Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference **8FM0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1. $$f(z) = z^3 + pz^2 + qz - 15$$

where p and q are real constants.

Given that the equation $f(z) = 0$ has roots

$$\alpha, \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right)$$

(a) solve completely the equation $f(z) = 0$

(5)

(b) Hence find the value of p.

(2)

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Question 1 continued

(Total for Question 1 is 7 marks)

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2. The plane Π passes through the point A and is perpendicular to the vector $\mathbf{n}$

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

where O is the origin,

(a) find a Cartesian equation of Π.

(2)

With respect to the fixed origin O, the line l is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$$

The line l intersects the plane Π at the point X.

(b) Show that the acute angle between the plane Π and the line l is 21.2° correct to one decimal place.

(4)

(c) Find the coordinates of the point X.

(4)

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Question 2 continued

(Total for Question 2 is 10 marks)

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3. Tyler invested a total of £5000 across three different accounts; a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%
- the property bond account had increased in value by 3.5%
- the share dealing account had **decreased** in value by 2.5%

- the total value across Tyler's three accounts had increased by £79

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

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Question 3 continued

(Total for Question 3 is 7 marks)

4. The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

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Question 4 continued

(Total for Question 4 is 5 marks)

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5.

$$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that **M** is non-singular.

(2)

The hexagon R is transformed to the hexagon S by the transformation represented by the matrix **M**.

Given that the area of hexagon R is 5 square units,

(b) find the area of hexagon S.

(1)

The matrix **M** represents an enlargement, with centre (0, 0) and scale factor k, where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(c) Find the value of k.

(2)

(d) Find the value of θ.

(2)

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Question 5 continued

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(Total for Question 5 is 7 marks)

6. (a) Prove by induction that for all positive integers n,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

(b) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r+6)(r-6) = \frac{1}{4}n(n+1)(n-8)(n+9)$$

(4)

(c) Hence find the value of n that satisfies

$$\sum_{r=1}^{n} r(r+6)(r-6) = 17\sum_{r=1}^{n} r^2$$

(5)

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Question 6 continued

(Total for Question 6 is 15 marks)

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7.

Diagrams not drawn to scale

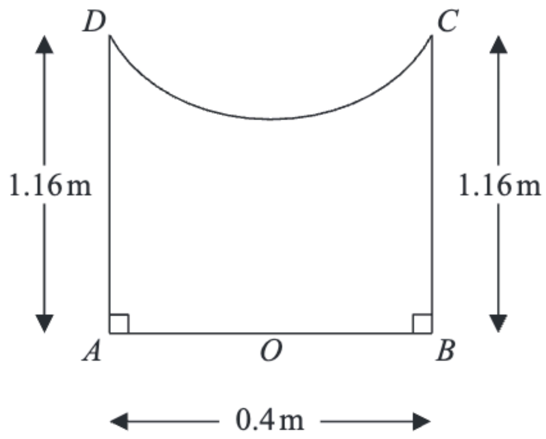

Figure 1

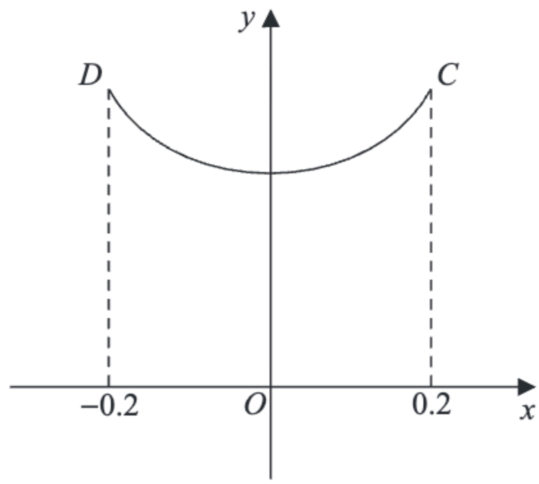

Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where k is a constant and where O is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of k.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

It was later discovered that the volume of concrete used to make the bird bath was $0.127\,\text{m}^3$ correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

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Question 7 continued

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(Total for Question 7 is 12 marks)

8. (a) Shade on an Argand diagram the set of points

$$\left\{z \in \mathbb{C} : |z - 4\mathrm{i}| \leqslant 3\right\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\mathrm{i}) \leqslant \frac{\pi}{4}\right\}$$

(6)

The complex number w satisfies

$$|w - 4\mathrm{i}| = 3$$

(b) Find the maximum value of $\arg w$ in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)

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Question 8 continued

(Total for Question 8 is 8 marks)

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9. An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish F swims from a point A to a point B.

