London Academy of Maths

Loci and Argand Diagrams

Question Paper

1 **(a)** A locus C_1 is defined by $C_1 = \{z : |z+\mathrm{i}| \leqslant |z-2|\}$.

(i) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_1. **[2]**

(ii) Find the cartesian equation of the boundary line of the region representing C_1, giving your answer in the form $ax+by+c=0$. **[2]**

(b) A locus C_2 is defined by $C_2 = \{z : |z+1| \leqslant 3\} \cap \{z : |z-2\mathrm{i}| \geqslant 2\}$.

Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_2. **[3]**

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2 Draw the region of the Argand diagram for which $|z-3-4\mathrm{i}| \le 5$ and $|z| \le |z-2|$. **[4]**

3 **(a)** Sketch on a single Argand diagram the loci given by

(i) $|z-1+2\mathrm{i}|=3$, **[2]**

(ii) $|z+1|=|z-2|$. **[2]**

(b) Indicate, by shading, the region of the Argand diagram for which $|z-1+2\mathrm{i}|\leqslant 3$ and $|z+1|\leqslant|z-2|$. **[2]**

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4 Indicate by shading on an Argand diagram the region

$\{z : |z| \leqslant |z-4|\} \cap \{z : |z-3-2\mathrm{i}| \leqslant 2\}$. **[3]**

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5 Draw the region in an Argand diagram for which $|z| \le 2$ and $|z| > |z - 3\mathrm{i}|$. **[3]**

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6 Fig. 2 shows two complex numbers z_1 and z_2 represented on an Argand diagram.

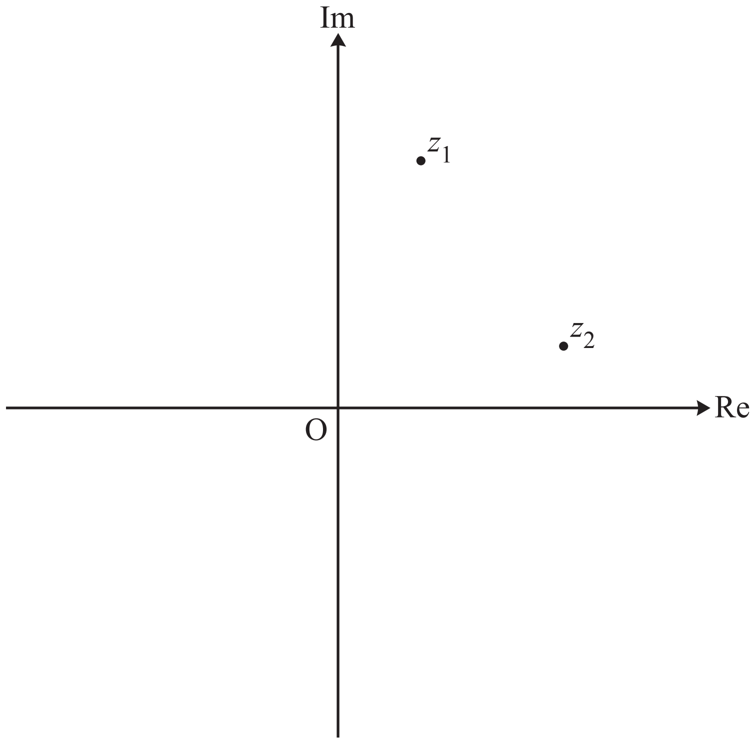

Fig. 2

(a) On the copy of Fig. 2 in the Printed Answer Booklet, mark points representing each of the following complex numbers.

- z_1^*
- $z_2 - z_1$ **[2]**

(b) **In this question you must show detailed reasoning.**

In the case where $z_1 = 1+2\mathrm{i}$ and $z_2 = 3+\mathrm{i}$, find $\dfrac{z_2 - z_1}{z_1^*}$ in the form $a+\mathrm{i}b$, where a and b are real numbers. **[2]**

7 **(i)** Write down, in complex form, the equation of the locus represented by the circle in the Argand diagram shown in Fig. 3. **[2]**

