$

$$= \begin{pmatrix} 6 \\ -8 \end{pmatrix} = (6\mathbf{i} - 8\mathbf{j}) \text{ms}^{-1}$$

b)  $\underline{s} = \underline{r}_A = \lambda\mathbf{i} - 4.5\mathbf{j}$

since  $\underline{r}_0 = \underline{0}$

$$\underline{u} = -2\mathbf{i} + 2\mathbf{j}$$

$$\underline{v} = /$$

$$\underline{a} = 4\mathbf{i} - 5\mathbf{j}$$

$$t = T$$

$$\begin{pmatrix} \lambda \\ -4.5 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} T + \frac{1}{2} \begin{pmatrix} 4 \\ -5 \end{pmatrix} T^2$$

$$\textcircled{2} \quad -4.5 = 2T - 2.5T^2$$

$$2.5T^2 - 2T - 4.5 = 0$$

$$5T^2 - 4T - 9 = 0$$

$$(5T - 9)(T + 1) = 0$$

$$\boxed{T = 9/5} \quad \text{or } T = -1$$

$$\boxed{T = 1.8 \text{ s}}$$

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## Question 6 continued

c) from part (b),

$$(1) : \lambda = -2T + 2T^2$$

$$\text{Sub in } T=1.8 : \lambda = -2(1.8) + 2(1.8)^2$$

$$= \boxed{2.88} = \boxed{\frac{72}{25}}$$

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7. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors.]

A particle  $P$  of mass  $2\text{kg}$  moves under the action of two forces,  $(p\mathbf{i} + q\mathbf{j})\text{N}$  and  $(2q\mathbf{i} + p\mathbf{j})\text{N}$ , where  $p$  and  $q$  are constants.

Given that the acceleration of  $P$  is  $(\mathbf{i} - \mathbf{j})\text{ms}^{-2}$

(a) find the value of  $p$  and the value of  $q$ . (5)

(b) Find the size of the angle between the direction of the acceleration and the vector  $\mathbf{j}$ . (2)

At time  $t = 0$ , the velocity of  $P$  is  $(3\mathbf{i} - 4\mathbf{j})\text{ms}^{-1}$

At  $t = T$  seconds,  $P$  is moving in the direction of the vector  $(11\mathbf{i} - 13\mathbf{j})$ .

(c) Find the value of  $T$ . (5)

a)  $F_{\text{res}} = ma$

$$\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 2q \\ p \end{pmatrix} = 2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Note  $2g$  isn't involved

in  $F_{\text{res}}$  because the

particle "moves under

the action of two forces"

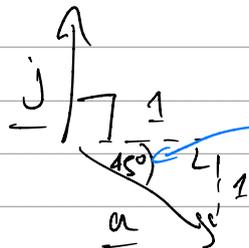
①  $p + 2q = 2$

②  $q + p = -2$

① - ②  $\boxed{q = 4}$

$4 + p = -2 \Rightarrow \boxed{p = -6}$

b)



angle =  $90 + 45^\circ$

=  $\boxed{135^\circ}$

because the triangle is isosceles.

c) P.T.O

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## Question 7 continued

$$c) S = /$$

$$u = 3\hat{i} - 4\hat{j}$$

$$v = 11k\hat{i} - 13k\hat{j}$$

$$a = \hat{i} - \hat{j}$$

$$t = T$$

$$\begin{pmatrix} 11k \\ -13k \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} T$$

$$11k = 3 + T \quad \textcircled{A}$$

$$-13k = -4 - T \quad \textcircled{B}$$

$$\textcircled{A} + \textcircled{B} \quad -2k = -1$$

$$k = 1/2$$

$$11(1/2) = 3 + T$$

$$T = 11/2 - 3 = 5/2 = 2.5 \text{ s}$$

8. A particle  $P$  is moving with constant acceleration  $(-4\mathbf{i} + \mathbf{j})\text{ms}^{-2}$

At time  $t = 0$ ,  $P$  has velocity  $(14\mathbf{i} - 5\mathbf{j})\text{ms}^{-1}$

(a) Find the speed of  $P$  at time  $t = 2$  seconds.

(3)

(b) Find the size of the angle between the direction of  $\mathbf{i}$  and the direction of motion of  $P$  at time  $t = 2$  seconds.