The octopus is modelled as a fixed particle at the origin O.

Fish F is modelled as a particle moving in a straight line from A to B.

Relative to O, the coordinates of A are $(-3, 1, -7)$ and the coordinates of B are $(9, 4, 11)$, where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish F.

(7)

(b) Criticise the model in relation to fish F.

(1)

(c) Criticise the model in relation to the octopus.

(1)

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Question 9 continued

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 80 MARKS

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Paper 1: Core Pure Mathematics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	$\alpha\left(\dfrac{5}{\alpha}\right)\left(\alpha+\dfrac{5}{\alpha}-1\right)=15$	M1	1.1b
		A1	1.1b
	$\Rightarrow 5\alpha+\dfrac{25}{\alpha}-5=15 \Rightarrow \alpha^2-4\alpha+5=0$ $\Rightarrow \alpha=\dfrac{--4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$ or $(\alpha-2)^2-4+5=0 \Rightarrow \alpha=...$	M1	3.1a
	$\Rightarrow \alpha = 2\pm \text{i}$	A1	1.1b
	Hence the roots of $\text{f}(z)=0$ are $2+\text{i}, 2-\text{i}$ and 3	A1	2.2a
		(5)	
(b)	$p=-\left(\text{"}(2+\text{i})\text{"}+\text{"}(2-\text{i})\text{"}+\text{"}3\text{"}\right) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ cso	A1	1.1b
		(2)	
	1(b) alternative		
	$\text{f}(z)=(z-3)(z^2-4z+5) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ cso	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Multiplies the three given roots together and sets the result equal to 15 or -15

A1: Obtains a correct equation in α

M1: Forms a quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha = ...$

A1: $\alpha = 2\pm\text{i}$

A1: Deduces the roots are $2+\text{i}, 2-\text{i}$ and 3

(b)

M1: Applies the process of finding $-\sum\left(\text{of their three roots found in part } (a)\right)$ to give $p=...$

A1: $p=-7$ by correct solution only

(b) Alternative

M1: Applies the process expanding $(z-\text{"}3\text{"})(z-(\text{their sum})z+\text{their product})$ in order to find $p=...$

A1: $p=-7$ by correct solution only

Question	Scheme	Marks	AOs
2(a)	$\mathbf{r}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}5\\-3\\-4\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}$	M1	1.1b
	$3x-y+2z=10$	A1	2.5
		(2)	
(b)	$\begin{pmatrix}3\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\-5\\3\end{pmatrix}=8$	B1	1.1b
	$\sqrt{(3)^2+(-1)^2+(2)^2}\,.\sqrt{(-1)^2+(-5)^2+(3)^2}\,\cos\alpha="-3+5+6"$	M1	1.1b
	$\theta=90^\circ-\arccos\left(\frac{8}{\sqrt{14}.\sqrt{35}}\right)$ or $\sin\theta=\frac{8}{\sqrt{14}.\sqrt{35}}$	M1	2.1
	$\theta=21.2°$ (1 dp) * cso	A1*	1.1b
		(4)	
(c)	$3(7-\lambda)-(3-5\lambda)+2(-2+3\lambda)=10\Rightarrow\lambda=\ldots$	M1	3.1a
	$\lambda=-\frac{1}{2}$	A1	1.1b
	$\overrightarrow{OX}=\begin{pmatrix}7\\3\\-2\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix}=\begin{pmatrix}\ldots\\\ldots\\\ldots\end{pmatrix}$	M1	1.1b
	$X(7.5,5.5,-3.5)$	A1ft	1.1b
		(4)	
			(10 marks)

Notes:

(a)

M1: Attempts to apply the formula $\mathbf{r.n}=\mathbf{a.n}$

A1: Correct Cartesian notation. e.g. $3x-y+2z=10$ or $-3x+y-2z=-10$

Note: Do not allow final answer given as $\mathbf{r}\bullet(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=10$, o.e.