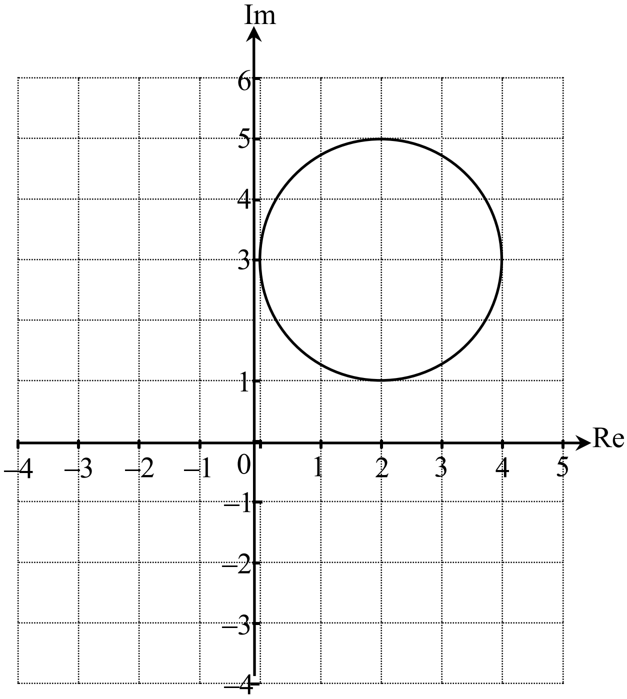

Fig. 3

(ii) On the copy of Fig. 3 in the Printed Answer Booklet mark with a cross any point(s) on the circle for which $\arg(z-2\text{i})=\frac{\pi}{4}$. **[2]**

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8 On separate Argand diagrams, sketch the set of points represented by each of the following.

(a) $|z-1-2\mathrm{i}| \leqslant 4$. **[3]**

(b) $\arg(z+\mathrm{i}) = \frac{1}{3}\pi$. **[3]**

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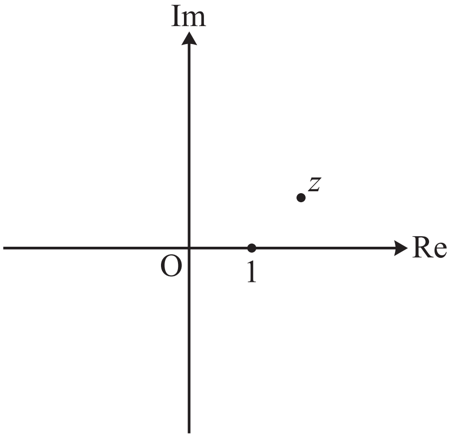
9 The Argand diagram below shows the points representing 1 and z, where $|z| = 2$.



Mark the points representing the following complex numbers on the copy of the diagram in the Printed Answer Booklet, labelling them clearly.

- z^*
- $\frac{1}{z}$
- $1 + z$
- $\mathrm{i}z$ **[4]**

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10 **(a)** On a single Argand diagram, sketch the loci defined by

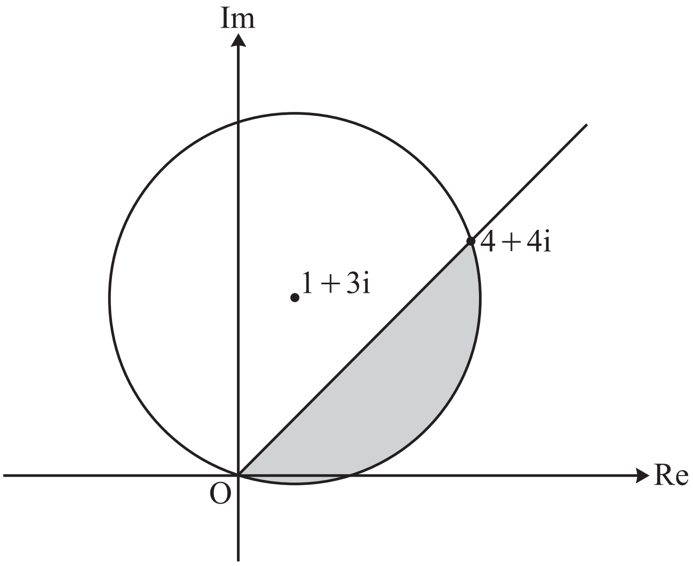
- $\arg(z-2) = \frac{3}{4}\pi$,
- $|z| = |z+2-\mathrm{i}|$. **[4]**

(b) **In this question you must show detailed reasoning.**

The point of intersection of the two loci in part **(a)** represents the complex number w.

Find w, giving your answer in exact form. **[5]**

11 An Argand diagram is shown below. The circle has centre at the point representing $1+3\text{i}$, and the half line intersects the circle at the origin and at the point representing $4+4\text{i}$.



