Integration Techniques

Worked Solutions

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1. Find

$$\int \frac{x^2 - 5}{2x^3}\,\mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(3)

$$\int \frac{x^2-5}{2x^3}\,dx = \int \frac{x^2}{2x^3} - \frac{5}{2x^3}\,dx$$

$$= \int \frac{1}{2x} - \frac{5}{2}x^{-3}\,dx$$

$$= \frac{1}{2}\int \frac{1}{x}\,dx - \frac{5}{2}\int x^{-3}\,dx$$

$$= \boxed{\frac{1}{2}\ln(x) + \frac{5}{4}x^{-2} + c}$$

$$= \boxed{\frac{1}{2}\ln(x) + \frac{5}{4x^2} + c}$$

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2. Find

(a) $\int (2x + 3)^{12}\,dx$

(2)

(b) $\int \frac{5x}{4x^2 + 1}\,dx$

(2)

a) $\int (2x + 3)^{12}\,dx$

Guess : $(2x+3)^{13}$ $\times \frac{1}{26}$

Check : $26(2x+3)^{12}$ $\times \frac{1}{26}$

$= \boxed{\frac{1}{26}(2x+3)^{13} + C}$

b) $\int \frac{5x}{4x^2+1}\,dx$

Guess : $\ln|4x^2+1|$ $\times \frac{5}{8}$

Check : $\frac{8x}{4x^2+1}$ $\times \frac{5}{8}$

$= \boxed{\frac{5}{8}\ln|4x^2+1| + C}$

3. $$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x-2}$$

where A, B and C are integers to be found.

(3)

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x)\,dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

a) $2x^2 - 5x + 8 = (Ax + B)(x - 2) + C$

$x = 2$: $6 = C$

$x = 0$: $8 = -2B + C \Rightarrow 8 = -2B + 6 \Rightarrow B = -1$

$x = 1$: $5 = -(A + B) + C \Rightarrow 5 = -A + 1 + 6 \Rightarrow A = 2$

$g(x) = 2x - 1 + \frac{6}{x-2}$

b) $\int_4^8 2x - 1 + \frac{6}{x-2}\,dx$

$= \left[x^2 - x + 6\ln(x-2)\right]_4^8$

$= \left(8^2 - 8 + 6\ln(6)\right) - \left(4^2 - 4 + 6\ln(2)\right)$

$= 56 + 6\ln(6) - 12 - 6\ln(2)$

$= \boxed{44 + 6\ln(3)}$

4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\,dx$$

giving your answer in the form $p + q\ln 2$, where p and q are integers.

(5)

a) $4x^3 + 2x^2 + 3x + 8 = (Ax+B)(x^2+4) + Cx + D$

due to x^2+4, I will compare coefficients as I can't sub anything in to make that cancel out.

$= Ax^3 + Bx^2 + (4A+C)x + 4B + D$

$[x^3]$: $4 = A$

$[x^2]$: $2 = B$

$[x]$: $3 = 4A + C \Rightarrow 3 = 16 + C \Rightarrow C = -13$

$[k]$: $8 = 4B + D \Rightarrow 8 = 8 + D \Rightarrow D = 0$

b) $\int_1^4 4x + 2 - \frac{13x}{x^2+4}\,dx$

Guess : $\ln(x^2+4) \times \frac{13}{2}$

Check : $\frac{2x}{x^2+4} \times \frac{13}{2}$

$= \left[2x^2 + 2x - \frac{13}{2}\ln(x^2+4)\right]_1^4$

$= \left(2(4)^2 + 2(4) - \frac{13}{2}\ln(4^2+4)\right) - \left(2 + 2 - \frac{13}{2}\ln(5)\right)$

$= 40 - \frac{13}{2}\ln(20) - 4 + \frac{13}{2}\ln(5)$

Question 4 continued

$= 36 + \frac{13}{2}\ln\left(\frac{1}{4}\right)$

$= 36 + \frac{13}{2}\ln\left(2^{-2}\right)$

$= 36 - 13\ln(2)$

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5. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

$$\int_a^{2a} \frac{t+1}{t}\,dt = \int_a^{2a} 1 + \frac{1}{t}\,dt$$

$$= \Big[t + \ln(t)\Big]_a^{2a}$$

$$= \big(2a + \ln(2a)\big) - \big(a + \ln(a)\big)$$

$$= a + \ln\left(\frac{2a}{a}\right)$$

$$a + \ln(2)$$

So $a + \ln(2) = \ln(7)$

$$\Rightarrow \boxed{a = \ln\left(\frac{7}{2}\right) \Rightarrow k = \frac{7}{2}}$$

(Total for Question 4 is 4 marks)

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6. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3}\,\mathrm{d}x$$

(4)

(ii) Find, in simplest form,

$$\int x\left(x^2+3\right)^7\,\mathrm{d}x$$

(2)

i) $\int_2^4 \frac{8}{(2x-3)^3}\,dx$

Guess : $(2x-3)^{-2}$ $\times -2$

Check : $-4(2x-3)^{-3}$ $\times -2$

$= \int_2^4 8(2x-3)^{-3}\,dx$

$= -2\left[\frac{1}{(2x-3)^2}\right]_2^4$

$= -2\left(\frac{1}{5^2} - \frac{1}{1^2}\right)$

$= -2\left(\frac{1}{25} - 1\right)$

$= \boxed{\frac{48}{25}}$

ii) $\int x(x^2+3)^7\,dx$

Guess : $(x^2+3)^8$ $\times \frac{1}{16}$

Check : $16x(x^2+3)^7$ $\times \frac{1}{16}$

$= \boxed{\frac{1}{16}(x^2+3)^8 + C}$

7. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \ \ x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2}\, \mathrm{d}x$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

a) $x^2+8x-3 = (Ax+B)(x+2) + C$

$x=-2: \quad -15 = C$

$x=0: \quad -3 = 2B + C \quad \Rightarrow \quad -3 = 2B - 15 \quad \Rightarrow B = 6$

$A=1$ by comparing coefficients.

b) $\displaystyle\int_0^6 \frac{x^2+8x-3}{x+2}\, dx = \int x+6 - \frac{15}{x+2}\, dx$

$= \left[\frac{1}{2}x^2 + 6x - 15\ln|x+2|\right]_0^6$

$= \left(\frac{1}{2}(6)^2 + 6(6) - 15\ln(6+2)\right) - \left(-15\ln(2)\right)$

$= 54 - 15\ln(8) + 15\ln(2)$

$= 54 + 15\ln(1/4)$

$= 54 + 15\ln(2^{-2})$

$= \boxed{54 - 30\ln(2)}$

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8. (i) Find

$$\int\left((3x+5)^9 + e^{5x}\right)dx$$

(3)

(ii) Given that b is a constant greater than 2, and

$$\int_2^b \frac{x}{x^2+5}\,dx = \ln\left(\sqrt{6}\right)$$

use integration to find the value of b.