(b)

B1: $\overrightarrow{OA}\bullet\mathbf{n}=8$

M1: An attempt to apply the correct dot product formula between **n** and **d**

M1: Depends on previous M mark. Applies the dot product formula to find the angle between Π and l

A1*: 21.2° cso

Question 2 notes continued:	
(c)	
M1:	Substitutes l into Π and solves the resulting equation to give $\lambda = ...$
A1:	$\lambda = -\frac{1}{2}$ o.e.
M1:	Depends on previous M mark. Substitutes their λ into l and finds at least one of the coordinates
A1ft:	$(7.5, 5.5, -3.5)$ but follow through on their value of λ

Question	Scheme	Marks	AOs
3	x = value of savings account, y = value of property bond account, z = value of share dealing account	M1	3.1b
	$x+y+z=5000$ $x+400=y$ $0.015x+0.035y-0.025z=79$ **or** $1.015x+1.035y+0.975z=5079$	A1	1.1b
	Let $\mathbf{A}=\begin{pmatrix}1&1&1\\1&-1&0\\0.015&0.035&-0.025\end{pmatrix}$ or $\begin{pmatrix}1&1&1\\1&-1&0\\1.015&1.035&0.975\end{pmatrix}$		
	e.g. $\begin{pmatrix}1&1&1\\1&-1&0\\0.015&0.035&-0.025\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5000\\-400\\79\end{pmatrix}$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1&1&1\\1&-1&0\\0.015&0.035&-0.025\end{pmatrix}^{-1}\begin{pmatrix}5000\\-400\\79\end{pmatrix}=\begin{pmatrix}...\\...\\...\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1800\\2200\\1000\end{pmatrix}$	A1	1.1b
	Tyler invested £1800 in the savings account, £2200 in the property bond account and £1000 in the share dealing account	A1ft	3.2a
			(7 marks)

Notes:

M1: Attempts to set up 3 equations with 3 unknowns

A1: At least 2 equations are correct with the appropriate variables defined

M1: Sets up a matrix equation of the form, e.g. $\begin{pmatrix}...&...&...\\...&...&...\\...&...&...\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}...\\...\\...\end{pmatrix}$, where "…" are numerical values

A1: Correct matrix equation (or equivalent)

M1: Depends on previous M mark. Applies $(\text{their } \mathbf{A})^{-1}\begin{pmatrix}5000\\\text{their "}-400\text{"}\\\text{their "79"}\end{pmatrix}$ and obtains at least one value of x, y or z

A1: Correct answer

A1ft: Correct follow through answer in context

Question	Scheme	Marks	AOs
4	$\{w = x - 1 \Rightarrow\}\ x = w + 1$	B1	3.1a
	$(w+1)^3 + 3(w+1)^2 - 8(w+1) + 6 = 0$	M1	3.1a
	$w^3 + 3w^2 + 3w + 1 + 3(w^2 + 2w + 1) - 8w - 8 + 6 = 0$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
	Alternative		
	$\alpha + \beta + \gamma = -3,\ \alpha\beta + \beta\gamma + \alpha\gamma = -8,\ \alpha\beta\gamma = -6$	B1	3.1a
	sum roots $= \alpha - 1 + \beta - 1 + \gamma - 1$	M1	3.1a
	$= \alpha + \beta + \gamma - 3 = -3 - 3 = -6$		
	pair sum $= (\alpha - 1)(\beta - 1) + (\alpha - 1)(\gamma - 1) + (\beta - 1)(\gamma - 1)$		
	$= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha + \beta + \gamma) + 3$		
	$= -8 - 2(-3) + 3 = 1$		
	product $= (\alpha - 1)(\beta - 1)(\gamma - 1)$		
	$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) - 1$		
	$= -6 - (-8) - 3 - 1 = -2$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
		(5 marks)	

Notes:

B1: Selects the method of making a connection between x and w by writing $x = w + 1$

M1: Applies the process of substituting their $x = w + 1$ into $x^3 + 3x^2 - 8x + 6 = 0$

M1: Depends on previous M mark. Manipulating their equation into the form $w^3 + pw^2 + qw + r = 0$

A1: At least two of p, q, r are correct

A1: Correct final equation

Alternative

B1: Selects the method of giving three correct equations each containing α, β and γ

M1: Applies the process of finding sum roots, pair sum and product

M1: Depends on previous M mark. Applies $w^3 - (\text{their sum roots})w^2 + (\text{their pair sum})w - \text{their } \alpha\beta\gamma = 0$