(3)

At time  $t = T$  seconds,  $P$  is moving in the direction of vector  $(2\mathbf{i} - 3\mathbf{j})$

(c) Find the value of  $T$

(4)

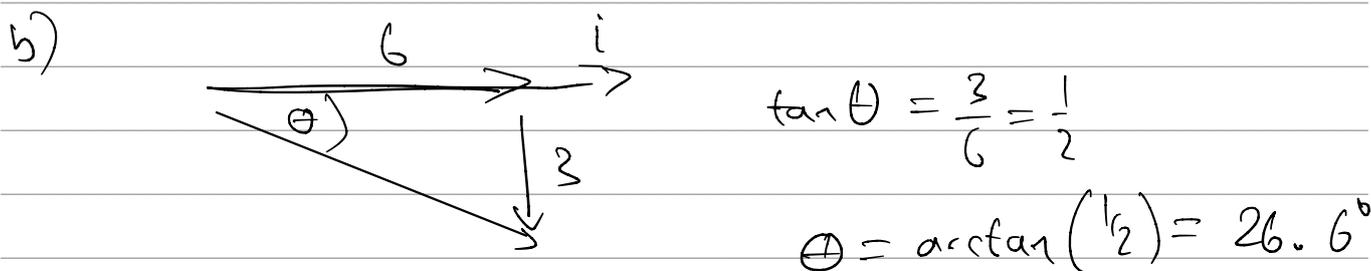
a)  $\underline{s} = /$   
 $\underline{u} = 14\mathbf{i} - 5\mathbf{j}$   
 $\underline{v} = ?$   
 $\underline{a} = -4\mathbf{i} + \mathbf{j}$   
 $t = 2$

$$\underline{v} = \begin{pmatrix} 14 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} 2$$

$$= \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$\text{Speed} = |\underline{v}| = \sqrt{6^2 + 3^2} = \sqrt{45}$$

$$= 6.71 \text{ms}^{-1} \text{ to 3sf}$$



c)  $\underline{s} = /$   
 $\underline{u} = 14\mathbf{i} - 5\mathbf{j}$   
 $\underline{v} = 2k\mathbf{i} - 3k\mathbf{j}$   
 $\underline{a} = -4\mathbf{i} + \mathbf{j}$   
 $t = T$

$$\begin{pmatrix} 2k \\ -3k \end{pmatrix} = \begin{pmatrix} 14 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} T$$

$$2k = 14 - 4T \Rightarrow 6k = 42 - 12T \quad \textcircled{A}$$

$$-3k = -5 + T \Rightarrow -6k = -10 + 2T \quad \textcircled{B}$$

$$\textcircled{A} + \textcircled{B} \quad 0 = 32 - 10T$$

$$\Rightarrow \underline{T = 3.2 \text{ s}}$$

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9. A particle  $P$  is moving with constant acceleration.

At time  $t = 1$  second,  $P$  has velocity  $(-i + 4j) \text{ ms}^{-1}$

At time  $t = 4$  seconds,  $P$  has velocity  $(5i - 8j) \text{ ms}^{-1}$

Find the speed of  $P$  at time  $t = 3.5$  seconds.

(6)

$$\underline{s} = /$$

$$\underline{u} = -i + 4j$$

$$\underline{v} = 5i - 8j$$

$$\underline{a} = ?$$

$$t = 3$$

$$\begin{pmatrix} 5 \\ -8 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + 3\underline{a}$$

$$3\underline{a} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$\underline{a} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\underline{s} = /$$

$$\underline{u} = -i + 4j$$

$$\underline{v} = ?$$

$$\underline{a} = 2i - 4j$$

$$t = 2.5s$$

$$\underline{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} 2.5$$

$$= \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$\text{Speed} = |\underline{v}| = \sqrt{4^2 + 6^2} = \sqrt{52} = 7.21 \text{ ms}^{-1}$$

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[In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal perpendicular unit vectors.]

10. A particle  $P$  rests in equilibrium on a smooth horizontal plane.

A system of **three** forces,  $\mathbf{F}_1$  N,  $\mathbf{F}_2$  N and  $\mathbf{F}_3$  N where

$$\mathbf{F}_1 = (3c\mathbf{i} + 4c\mathbf{j})$$

$$\mathbf{F}_2 = (-14\mathbf{i} + 7\mathbf{j})$$

is applied to  $P$ .