(5)

i) $\int\left((3x+5)^9 + e^{5x}\right)\ dx$

Guess: $(3x+5)^{10}$ $\times\frac{1}{30}$

Check: $30(3x+5)^9$ $\times\frac{1}{30}$

$= \frac{1}{30}(3x+5)^{10} + \frac{1}{5}e^{5x} + C$

ii) $\int_2^b \frac{x}{x^2+5}\ dx = \ln(\sqrt{6})$

Guess: $\ln|x^2+5|$ $\times\frac{1}{2}$

Check: $\frac{2x}{x^2+5}$ $\times\frac{1}{2}$

$\cancel{\frac{1}{2}}\left[\ln|x^2+5|\right]_2^b = \cancel{\frac{1}{2}}\ln(6)$

$\ln(b^2+5) - \ln(9) = \ln(6)$

$\ln(b^2+5) = \ln(6\times 9)$

$b^2+5 = 54$

$b^2 = 49$

$b = \pm 7$ $\quad b > 2$ so $\boxed{b = 7}$

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9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x+3)}{x^2 + 3x + 4}$$

where A, B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x)\,dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

a) $2x^3 - 4x - 15 = (Ax+B)(x^2+3x+4) + C(2x+3)$

$= Ax^3 + (B+3A)x^2 + (4A+3B+2C)x + 4B+3C$

$[x^3]$: $A = 2$

$[x^2]$: $0 = B + 3A \Rightarrow 0 = B + 6 \Rightarrow B = -6$

$[x]$: $-15 = 4B + 3C \Rightarrow -15 = -24 + 3C \Rightarrow C = 3$

b) $\int_3^5 2x - 6 + \frac{3(2x+3)}{x^2+3x+4}\,dx$

Guess: $(\ln|x^2+3x+4|) \times 3$

Check: $\frac{2x+3}{x^2+3x+4} \times 3$

$= \left[x^2 - 6x + 3\ln|x^2+3x+4|\right]_3^5$

$= (5^2 - 6(5) + 3\ln(5^2+3(5)+4)) - (3^2 - 6(3) + 3\ln(3^2+3(3)+4))$

Question 9 continued

$$= \left(-5 + 3\ln(44)\right) - \left(-9 + 3\ln(22)\right)$$

$$= 4 + 3\ln(2)$$

$$= 4 + \ln(2^3)$$

$$= \boxed{4 + \ln(8)}$$

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10. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1}\,dx$$

giving your answer in simplest form.

(4)

(ii) $$h(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $h(x) = Ax + B + \dfrac{C}{(x-1)^2}$ where A, B and C are constants to be found, find

$$\int h(x)\,dx$$

(6)

i) $\int_3^{42} \frac{2}{3x-1}\,dx = \left[\frac{2}{3}\ln|3x-1|\right]_3^{42}$

$= \frac{2}{3}\left(\ln(125) - \ln(8)\right)$

$= \frac{2}{3}\ln\left(\frac{125}{8}\right)$

$= \ln\left(\left(\frac{125}{8}\right)^{2/3}\right)$

$= \ln\left(\frac{25}{4}\right)$

ii) $2x^3 - 7x^2 + 8x + 1 = (Ax+B)(x-1)^2 + C$

$x=1$: $4 = C$

$x=0$ $1 = B + C \Rightarrow 1 = B + 4 \Rightarrow B = -3$

$A = 2$ by comparing coefficients of x^3

Question 10 continued

so $\int h(x)\,dx$

$= \int 2x - 3 + \frac{4}{(x-1)^2}\,dx$

Guess: $(x-1)^{-1}$ $\times -4$

Check: $-1(x-1)^{-2}$ $\times -4$

$= \int 2x - 3 + 4(x-1)^{-2}\,dx$

$= \boxed{x^2 - 3x - 4(x-1)^{-1} + c}$

$= \boxed{x^2 - 3x - \frac{4}{x-1} + c}$

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11. (i) Find

$$\int \frac{12}{(2x-1)^2}\,dx$$

giving your answer in simplest form.

(2)

(ii) (a) Write $\dfrac{4x+3}{x+2}$ in the form

$A + \dfrac{B}{x+2}$ where A and B are constants to be found

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$$

giving your answer in simplest form.

(6)

i) $\int 12(2x-1)^{-2}\,dx$ Guess: $(2x-1)^{-1}$ $\times -6$

Check: $-2(2x-1)^{-2}$ $\times -6$

$= \boxed{-6(2x-1)^{-1} + c}$

$= \boxed{\dfrac{-6}{2x-1} + c}$

ii) a) $4x+3 = A(x+2) + B$

$x=-2$: $-5 = B$

$x=0$: $3 = 2A + B$ $\quad 3 = 2A - 5 \quad \Rightarrow A = 4$

b) $\int_{-8}^{-5} \dfrac{4x+3}{x+2}\,dx = \int_{-8}^{-5} 4 - \dfrac{5}{x+2}\,dx$

$= \Big[4x - 5\ln|x+2|\Big]_{-8}^{-5}$

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Question 11 continued

$= \left(4(-5) - 5\ln|-3|\right) - \left(4(-8) - 5\ln|-6|\right)$

$= 12 - 5\ln(3) + 5\ln(6)$

$= 12 + 5\ln(2)$

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12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, dx$$