A1: At least two of p, q, r are correct

A1: Correct final equation

Question	Scheme	Marks	AOs
5(a)	$\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$	M1	1.1a
	M is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area}(S) = 4(5) = 20$	B1ft	1.2
		(1)	
(c)	$k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$	M1	1.1b
	$= 2$	A1ft	1.1b
		(2)	
(d)	$\cos\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$	M1	1.1b
	$\theta = 60°$ or $\frac{\pi}{3}$	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)
M1: An attempt to find $\det(\mathbf{M})$.
A1: $\det(\mathbf{M}) = 4$ **and** reference to zero, e.g. $4 \neq 0$ **and** conclusion.

(b)
B1ft: 20 or a correct ft based on their answer to part (a).

(c)
M1: $\sqrt{(\text{their } \det\mathbf{M})}$
A1ft: 2

(d)
M1: **Either** $\cos\theta = \frac{1}{(\text{their } k)}$ **or** $\sin\theta = \frac{\sqrt{3}}{(\text{their } k)}$ **or** $\tan\theta = \sqrt{3}$
A1: $\theta = 60^{\circ}$ or $\frac{\pi}{3}$. Also accept any value satisfying $360n + 60^{\circ}$, $n \in \mathbb{Z}$, o.e.

Question	Scheme	Marks	AOs
6(a)	$n=1,\ \sum_{r=1}^{1} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$	B1	2.2a
	Assume general statement is true for $n=k$ So assume $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true	M1	2.4
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1	2.1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$	A1	1.1b
	$= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$	A1	1.1b
	Then the general result is true for $n=k+1$ As the general result has been shown to be true for $n=1$, then the general result is true for all $n \in \mathbb{Z}^+$	A1	2.4
		(6)	
(b)	$\sum_{r=1}^{n} r(r+6)(r-6) = \sum_{r=1}^{n} (r^3 - 36r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{36}{2}n(n+1)$	M1 A1	2.1 1.1b
	$= \frac{1}{4}n(n+1)\left[n(n+1) - 72\right]$	M1	1.1b
	$= \frac{1}{4}n(n+1)(n-8)(n+9)$ * cso	A1*	1.1b
		(4)	
(c)	$\frac{1}{4}n(n+1)(n-8)(n+9) = \frac{17}{6}n(n+1)(2n+1)$	M1	1.1b
	$\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$	M1	1.1b
	$3n^2 - 65n - 250 = 0$	A1	1.1b
	$(3n+10)(n-25) = 0$	M1	1.1b
	(As n must be a positive integer,) $n = 25$	A1	2.3
		(5)	
			(15 marks)

Question 6 notes:
(a) **B1:** Checks $n=1$ works for both sides of the general statement **M1:** Assumes (general result) true for $n=k$ **M1:** Attempts to add $(k+1)^{\text{th}}$ term to the sum of k terms **A1:** Correct algebraic work leading to **either** $\frac{1}{6}(k+1)(2k^2+7k+6)$ **or** $\frac{1}{6}(k+2)(2k^2+5k+3)$ **or** $\frac{1}{6}(2k+3)(k^2+3k+2)$ **A1:** Correct algebraic work leading to $\frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ **A1:** cso leading to a correct induction statement conveying all three underlined points
(b) **M1:** Substitutes at least one of the standard formulae into their expanded expression **A1:** Correct expression **M1:** Depends on previous M mark. Attempt to factorise at least $n(n+1)$ having used **A1*:** Obtains $\frac{1}{4}n(n+1)(n-8)(n+9)$ by cso
(c) **M1:** Sets their part (a) answer equal to $\frac{17}{6}n(n+1)(2n+1)$ **M1:** Cancels out $n(n+1)$ from both sides of their equation **A1:** $3n^2 - 65n - 250 = 0$ **M1:** A valid method for solving a 3 term quadratic equation **A1:** Only one solution of $n = 25$