Given that  $P$  remains in equilibrium,

- (a) find  $\mathbf{F}_3$  in terms of  $c$ ,  $\mathbf{i}$  and  $\mathbf{j}$ . (2)

The force  $\mathbf{F}_3$  is **removed** from the system.

Given that  $c = 2$

- (b) find the size of the angle between the direction of  $\mathbf{i}$  and the direction of the resultant force acting on  $P$ . (4)

The mass of  $P$  is  $m$  kg.

Given that the magnitude of the acceleration of  $P$  is  $8.5 \text{ ms}^{-2}$

- (c) find the value of  $m$ . (4)

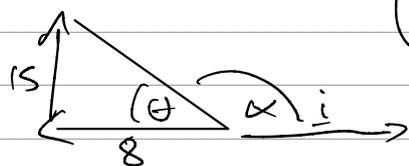
a)  $F_{res} = ma$

$F_1 + F_2 + F_3 = \underline{0}$       Since  $P$  remains in equilibrium ( $a=0$ )

$\begin{pmatrix} 3c \\ 4c \end{pmatrix} + \begin{pmatrix} -14 \\ 7 \end{pmatrix} + F_3 = \underline{0}$

$F_3 = \begin{pmatrix} 14 \\ -7 \end{pmatrix} + \begin{pmatrix} -3c \\ -4c \end{pmatrix} = \begin{pmatrix} 14-3c \\ -7-4c \end{pmatrix} = (14-3c)\underline{i} + (-7-4c)\underline{j}$

b)  $F_{res} = F_1 + F_2 = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + \begin{pmatrix} -14 \\ 7 \end{pmatrix} = \begin{pmatrix} -8 \\ 15 \end{pmatrix}$



$\theta = \tan^{-1}\left(\frac{15}{8}\right)$

$\alpha = 180 - \theta = 180 - \tan^{-1}\left(\frac{15}{8}\right) = \boxed{118^\circ}$

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## Question 10 continued

$$c) F_{rs} = ma$$

$$|F_{rs}| = m |a|$$

$$\sqrt{(-8)^2 + 15^2} = m \times 8.5$$

$$17 = 8.5m$$

$$\Rightarrow m = 2.$$

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11. A particle  $P$  is moving in a plane with constant acceleration.  
 The velocity,  $\text{v ms}^{-1}$ , of  $P$  at time  $t$  seconds is given by

$$\mathbf{v} = (7 - 5t)\mathbf{i} + (12t - 20)\mathbf{j}$$

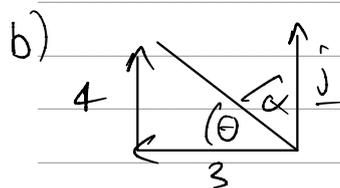
- (a) Find the speed of  $P$  when  $t = 2$  (3)
- (b) Find, to the nearest degree, the size of the angle between the direction of motion of  $P$  and the vector  $\mathbf{j}$ , when  $t = 2$  (3)

The constant acceleration of  $P$  is  $\mathbf{a ms}^{-2}$

- (c) Find  $\mathbf{a}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$  (3)
- (d) Find the value of  $t$  when  $P$  is moving in the direction of the vector  $(-5\mathbf{i} + 8\mathbf{j})$  (4)

a)  $\underline{v(2)} = (7 - 10)\underline{i} + (24 - 20)\underline{j} = -3\underline{i} + 4\underline{j}$

Speed =  $|\underline{v(2)}| = \sqrt{3^2 + 4^2} = \boxed{5} \text{ ms}^{-1}$



$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\alpha = 90 - \theta = \boxed{36.9^\circ} \text{ to 3sf}$$

c)  $\underline{a} = \frac{d\underline{v}}{dt} = \boxed{-5\underline{i} + 12\underline{j}}$

alternatively could set up a suvat equation, but good mathematicians are lazy (efficient).

d) No need to set up suvat here either.

$$(7 - 5t)\underline{i} + (12t - 20)\underline{j} = -5k\underline{i} + 8k\underline{j}$$

$$7 - 5t = -5k \quad \textcircled{1}$$

$$\textcircled{1} \times 4$$

$$28 - 20t = -20k \quad \textcircled{A}$$

$$\therefore 2 \left( \begin{array}{l} 12t - 20 = 8k \\ 6t - 10 = 4k \end{array} \right) \div 2$$

$$\textcircled{2} \times 5$$

$$30t - 50 = 20k \quad \textcircled{B}$$

$$6t - 10 = 4k \quad \textcircled{2}$$

$$\textcircled{A} + \textcircled{B}$$

$$10t - 22 = 0 \quad \boxed{t = 2.2\text{s}}$$

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12. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors.]