(2)

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(b) find the value of k

(4)

$$\int \frac{9x}{3x^2+k} \, dx$$

Guess: $\ln(3x^2+k) \times \frac{3}{2}$

Check: $\frac{6x}{3x^2+k} \times \frac{3}{2}$

$$= \boxed{\frac{3}{2}\ln|3x^2+k| + C}$$

b) $\frac{3}{2}\left[\ln|3x^2+k|\right]_2^5 = \ln(8)$

$\frac{3}{2}\left(\ln(75+k) - \ln(12+k)\right) = \ln(8)$

$\frac{3}{2}\ln\left(\frac{75+k}{12+k}\right) = \ln(8)$

$\ln\left(\frac{75+k}{12+k}\right) = \ln\left(8^{2/3}\right)$

$\frac{75+k}{12+k} = 4 \implies 75+k = 48+4k$

$27 = 3k$

$\boxed{k = 9}$

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13. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(ii) (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2-4\right)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2-4\right)^5 \mathrm{d}x = R$$

where R is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

i) $\frac{d}{dx}\left(\sin^2(3x)\right) = 2\sin(3x) \times 3\cos(3x)$

$= 6\sin(3x)\cos(3x)$ by chain rule.

$\frac{d}{dx}\left(\ln\left(\sin^2(3x)\right)\right) = \frac{6\sin(3x)\cos(3x)}{\sin^2(3x)}$

$= \frac{6\cos(3x)}{\sin(3x)} = \boxed{6\cot(3x)}$

ii) a) $\frac{d}{dx}\left(\left(3x^2-4\right)^6\right) = 6\left(3x^2-4\right)^5 \times 6x$

$= 36x\left(3x^2-4\right)^5$

b) $\int_0^{\sqrt{2}} x\left(3x^2-4\right)^5 dx = \frac{1}{36}\int_0^{\sqrt{2}} 36x\left(3x^2-4\right)^5 dx$

$= \frac{1}{36}\left[\left(3x^2-4\right)^6\right]_0^{\sqrt{2}}$ using (a)

$= \frac{1}{36}\left(\left(3(\sqrt{2})^2-4\right)^6 - (-4)^6\right) = \frac{1}{36}\left(-4032\right) = \boxed{-112}$

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14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin\theta} + \frac{\sin 2\theta}{\cos\theta}\right)^2 = 6\cot\theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right)\cot x \, dx$$

(2)

a) $\frac{\cos(2x)}{\sin(x)} + \frac{\sin(2x)}{\cos(x)} = \frac{\cos(2x)}{\sin(x)} + \frac{2\sin(x)\cos(x)}{\cos(x)}$

$= \frac{\cos(2x)}{\sin(x)} + 2\sin(x)$

$= \frac{\cos(2x) + 2\sin^2(x)}{\sin(x)}$

$= \frac{1 - 2\sin^2(x) + 2\sin^2(x)}{\sin(x)}$

$= \frac{1}{\sin(x)} = \operatorname{cosec}(x)$

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Question 14 continued

b) same as $\operatorname{cosec}^2(\theta) = 6\cot\theta - 4$ $\quad 0 < \theta < \frac{\pi}{2}$

$$\cot^2\theta + 1 = 6\cot\theta - 4$$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$$\cot^2\theta - 6\cot\theta + 5 = 0$$

$$(\cot\theta - 5)(\cot\theta - 1) = 0$$

$$\cot\theta = 5 \quad \text{or} \quad \cot\theta = 1$$

$$\tan\theta = \frac{1}{5} \quad \text{or} \quad \tan\theta = 1$$

$$\Rightarrow \underline{\theta = 0.197} \text{ to 3sf} \quad \text{or} \quad \underline{\theta = \frac{\pi}{4}}$$

c) $$\int_{\pi/6}^{\pi/4} \operatorname{cosec}(x)\cot(x)\,dx = \Big[-\operatorname{cosec}(x)\Big]_{\pi/6}^{\pi/4}$$

formula booklet

$$= -\operatorname{cosec}\left(\frac{\pi}{4}\right) + \operatorname{cosec}\left(\frac{\pi}{6}\right)$$

$$= \frac{-2}{\sqrt{2}} + 2$$

$$= 2 - \sqrt{2}$$

Sometimes you might need to use the differentiation section of the formula booklet for an integral question, like in this question.

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15. (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

(4)

a) $\sin(3x) \equiv \sin(2x)\cos(x) + \cos(2x)\sin(x)$

$= 2\sin(x)\cos(x)\cos(x) + (1 - 2\sin^2(x))\sin(x)$

$= 2\sin(x)\cos^2(x) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x)(1-\sin^2(x)) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x) - 2\sin^3(x) + \sin(x) - 2\sin^3(x)$

$= 3\sin(x) - 4\sin^3(x)$

b) $\int_0^{\pi/3} \sin^3(x)\,dx = \int_0^{\pi/3} \frac{3\sin x - \sin 3x}{4}\,dx$ by (a)

$= \frac{1}{4}\int_0^{\pi/3} 3\sin x - \sin 3x \,dx$

$= \frac{1}{4}\left[-3\cos x + \frac{1}{3}\cos 3x\right]_0^{\pi/3}$

$= \frac{1}{4}\left(\left(-3\cos(\pi/3) + \frac{1}{3}\cos(\pi)\right) - \left(-3\cos(0) + \frac{1}{3}\cos(0)\right)\right)$

$= \frac{1}{4}\left(\left(-\frac{3}{2} - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right) = \frac{1}{4}\left(\frac{5}{6}\right) = \frac{5}{24}$

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16. (a) Using the identity for $\cos(A+B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

(2)

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,dx$$

(4)

a) $\cos(2A) = \cos(A)\cos(A) - \sin(A)\sin(A)$

$= \cos^2(A) - \sin^2(A)$

$= \cos^2(A) - (1 - \cos^2(A))$

$= 2\cos^2(A) - 1$

b) $\int_{\pi/12}^{\pi/8} (5 - 4\cos^2(3x))\,dx$

$\cos(2x) = 2\cos^2(x) - 1$
$\cos(6x) = 2\cos^2(3x) - 1$
$2\cos(6x) + 2 = 4\cos^2(3x)$

