

Integration Techniques

Question Paper



Leave
blank

1. Find

$$\int \frac{x^2 - 5}{2x^3} dx \quad x > 0$$

giving your answer in simplest form.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined writing area for the answer.





3.

$$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where A , B and C are integers to be found.

(3)

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x) dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

- (a) (i) find the values of the constants A , B and C
(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} dx$$

giving your answer in the form $p + q \ln 2$, where p and q are integers.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



5. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t} dt = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 4 is 4 marks)

Question 5 continued

Lined writing area for the answer to Question 5.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave
blank

6. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3} dx \quad (4)$$

(ii) Find, in simplest form,

$$\int x(x^2+3)^7 dx \quad (2)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



7. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \quad x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants A , B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} dx$$

giving your answer in the form $a + b \ln 2$ where a and b are integers to be found.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



9. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A , B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x) dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Leave blank

12. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} dx \quad (2)$$

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} dx = \ln 8$$

(b) find the value of k (4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



13. (i) Find $\frac{d}{dx} \ln(\sin^2 3x)$ writing your answer in simplest form. (2)

(ii)(a) Find $\frac{d}{dx}(3x^2 - 4)^6$ (2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x(3x^2 - 4)^5 dx = R$$

where R is an integer to be found.

(Solutions relying on calculator technology are not acceptable.) (3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



14.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} \right)^2 = 6 \cot \theta - 4$$

giving your answers to 3 significant figures as appropriate. (5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \right) \cot x \, dx \quad (2)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Leave
blank

Question 15 continued

Lined area for writing the answer to Question 15.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

16. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2 \cos^2 A - 1 \tag{2}$$

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4 \cos^2 3x) dx \tag{4}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 18 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing the answer to Question 18.

19. $f(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x + 3)^2} \quad x \in \mathbb{R} \quad x > -3$

(a) Find the values of the constants A , B , C and D such that

$$f(x) = Ax^2 + Bx + C + \frac{D}{(x + 3)^2} \tag{4}$$

(b) Hence find,

$$\int f(x) dx \tag{3}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



21. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x - k)} dx$ is independent of k , (4)

(b) show that $\int_k^{2k} \frac{2}{(2x - k)^2} dx$ is inversely proportional to k . (3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 21 continued

A series of horizontal lines for writing, starting below the section header and ending above the footer.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave
blank

22. Given that

$$4x^3 + 2x^2 + 17x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$$

(a) find the values of the constants A , B , C and D .

(4)

(b) Hence find

$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4} dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(6)



25.

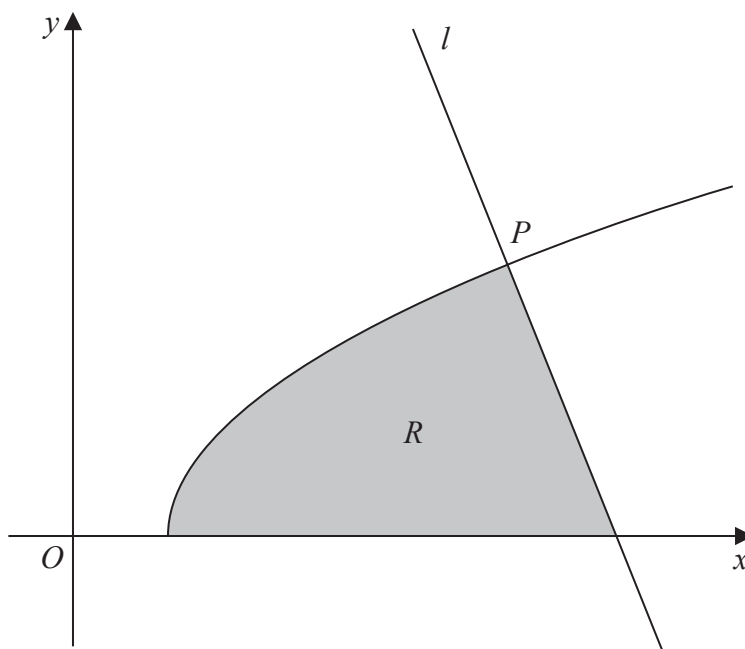


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l , shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0 \tag{5}$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and l .

(b) Use algebraic integration to find the exact area of R . (4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Leave blank

26. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \quad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = g(x)$, where

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \quad -3 < x < 3.5 \quad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
 Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