A particle  $P$  of mass  $4\text{ kg}$  is at rest at the point  $A$  on a smooth horizontal plane.

At time  $t = 0$ , two forces,  $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\text{ N}$  and  $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\text{ N}$ , where  $\lambda$  and  $\mu$  are constants, are applied to  $P$

Given that  $P$  moves in the direction of the vector  $(3\mathbf{i} + \mathbf{j})$

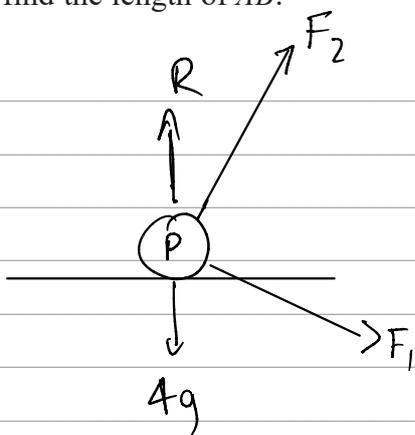
(a) show that

$$\lambda - 3\mu + 7 = 0 \quad (4)$$

At time  $t = 4$  seconds,  $P$  passes through the point  $B$ .

Given that  $\lambda = 2$

(b) find the length of  $AB$ .



a) Note that  $R = 4g$  as well because the particle is at rest initially. (5)

$$\mathbf{F}_{\text{res}} = m\mathbf{a}$$

$$\mathbf{F}_1 + \mathbf{F}_2 = 4\mathbf{a}$$

$$\begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 4 \begin{pmatrix} 3h \\ k \end{pmatrix}$$

Note that

$$\mathbf{a} = \begin{pmatrix} 3h \\ k \end{pmatrix}$$

$$4 + \lambda = 12h \quad (1)$$

$$-1 + \mu = 4k \quad (2)$$

because the particle must be accelerating in the same direction as it is moving, otherwise  $P$  wouldn't move in that direction.

$$(2) \times 3 \quad -3 + 3\mu = 12k$$

$$\Rightarrow 4 + \lambda = -3 + 3\mu$$

$$\lambda - 3\mu + 7 = 0$$

$$\begin{aligned} \text{b) } \lambda = 2 &\Rightarrow 9 - 3\mu = 0 \\ &\Rightarrow \mu = 3 \end{aligned}$$

P.7.0

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## Question 12 continued

from part (a)

$$\begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 4 \begin{pmatrix} 3h \\ k \end{pmatrix}$$

$$\Rightarrow \begin{cases} 6 = 12h \\ 2 = 4k \end{cases} \Rightarrow h = \frac{1}{2} \quad \therefore \underline{a} = \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix}$$

$$\underline{S} = ?$$

$$\underline{u} = \underline{0}$$

$$\underline{v} = \underline{r}$$

$$\underline{a} = \frac{3}{2}\underline{i} + \frac{1}{2}\underline{j}$$

$$\underline{t} = 4$$

$$\underline{S} = \frac{1}{2} \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix} 4^2$$

$$= \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$

$$d_{AB} = \sqrt{12^2 + 4^2}$$

$$= \boxed{12.6} \text{ arbitrary units to 3 sf}$$

$$\left( = \boxed{4\sqrt{10}} \right).$$

13. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively.]

A particle  $P$  moves with constant acceleration  $(-\lambda\mathbf{i} + 2\lambda\mathbf{j})\text{ms}^{-2}$ , where  $\lambda$  is a positive constant.