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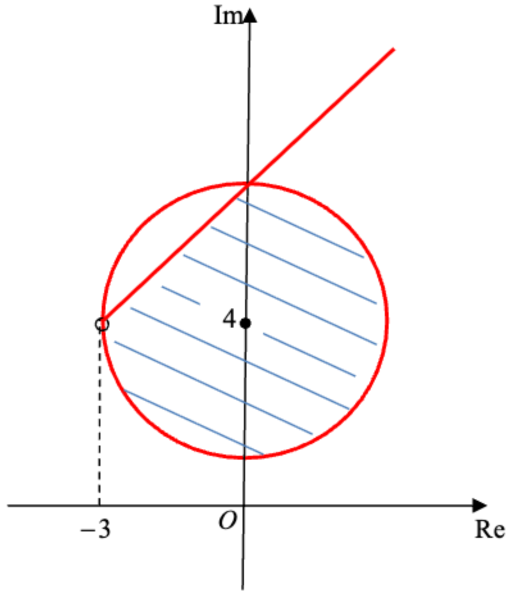
Question	Scheme		Marks	AOs
7(a)	Depth = 0.16 (m)		B1	2.2b
			(1)	
(b)	$y=1+kx^2 \Rightarrow 1.16=1+k(0.2)^2 \Rightarrow k=...$		M1	3.3
	$\Rightarrow k=4$ cao $\{\text{So } y=1+4x^2\}$		A1	1.1b
			(2)	
(c)	$\frac{\pi}{4}\int(y-1)\mathrm{d}y$	$\frac{\pi}{4}\int y\,\mathrm{d}y$	B1ft	1.1a
	$=\left\{\frac{\pi}{4}\right\}\int_1^{1.16}(y-1)\mathrm{d}y$	$=\left\{\frac{\pi}{4}\right\}\int_0^{0.16}y\mathrm{d}y$	M1	3.3
	$=\left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}-y\right]_1^{1.16}$	$=\left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}\right]_0^{0.16}$	M1 A1	1.1b 1.1b
	$=\frac{\pi}{4}\left(\left(\frac{1.16^2}{2}-1.16\right)-\left(\frac{1}{2}-1\right)\right)\ \{=0.0032\pi\}$	$=\frac{\pi}{4}\left(\left(\frac{0.16^2}{2}\right)-(0)\right)\ \{=0.0032\pi\}$		
	$V_{\text{cylinder}}=\pi(0.2)^2(1.16)\ \{=0.0464\pi\}$		B1	1.1b
	Volume $=0.0464\pi-0.0032\pi\ \{=0.0432\pi\}$		M1	3.4
	$=0.1357168026...=0.136(\text{m}^3)$ (3sf)		A1	1.1b
			(7)	
(d)	Any one of e.g. the measurements may not be accurate the inside surface of the bowl may not be smooth there may be wastage of concrete when making the bird bath		B1	3.5b
			(1)	
(e)	Some comment consistent with their values. We do need a reason e.g. $\left[\left(\frac{0.136-0.127}{0.127}\right)\times100=7.0866...\right]$ so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model **or** We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used		B1ft	3.5a
			(1)	
			(12 marks)	

Question 7 notes:	
(a)	
B1:	Infers that the maximum depth of the bird bath could be 0.16 (m)
(b)	
M1:	Substitutes $y = 1.16$ and $x = 0.2$ or $x = -0.2$ into $y = 1 + kx^2$ and rearranges to give $k = ...$
A1:	$k = 4$ cao
(c)	
B1ft:	Uses the model to obtain either $\frac{\pi}{(\text{their } k)}\int (y-1)\,\mathrm{d}y$ **or** $\frac{\pi}{(\text{their } k)}\int y\,\mathrm{d}y$
M1:	Chooses limits that are appropriate to their model
M1:	Integrates y (with respect to y) to give $\pm\lambda y^2$, where $\lambda \neq 0$ is a constant
A1:	Uses their model correctly to give either $y - 1 \to \frac{y^2}{2} - y$ or $y \to \frac{y^2}{2}$
B1:	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or 0.0464π **or** $\frac{29}{625}\pi$, o.e.
M1:	Depends on **both** previous M marks Uses the model to find $V_{\text{their cylinder}} -$ their integrated volume
A1:	0.136 cao
(d)	
B1:	States an acceptable limitation of the model
(e)	
B1ft:	Compares the actual volume with their answer to (c). Makes an assessment of the model. E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason

Question	Scheme	Marks	AOs
8(a)		M1	1.1b
		A1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(6)	
(b)	$(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$	M1	3.1a
	$= 2.42\ (2\text{dp})$ **cao**	A1	1.1b
		(2)	
			(8 marks)