$= \int_{\pi/12}^{\pi/8} 5 - 2\cos(6x) - 2 \; dx$

$= \int_{\pi/12}^{\pi/8} 3 - 2\cos(6x) \; dx$

$= \left[3x - \frac{1}{3}\sin(6x)\right]_{\pi/12}^{\pi/8}$

$= \left(\frac{3\pi}{8} - \frac{1}{3} \times \frac{\sqrt{2}}{2}\right) - \left(\frac{3\pi}{12} - \frac{1}{3} \times 1\right)$

$= \boxed{\frac{\pi}{8} + \frac{2 - \sqrt{2}}{6}}$

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17. Find

(i) $\displaystyle\int \frac{3x-2}{3x^2-4x+5}\,dx$ **(2)**

(ii) $\displaystyle\int \frac{e^{2x}}{(e^{2x}-1)^3}\,dx \qquad x \neq 0$ **(2)**

i) $\displaystyle\int \frac{3x-2}{3x^2-4x+5}\,dx$

Guess : $\ln|3x^2-4x+5| \times \frac{1}{2}$

Check : $\dfrac{6x-4}{3x^2-4x+5} \times \frac{1}{2}$

$= \boxed{\frac{1}{2}\ln|3x^2-4x+5| + C}$

ii) $\displaystyle\int e^{2x}(e^{2x}-1)^{-3}\,dx$

Guess : $(e^{2x}-1)^{-2} \times -\frac{1}{4}$

Check : $-4e^{2x}(e^{2x}-1)^{-3} \times \frac{-1}{4}$

$= \boxed{-\frac{1}{4}(e^{2x}-1)^{-2} + c}$

$= \boxed{\dfrac{-1}{4(e^{2x}-1)^2} + C}$

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18. (i) Find, in simplest form,

$$\int (2x-5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1+2\cos x} \, dx = \ln a$$

where a is a rational constant to be found.

(4)

i) $\int (2x-5)^7 \, dx$

Guess : $(2x-5)^8 \times \frac{1}{16}$

Check : $16(2x-5)^7 \times \frac{1}{16}$

$= \boxed{\frac{1}{16}(2x-5)^8 + C}$

ii) $\int_0^{\pi/3} \frac{4\sin(x)}{1+2\cos(x)} \, dx$

Guess : $\ln|1+2\cos(x)| \times -2$

Check : $\frac{-2\sin(x)}{1+2\cos(x)} \times -2$

$= -2\left[\ln|1+2\cos(x)|\right]_0^{\pi/3}$

$= -2\left(\ln\left(1+2\times\frac{1}{2}\right) - \ln\left(1+2\times 1\right)\right)$

$= -2\ln\left(\frac{2}{3}\right)$

$= \ln\left(\left(\frac{2}{3}\right)^{-2}\right) = \ln\left(\frac{9}{4}\right)$ so $\boxed{a = \frac{9}{4}}$

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19.
$$f(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2} \qquad x \in \mathbb{R} \quad x > -3$$

(a) Find the values of the constants A, B, C and D such that

$$f(x) = Ax^2 + Bx + C + \frac{D}{(x+3)^2}$$

(4)

(b) Hence find,

$$\int f(x)\,dx$$

x^2+6x+9

(3)

a) $2x^4 + 15x^3 + 35x^2 + 21x - 4 = (x+3)^2(Ax^2+Bx+C) + D$

$2x^4 + 15x^3 + 35x^2 + 21x - 4 = Ax^4 + (6A+B)x^3 + (6B+9A+C)x^2$
$+ (6C+9B)x + D+9C$

$[x^4]$: $2 = A$

$[x^3]$: $15 = 6A + B \implies 15 = 12 + B \implies B = 3$

$[x]$: $21 = 6C + 9B \implies 21 = 6C + 27 \implies C = -1$

$[k]$: $-4 = D + 9C \implies -4 = D - 9 \implies D = 5$

b) $\int f(x)\,dx = \int 2x^2 + 3x - 1 + \frac{5}{(x+3)^2}\,dx$

$= \frac{2}{3}x^3 + \frac{3}{2}x^2 - x + \int 5(x+3)^{-2}\,dx$

$= \frac{2}{3}x^3 + \frac{3}{2}x^2 - x - 5(x+3)^{-1} + c$

Guess: $(x+3)^{-1}$ $\times -5$

Check: $-(x+3)^{-2}$ $\times -5$

$= \frac{2}{3}x^3 + \frac{3}{2}x^2 - x - \frac{5}{x+3} + c$

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20. (i) Find

$$\int_5^{13} \frac{1}{(2x-1)}\,dx$$

writing your answer in its simplest form.

(4)

(ii) Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin 2x + \sec\frac{1}{3}x \tan\frac{1}{3}x\,dx$$

(3)

i) $\int_5^{13} \frac{1}{2x-1}\,dx = \frac{1}{2}\Big[\ln|2x-1|\Big]_5^{13}$

$= \frac{1}{2}\left(\ln(25) - \ln(9)\right)$

$= \frac{1}{2}\ln\left(\frac{25}{9}\right)$

$= \ln\left(\left(\frac{25}{9}\right)^{1/2}\right) = \ln\left(\frac{5}{3}\right)$

ii) $\int_0^{\pi/2} \sin(2x) + \sec(1/3x)\tan(1/3x)\,dx$

$= \Big[-\frac{1}{2}\cos(2x) + 3\sec(1/3x)\Big]_0^{\pi/2}$

by formula booklet, $\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$

So $\frac{d}{dx}\left(\sec(\frac{1}{3}x)\right) = \frac{1}{3}\sec(\frac{1}{3}x)\tan(\frac{1}{3}x)$

remember this !