At time  $t = 0$ , the velocity of  $P$  is  $(5\mathbf{i} - 8\mathbf{j})\text{ms}^{-1}$

(a) Find the velocity of  $P$  when  $t = 5$  s, giving your answer in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\lambda$ . (2)

The speed of  $P$  when  $t = 5$  s is  $13\text{ms}^{-1}$

(b) Show that (3)

$$25\lambda^2 - 42\lambda - 16 = 0$$

(c) Find the direction of motion of  $P$  when  $t = 4$  s, giving your answer as a bearing to the nearest degree. (5)

a)

$$\begin{aligned} \underline{s} &= 5\mathbf{i} - 8\mathbf{j} \\ \underline{v} &= ? \\ \underline{a} &= -\lambda\mathbf{i} + 2\lambda\mathbf{j} \\ t &= 5 \end{aligned}$$

$$v = \begin{pmatrix} 5 \\ -8 \end{pmatrix} + \begin{pmatrix} -\lambda \\ 2\lambda \end{pmatrix} 5$$

$$= \begin{pmatrix} 5 - 5\lambda \\ -8 + 10\lambda \end{pmatrix}$$

$$= (5 - 5\lambda)\mathbf{i} + (10\lambda - 8)\mathbf{j}$$

b)

$$\sqrt{(5 - 5\lambda)^2 + (10\lambda - 8)^2} = 13$$

$$(5 - 5\lambda)^2 + (10\lambda - 8)^2 = 169$$

$$25 - 50\lambda + 25\lambda^2 + 100\lambda^2 - 160\lambda + 64 = 169$$

$$\div 5 \left( \begin{aligned} 125\lambda^2 - 210\lambda - 80 &= 0 \\ 25\lambda^2 - 42\lambda - 16 &= 0 \end{aligned} \right) \div 5$$

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## Question 13 continued

$$\begin{aligned}
 c) \quad s &= 1 \\
 \underline{u} &= 5\mathbf{i} - 8\mathbf{j} \\
 \underline{v} &= ? \\
 \underline{a} &= -2\mathbf{i} + 4\mathbf{j} \\
 t &= 4
 \end{aligned}$$

$$25\lambda^2 - 42\lambda - 16 = 0$$

$$(25\lambda + 8)(\lambda - 2) = 0$$

$$\lambda = \frac{-8}{25} \quad \text{or} \quad \lambda = 2$$

$\lambda$  is a +ve constant so  $\lambda = 2$ .

$$\underline{v} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} 4$$

$$\Rightarrow \underline{a} = -2\mathbf{i} + 4\mathbf{j}$$

$$= \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$



$$\theta = \tan^{-1}\left(\frac{8}{3}\right)$$

$$\text{bearing} = 270^\circ + \theta$$

$$= 339^\circ \quad \text{to nearest degree}$$

Leave blank

14. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors.]

A particle  $A$  of mass  $0.5 \text{ kg}$  is at rest on a smooth horizontal plane.

At time  $t = 0$ , two forces,  $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j}) \text{ N}$  and  $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$ , where  $p$  and  $q$  are constants, are applied to  $A$ .

Given that  $A$  moves in the direction of the vector  $(\mathbf{i} - 2\mathbf{j})$ ,

(a) show that  $2p + q - 4 = 0$  (4)

Given that  $p = 5$

(b) find the speed of  $A$  at time  $t = 4$  seconds. (5)

a)  $F_{\text{res}} = ma$

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = 0.5 \begin{pmatrix} k \\ -2k \end{pmatrix}$$

See June 2022 a-level question from this pack if you're unsure as to why  $0.5g$  isn't included

(A)  $-3 + p = 0.5k$

(B)  $2 + q = -k$

(A)  $\times 2$   $-6 + 2p = k$  (C)

(C) + (B)  $2p + q - 4 = 0$

b)  $p = 5 \Rightarrow 10 + q - 4 = 0$   
 $\Rightarrow q = -6$

$s =$

$u = 0$

$v = ?$

$a = 4\mathbf{i} - 8\mathbf{j}$

$t = 4$

using part a),

$k = -6 + 2(5) = 4$

$\therefore a = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$

$v = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ -8 \end{pmatrix} 4 = \begin{pmatrix} 16 \\ -32 \end{pmatrix}$

Speed =  $|v| = \sqrt{16^2 + 32^2} = 16\sqrt{5} = 35.8 \text{ m s}^{-1}$

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