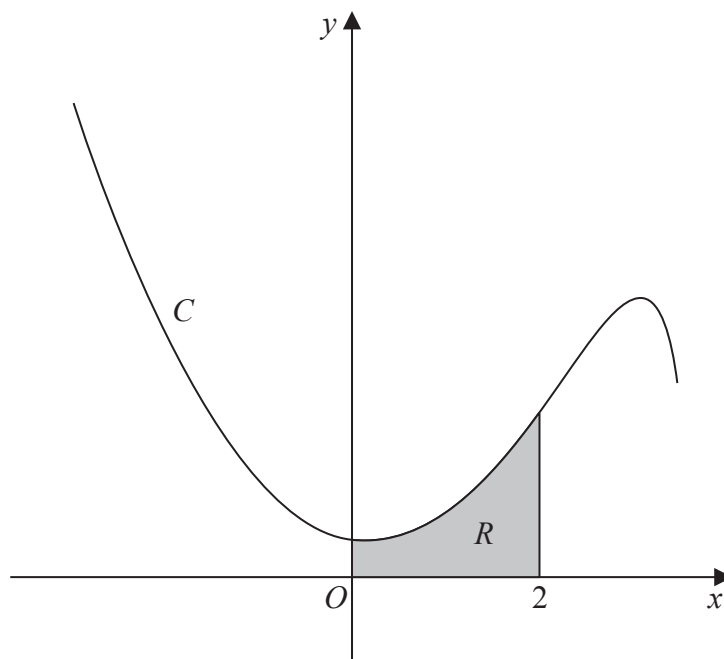


Figure 4

Figure 4 shows a sketch of the curve C .
 The region R , shown shaded in Figure 4, is bounded by C , the y -axis, the x -axis and the line with equation $x = 2$

(c) Find the exact area of R , writing your answer in the form $a + b \ln 2$, where a and b are constants to be found.

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

27.

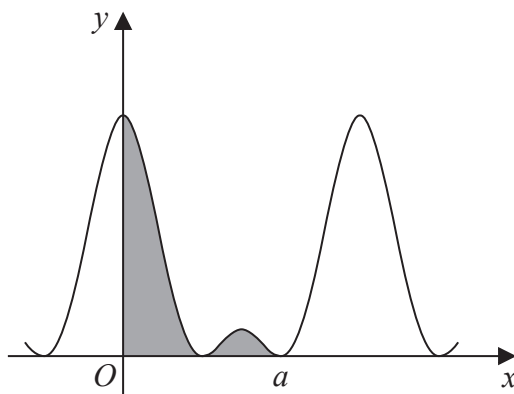


Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2 \cos 2x)^2$$

(a) Show that

$$(1 + 2 \cos 2x)^2 \equiv p + q \cos 2x + r \cos 4x$$

where p , q and r are constants to be found.

(2)

The curve touches the positive x -axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the y -axis and the x -axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



28. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{16}{9(3x - k)} \quad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

- (a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of k . (2)

The point P with x coordinate 1 lies on C .

Given that the gradient of the curve at P is -12

- (b) find the two possible values of k (3)

Given also that $k < 3$

- (c) find the equation of the normal to C at P , writing your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found. (3)

- (d) show, using algebraic integration that,

$$\int_1^3 \frac{16}{9(3x - k)} dx = \lambda \ln 10$$

where λ is a constant to be found. (4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



29.

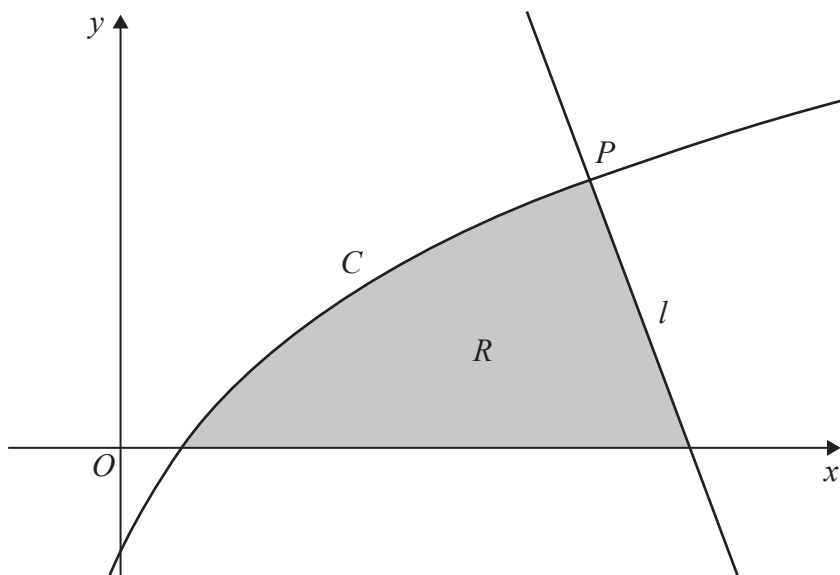


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{6x^2 + 4x - 2}{2x + 1} \quad x > -\frac{1}{2}$$

- (a) Find $f'(x)$, giving the answer in simplest form. (3)

The line l is the normal to C at the point $P(2, 6)$

- (b) Show that an equation for l is (3)
- $$16y + 5x = 106$$

- (c) Write $f(x)$ in the form $Ax + B + \frac{D}{2x + 1}$ where A , B and D are constants. (3)

The region R , shown shaded in Figure 5, is bounded by C , l and the x -axis.

- (d) Use algebraic integration to find the exact area of R , giving your answer in the form $P + Q \ln 3$, where P and Q are rational constants. (5)
- (Solutions based entirely on calculator technology are not acceptable.)*

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 29 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined writing area consisting of 25 horizontal lines.