$= \left(-\frac{1}{2}\cos(\pi) + 3\sec(\pi/6)\right) - \left(-\frac{1}{2}\cos(0) + 3\sec(0)\right)$

$= \frac{1}{2} + 2\sqrt{3} - \left(-\frac{1}{2} + 3\right) = 4 + 2\sqrt{3}$

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21. Given that $k \in \mathbb{Z}^+$

(a) show that $\displaystyle\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of k, **(4)**

(b) show that $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to k. **(3)**

a) $\displaystyle\int_k^{3k} \frac{2}{3x-k}\,dx = \frac{2}{3}\Big[\ln|3x-k|\Big]_k^{3k}$

$= \frac{2}{3}\Big(\ln(3(3k)-k) - \ln(3(k)-k)\Big)$

$= \frac{2}{3}\big(\ln(8k) - \ln(2k)\big)$

$= \frac{2}{3}\ln(4)$

b) $\displaystyle\int_k^{2k} 2(2x-k)^{-2}\,dx$

Guess: $(2x-k)^{-1}$ $\times -1$

Check: $-2(2x-k)^{-2}$ $\times -1$

$= \Big[-(2x-k)^{-1}\Big]_k^{2k}$

$= \dfrac{-1}{4k-k} - \dfrac{-1}{2k-k}$

$= \dfrac{1}{k} - \dfrac{1}{3k} = \boxed{\dfrac{2}{3k} \propto \dfrac{1}{k}}$

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22. Given that

$$4x^3 + 2x^2 + 17x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$$

(a) find the values of the constants A, B, C and D.

(4)

(b) Hence find

$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4}\,dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(6)

a) $4x^3 + 2x^2 + 17x + 8 = Ax^3 + Bx^2 + (4A + C)x + 4B + D$

$[x^3]$: $4 = A$

$[x^2]$: $2 = B$

$[x]$: $17 = 4A + C \Rightarrow 17 = 16 + C \Rightarrow C = 1$

$[k]$: $8 = 4B + D \Rightarrow 8 = 8 + D \Rightarrow D = 0$

b) $$\frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4} = \frac{(4x + 2)(x^2 + 4) + x}{x^2 + 4}$$

$$= 4x + 2 + \frac{x}{x^2 + 4}$$

$$\int_1^4 4x + 2 + \frac{x}{x^2+4}\,dx$$

Guess: $\ln|x^2 + 4| \times \frac{1}{2}$

Check: $\frac{2x}{x^2 + 4} \times \frac{1}{2}$

$$= \left[2x^2 + 2x + \tfrac{1}{2}\ln|x^2 + 4|\right]_1^4$$

$$= \left(2(4)^2 + 2(4) + \tfrac{1}{2}\ln(4^2 + 4)\right) - \left(2(1)^2 + 2(1) + \tfrac{1}{2}\ln(1^2 + 4)\right)$$

$$= 40 + \tfrac{1}{2}\ln(20) - 4 - \tfrac{1}{2}\ln(5) = 36 + \tfrac{1}{2}\ln(4) = \boxed{36 + \ln(2)}$$

23. Find, in simplest form,

$$\int (2\cos x - \sin x)^2 \, dx$$

(5)

$$\int 4\cos^2(x) - 4\sin(x)\cos(x) + \sin^2(x) \, dx$$

$$= \int 4(1-\sin^2(x)) - 2\sin(2x) + \sin^2(x) \, dx$$

$$= \int 4 - 2\sin(2x) - 3\sin^2(x) \, dx$$

$\cos(2x) = 1 - 2\sin^2(x)$

$\frac{1-\cos(2x)}{2} = \sin^2(x)$

$$= 4x + \cos(2x) - \int 3\left(\frac{1-\cos(2x)}{2}\right) dx$$

$$= 4x + \cos(2x) - \frac{3}{2}\int 1 - \cos(2x) \, dx$$

$$= 4x + \cos(2x) - \frac{3}{2}\left(x - \frac{1}{2}\sin(2x)\right) + C$$

$$= \frac{5}{2}x + \cos(2x) + \frac{3}{4}\sin(2x) + C$$

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24. (a) Using the formula for $\sin(A+B)$ and the relevant double angle formulae, find an identity for $\sin 3x$, giving your answer in the form

$$\sin(3x) \equiv P\sin x + Q\sin^3 x$$

where P and Q are constants to be determined.

(4)

(b) Hence, showing each step of your working, evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 3x \cos x \, dx$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

a) $\sin(3x) \equiv \sin(2x+x)$

$= \sin(2x)\cos(x) + \cos(2x)\sin(x)$

$= 2\sin(x)\cos^2(x) + \left(1-2\sin^2(x)\right)\sin(x)$

$= 2\sin(x)\left(1-\sin^2(x)\right) + \sin(x) - 2\sin^3(x)$

$= 2\sin(x) - 2\sin^3(x) + \sin(x) - 2\sin^3(x)$

$= 3\sin(x) - 4\sin^3(x)$

b) $\int_{\pi/6}^{\pi/2} \sin(3x)\cos(x)\,dx$

$= \int_{\pi/6}^{\pi/2} \left(3\sin(x) - 4\sin^3(x)\right)\cos(x)\,dx$

$= \int_{\pi/6}^{\pi/2} 3\sin(x)\cos(x)\,dx - \int_{\pi/6}^{\pi/2} 4\sin^3(x)\cos(x)\,dx$

Guess: $\sin^2(x) \times \frac{3}{2}$ Guess: $\sin^4(x)$

Check: $2\sin(x)\cos(x) \times \frac{3}{2}$ Check: $4\sin^3(x)\cos(x)$

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Question 24 continued

$$= \left[\frac{3}{2}\sin^2(x) - \sin^4(x) \right]_{\pi/6}^{\pi/2}$$

note to integrate
$3\sin(x)\cos(x)$
you can also
rewrite it as
$\frac{3}{2}\sin(2x)$
and integrate that.

$$= \left(\frac{3}{2}\sin^2\left(\frac{\pi}{2}\right) - \sin^4\left(\frac{\pi}{2}\right) \right) - \left(\frac{3}{2}\sin^2\left(\frac{\pi}{6}\right) - \sin^4\left(\frac{\pi}{6}\right) \right)$$

$$= \left(\frac{3}{2} - 1 \right) - \left(\frac{3}{2} \times \frac{1}{4} - \frac{1}{16} \right)$$

$$= \boxed{\frac{3}{16}}$$

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25.