Notes:

(a)
M1: Circle
A1: Centre (0, 4) and above the real axis
M1: Half-line
A1: (−3, 4) positioned correctly and the half-line intersects the top of the circle on the *y*-axis
M1: Depends on **both** previous M marks Shades in a region inside the circle and below the half-line
A1: cso

Note: Final A1 mark is dependent on all previous marks being scored in part (a)

(b)
M1: Uses trigonometry to give an expression for an angle in the range $\left(\frac{\pi}{2}, \pi\right)$ or (90°, 180°)
A1: 2.42 cao

Question	Scheme	Marks	AOs
9(a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{ \overrightarrow{OF} \bullet \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$ $\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$	dM1	1.1b
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{ \overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	dM1	3.1a
	$= \sqrt{6}$ **or** $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	
	9(a) Alternative 1		
	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OA} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \text{ and } \overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \Rightarrow \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\cos\theta \left\{ = \dfrac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\lvert\overrightarrow{OA}\rvert . \lvert\overrightarrow{AB}\rvert} \right\} = \dfrac{\pm\left(\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right)}{\sqrt{(-3)^2 + (1)^2 + (-7)^2} . \sqrt{(12)^2 + (3)^2 + (18)^2}}$	dM1	1.1b
	$\left\{ \cos\theta = \dfrac{-36 + 3 - 126}{\sqrt{59} . \sqrt{477}} = \dfrac{-159}{\sqrt{59} . \sqrt{477}} \right\}$		
	$\theta = 161.4038029...$ or $18.59619709...$ or $\sin\theta = 0.3188964021...$	A1	1.1b
	minimum distance $= \sqrt{(-3)^2 + (1)^2 + (-7)^2} \sin(18.59619709...)$	dM1	3.1a
	$= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	
	9(a) Alternative 2		
	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\lvert\overrightarrow{OF}\rvert^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$	dM1	1.1b
	$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$		
	$= 477\lambda^2 - 318\lambda + 59$	A1	1.1b
	$= 53(3\lambda - 1)^2 + 6$	dM1	3.1a
	minimum distance $= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	AOs
9(b)	e.g. Fish F may not swim in an exact straight line from A to B Fish F may hit an obstacle whilst swimming from A to B Fish F may deviate his path to avoid being caught by the octopus	B1	3.5b
		(1)	
(c)	e.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed Octopus may during the fish F's motion move away from its fixed location at O	B1	3.5b
		(1)	
			(9 marks)

Question 9 notes:

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Applies $\overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent

M1: Depends on previous M mark. Writes down
(their $\overrightarrow{OF}$ which is in terms of λ)$\bullet$(their $\overrightarrow{AB}$) $= 0$. Can be implied

A1: Lambda is correct. e.g. $\lambda = \frac{1}{3}$ for $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\lambda = 1$ for $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$

M1: Depends on previous M mark. Complete method for finding $\left|\overrightarrow{OF}\right|$

A1: $\sqrt{6}$ **or** awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Alternative 1

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)

M1: Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$ (o.e.)

A1: $\theta =$ awrt 161.4 or awrt 18.6 or $\sin\theta =$ awrt 0.319

M1: Depends on previous M mark. (their OA)sin(their θ)

A1: $\sqrt{6}$ or awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Question 9 notes continued:
Alternative 2 **(a)** **M1:** Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OA}-\overrightarrow{OB}$ **or** the direction vector **d** **M1:** Applies $\overrightarrow{OA}+\lambda(\text{their }\overrightarrow{AB}\text{ or their }\overrightarrow{BA}\text{ or their }\mathbf{d})$ or equivalent **M1:** Depends on previous M mark. Applies Pythagoras by finding $\lvert\overrightarrow{OF}\rvert^2$, o.e. **A1:** $\lvert\overrightarrow{OF}\rvert^2 = 477\lambda^2 - 318\lambda + 59$ **M1:** Depends on previous M mark. Method of completing the square or differentiating their $\lvert\overrightarrow{OF}\rvert^2$ w.r.t. λ **A1:** $\sqrt{6}$ or awrt 2.4 **A1ft :** Correct follow through conclusion, which is in context with the question
(b) **B1:** An acceptable criticism for fish F, which is in context with the question
(c) **B1:** An acceptable criticism for the octopus, which is in context with the question