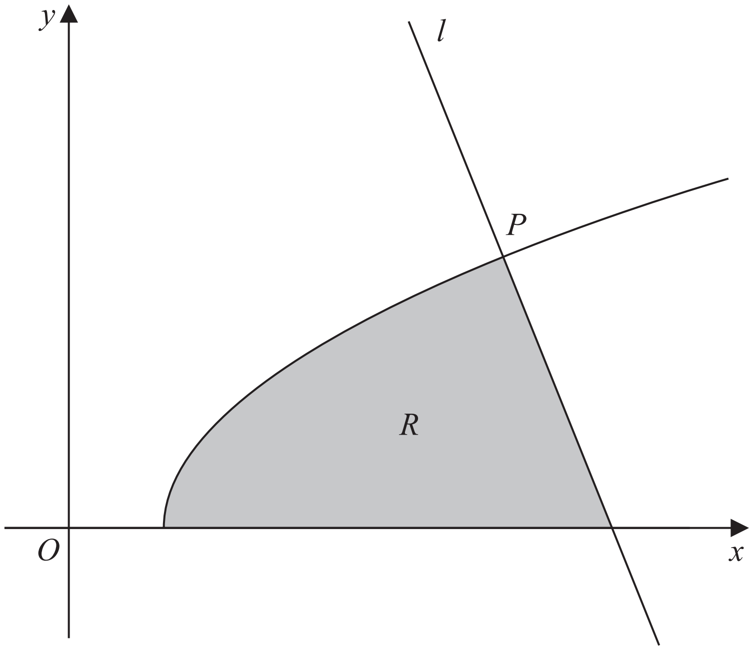

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

a) $y = (4x-7)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(4x-7)^{-1/2}(4)$

$= 2(4x-7)^{-1/2}$

$m_T = \left.\frac{dy}{dx}\right|_{x=8} = 2(4(8)-7)^{-1/2} = \frac{2}{5} \Rightarrow m_N = -\frac{5}{2}$

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Question 25 continued

$y = \frac{-5}{2}x + c$

sub in $(8, 5)$:

$5 = \frac{-5}{2}(8) + c \implies 5 = -20 + c \implies c = 25$

$y = \frac{-5}{2}x + 25$

$\implies 5x + 2y - 50 = 0$

b) First find x-intercepts of curve & normal!

$0 = \sqrt{4x-7}$ $\qquad$ $5x + 2(0) - 50 = 0$

$\implies x = 7/4$ $\qquad$ $\implies x = 10$

So Area $= \int_{7/4}^{8} \sqrt{4x-7}\,dx + \underbrace{(10-8) \times 5 \times \frac{1}{2}}_{\text{area of triangle.}}$

$= \int_{7/4}^{8} (4x-7)^{1/2}\,dx + 5$

Guess: $(4x-7)^{3/2} \times \frac{1}{6}$

$= \frac{1}{6}\left[(4x-7)^{3/2}\right]_{7/4}^{8} + 5$

Check: $6(4x-7)^{1/2} \times \frac{1}{6}$

$= \frac{1}{6}\left((25)^{3/2} - (0)^{3/2}\right) + 5$

$= \frac{125}{6} + 5 = \boxed{\frac{155}{6}}$

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26. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x-4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

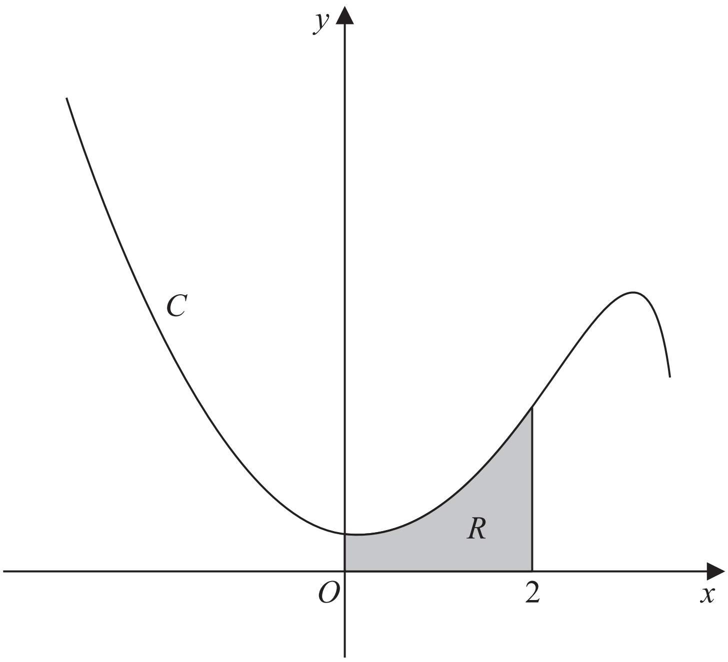

Figure 4

Figure 4 shows a sketch of the curve C.
The region R, shown shaded in Figure 4, is bounded by C, the y-axis, the x-axis and the line with equation $x = 2$

(c) Find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

a) $x^4 - x^3 - 10x^2 + 3x - 9 = (x^2+P)(x-4)(x+3) + Q(x+3)$

Since $x^2 - x - 12 = (x-4)(x+3)$

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Question 26 continued

could also do by substitution

$$x^4 - x^3 - 10x^2 + 3x - 9 = (x^2+P)(x^2-x-12) + Q(x+3)$$

$$= x^4 - x^3 + (P-12)x^2 + (Q-P)x - 12P + 3Q$$

$[x^2]$: $-10 = P - 12 \Rightarrow \boxed{P = 2}$

$[x]$: $3 = Q - P \Rightarrow 3 = Q - 2 \Rightarrow Q = 5$

b) $g(x) = x^2 + 2 + \frac{5}{x-4} = x^2 + 2 + 5(x-4)^{-1}$

$g'(x) = 2x - 5(x-4)^{-2}$

$m_T = g'(2) = 2(2) - 5(2-4)^{-2} = 4 - \frac{5}{4} = \frac{11}{4}$

so $y = \frac{11}{4}x + c$

Now $g(2) = 4 + 2 + \frac{5}{-2} = \frac{7}{2}$

$\therefore \frac{7}{2} = \frac{11}{4}(2) + c \Rightarrow c = \frac{7}{2} - \frac{11}{2} = -2$

$\therefore \boxed{y = \frac{11}{4}x - 2}$

c) Area $= \int_0^2 \left(x^2 + 2 + \frac{5}{x-4}\right) dx$

$= \left[\frac{1}{3}x^3 + 2x + 5\ln|x-4|\right]_0^2$

$= \left(\frac{1}{3}(2)^3 + 2(2) + 5\ln|-2|\right) - \left(5\ln|-4|\right)$

$\frac{1}{2} = 2^{-1}$

$= \frac{20}{3} + 5\ln(2) - 5\ln(4) = \frac{20}{3} + 5\ln\left(\frac{1}{2}\right) = \boxed{\frac{20}{3} - 5\ln(2)}$

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27.

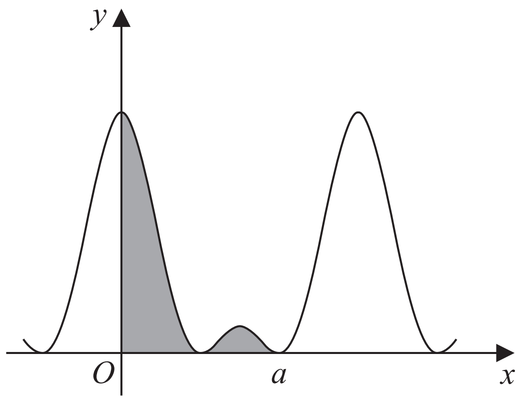

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where p, q and r are constants to be found.

(2)

The curve touches the positive x-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the y-axis and the x-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

(5)

a) $(1+2\cos(2x))^2 = 1 + 4\cos(2x) + 4\cos^2(2x)$

$= 1 + 4\cos(2x) + 2\cos(4x) + 2$

$\cos(2x) = 2\cos^2(x) - 1$

so $\cos(4x) = 2\cos^2(2x) - 1$ $\qquad = 3 + 4\cos(2x) + 2\cos(4x)$

$\Rightarrow 2\cos(4x) + 2 = 4\cos^2(2x)$

b) Find a first: it's where $y = 0$

i.e. $(1 + 2\cos(2x))^2 = 0$

$\Rightarrow \cos(2x) = -\frac{1}{2}$

$2x = \frac{2}{3}\pi, \; 2\pi - \frac{2}{3}\pi \ldots \quad x = \frac{1}{3}\pi, \; \frac{2}{3}\pi, \ldots$

Question 27 continued

by inspection a is the second smallest positive x intercept, so $a = \frac{2}{3}\pi$.

so Area $= \int_0^{2/3\pi} (1 + 2\cos(2x))^2 \, dx$

$= \int_0^{2/3\pi} 3 + 4\cos(2x) + 2\cos(4x) \quad dx$ by (a)

$= \left[3x + 2\sin(2x) + \frac{1}{2}\sin(4x) \right]_0^{2/3\pi}$

$= 3(2/3\pi) + 2\sin(4/3\pi) + \frac{1}{2}\sin(8/3\pi)$ $\sin(0) = 0$

$= 2\pi - \sqrt{3} + \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= 2\pi - \frac{3\sqrt{3}}{4}$

****

28. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{16}{9(3x - k)} \qquad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ giving your answer in simplest form in terms of k.

(2)

The point P with x coordinate 1 lies on C.

Given that the gradient of the curve at P is –12

(b) find the two possible values of k

(3)

Given also that $k < 3$

(c) find the equation of the normal to C at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(d) show, using algebraic integration that,

$$\int_1^3 \frac{16}{9(3x - k)}\,\mathrm{d}x = \lambda \ln 10$$

where λ is a constant to be found.

(4)

a) $y = \frac{16}{9}(3x-k)^{-1}$

$\frac{dy}{dx} = \frac{-16}{9} \times 3(3x-k)^{-2}$ by Chain rule

$= \frac{-16}{3(3x-k)^2}$

b) $\left.\frac{dy}{dx}\right|_{x=1} = -12 \Rightarrow -12 = \frac{-16}{3(3-k)^2}$ P.T.O

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Question 28 continued

$-36(3-k)^2 = -16$

$(3-k)^2 = \frac{4}{9}$

$3-k = \pm\frac{2}{3}$

$k = 3 \pm \frac{2}{3} = \underline{\frac{7}{3}}$ or $\underline{\frac{11}{3}}$

c) $k < 3$ so $k = \frac{7}{3}$

$m_T = \left.\frac{dy}{dx}\right|_{x=1} = -12$

so $m_N = \frac{1}{12}$

$\Rightarrow \quad y = \frac{1}{12}x + c \qquad$ A

Find y coordinate of P by subbing $x=1$ into $y = \frac{16}{9(3x-\frac{7}{3})}$

$x=1, y = \frac{16}{9(3-\frac{7}{3})} = \frac{8}{3} \qquad P = (1, \frac{8}{3})$

sub P into A:

$\frac{8}{3} = \frac{1}{12} + c \quad \Rightarrow \quad c = \frac{31}{12}$

$\Rightarrow \quad y = \frac{1}{12}x + \frac{31}{12}$

$x - 12y + 31 = 0$

Question 28 continued

d) $\displaystyle\int_1^3 \frac{16}{9\left(3x-\frac{7}{3}\right)}\,dx$

$= \dfrac{16}{9}\displaystyle\int_1^3 \frac{1}{3x-\frac{7}{3}}\,dx$

$= \dfrac{16}{9}\left[\dfrac{1}{3}\ln\left|3x-\dfrac{7}{3}\right|\right]_1^3$

$= \dfrac{16}{27}\left[\ln\left(3x-\dfrac{7}{3}\right)\right]_1^3$

$= \dfrac{16}{27}\left(\ln\left(\dfrac{20}{3}\right)-\ln\left(\dfrac{2}{3}\right)\right)$

$= \dfrac{16}{27}\ln(10)$

$\Rightarrow \lambda = \boxed{\dfrac{16}{27}}$

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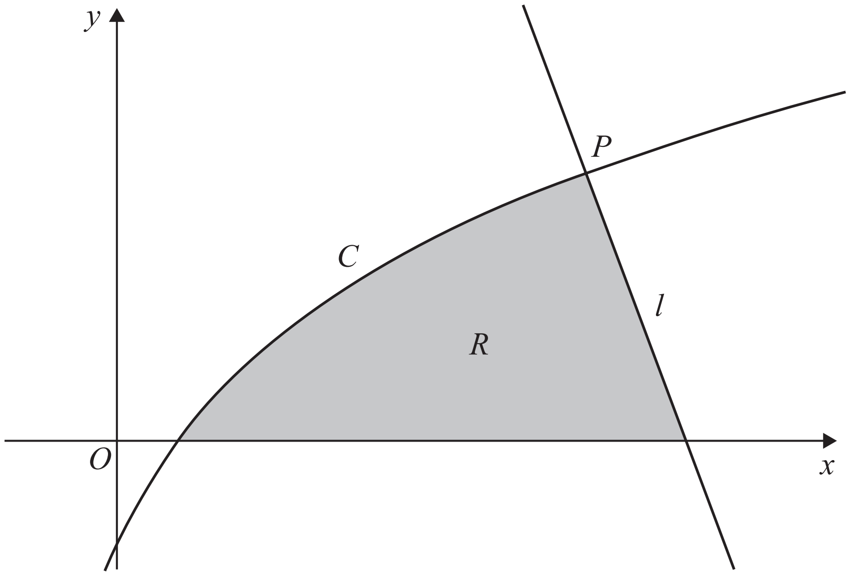

Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{6x^2 + 4x - 2}{2x + 1} \qquad x > -\frac{1}{2}$$

(a) Find $f'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

(b) Show that an equation for l is

$$16y + 5x = 106$$

(3)

(c) Write $f(x)$ in the form $Ax + B + \dfrac{D}{2x+1}$ where A, B and D are constants.

(3)

The region R, shown shaded in Figure 5, is bounded by C, l and the x-axis.

(d) Use algebraic integration to find the exact area of R, giving your answer in the form $P + Q\ln 3$, where P and Q are rational constants.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

a) Quotient rule : $u = 6x^2 + 4x - 2$ $\quad u' = 12x + 4$

$v = 2x + 1$ $\quad v' = 2$

$$f'(x) = \frac{(12x+4)(2x+1) - 2(6x^2+4x-2)}{(2x+1)^2}$$

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Question 29 continued

$$= \frac{24x^2 + 20x + 4 - 12x^2 - 8x + 4}{(2x+1)^2}$$

$$= \boxed{\frac{12x^2 + 12x + 8}{(2x+1)^2}} = \frac{4(3x^2 + 3x + 2)}{(2x+1)^2}$$

This step is to see if we can simplify at all. We can't though.

b) $m_T = f'(2) = \dfrac{4(3(2)^2 + 3(2) + 2)}{(2(2)+1)^2} = \dfrac{80}{25} = \dfrac{16}{5}$

$m_N = -\dfrac{5}{16}$

$y = -\dfrac{5}{16}x + c$ (A)

sub in $(2, 6)$ into (A):

$6 = -\dfrac{5}{16}(2) + c$

$6 = -\dfrac{5}{8} + c \Rightarrow c = \dfrac{53}{8}$.

$y = -\dfrac{5}{16}x + \dfrac{53}{8}$

$16y = -5x + 106$

$16y + 5x = 106$

c) $6x^2 + 4x - 2 = (Ax + B)(2x+1) + D$

$x = -\frac{1}{2}$: $-\frac{5}{2} = D$

$x = 0$: $-2 = B + D \Rightarrow -2 = B - \frac{5}{2} \Rightarrow B = \frac{1}{2}$

$6 = 2A \Rightarrow A = 3$ by comparing x^2 coefficient.

****

Question 29 continued

$f(x) = \boxed{3x + \tfrac{1}{2} - \dfrac{5}{2(2x+1)}}$

d) First find x-intercepts for $f(x)$ and the normal:

$f(x)=0 \Rightarrow 6x^2 + 4x - 2 = 0$

$3x^2 + 2x - 1 = 0$

$(3x-1)(x+1) = 0$

$x = \tfrac{1}{3}$ or $x = -1$ $\quad x > 0 \quad$ so $x = \tfrac{1}{3}$

$16(0) + 5x = 106 \Rightarrow x = 106/5$

Area of triangle

$$\text{so Area} = \int_{1/3}^{2} \left(3x + \tfrac{1}{2} - \frac{5}{2(2x+1)}\right) dx + \left(\frac{106}{5} - 2\right) \times 6 \times \frac{1}{2}$$

$$= \left[\frac{3}{2}x^2 + \frac{1}{2}x - \frac{5}{4}\ln|2x+1|\right]_{1/3}^{2} + \frac{288}{5}$$

$$= \left(\frac{3}{2}(2)^2 + \frac{1}{2}(2) - \frac{5}{4}\ln(2(2)+1)\right) - \left(\frac{3}{2}(\tfrac{1}{3})^2 + \tfrac{1}{2}(\tfrac{1}{3}) - \tfrac{5}{4}\ln(2(\tfrac{1}{3})+1)\right) + \frac{288}{5}$$

$$= \left(7 - \frac{5}{4}\ln(5)\right) - \left(\frac{1}{3} - \frac{5}{4}\ln\left(\tfrac{5}{3}\right)\right) + \frac{288}{5}$$

$$= \frac{964}{15} + \frac{5}{4}\ln\left(\frac{1}{3}\right)$$

$\tfrac{1}{3} = 3^{-1}$

$$= \frac{964}{15} - \frac{5}{4}\ln(3)$$

so, $\ln(\tfrac{1}{3}) = \ln(3^{-1})$

$= -\ln(3)$