State the **two** conditions that define the set of complex numbers represented by points in the shaded segment, including its boundaries. **[5]**

12 Fig. 9 shows a sketch of the region OPQ of the Argand diagram defined by

$$\left\{z : |z| \leqslant 4\sqrt{2}\right\} \cap \left\{z : \tfrac{1}{4}\pi \leqslant \arg z \leqslant \tfrac{1}{3}\pi\right\}.$$

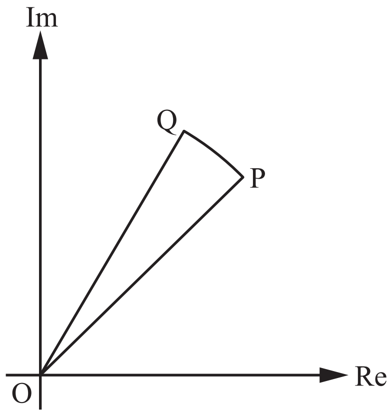


Fig. 9

(i) Find, in modulus-argument form, the complex number represented by the point P. **[2]**

(ii) Find, in the form $a + \mathrm{i}b$, where a and b are exact real numbers, the complex number represented by the point Q. **[3]**

(iii) **In this question you must show detailed reasoning.**

Determine whether the points representing the complex numbers

- $3 + 5\mathrm{i}$
- $5.5(\cos 0.8 + \mathrm{i}\sin 0.8)$

lie within this region. **[4]**

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13 **(i)** On an Argand diagram draw the locus of points which satisfy $\arg(z-4\mathrm{i})=\frac{\pi}{4}$. **[2]**

(ii) Give, in complex form, the equation of the circle which has centre at $6+4\mathrm{i}$ and touches the locus in part **(i)**. **[4]**

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14 Two sets of complex numbers are given by $\left\{z : \arg(z-10) = \frac{3}{4}\pi\right\}$ and $\{z : |z-3-6\mathrm{i}| = k\}$, where k is a positive constant. In an Argand diagram, one of the points of intersection of the two loci representing these sets lies on the imaginary axis.

(a) Sketch the loci on an Argand diagram. **[4]**

(b) **In this question you must show detailed reasoning.**

Find the complex numbers represented by the points of intersection. **[7]**

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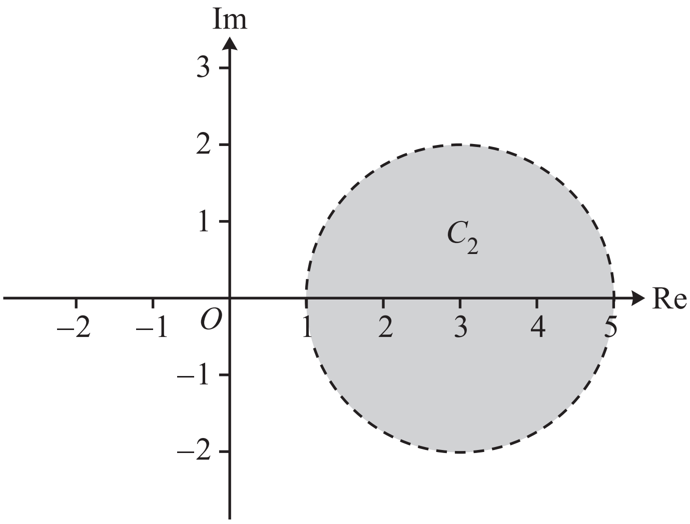
15 The locus C_1 is defined by $C_1 = \left\{ z : 0 \leqslant \arg(z+\mathrm{i}) \leqslant \frac{1}{4}\pi \right\}$.

(a) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_1. **[2]**

(b) Determine whether the complex number $1.2 + 0.8\mathrm{i}$ is in C_1. **[2]**

The locus C_2 is the set of complex numbers represented by the interior of the circle with radius 2 and centre 3. The locus C_2 is illustrated on the Argand diagram below.



(c) Use set notation to define C_2. **[2]**

(d) Determine whether the complex number $1.2 + 0.8\mathrm{i}$ is in C_2. **[2]**

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16 In an Argand diagram, the point P representing the complex number w lies on the locus defined by $\left\{z : \arg(z-7) = \frac{3}{4}\pi\right\}$. You are given that $\mathrm{Re}(w) = 1$.

(a) Find w. **[2]**

The point P also lies on the locus defined by $\{z : |z+3-9\mathrm{i}| = k\}$, where k is a constant.

(b) Find the complex number represented by the other point of intersection of the loci defined by $\{z : |z+3-9\mathrm{i}| = k\}$ and $\left\{z : \arg(z-7) = \frac{3}{4}\pi\right\}$. **[7]**

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17 **(a** On separate Argand diagrams, show the set of points representing each of the following inequalities.

(i) $|z| \leqslant \sqrt{5}$ **[3]**

(ii) $|z+2-4\mathrm{i}| \geqslant |z-2-6\mathrm{i}|$ **[3]**

(b) Show that there is a unique value of z, which should be determined, for which both $|z| \leqslant \sqrt{5}$ and $|z+2-4\mathrm{i}| \geqslant |z-2-6\mathrm{i}|$. **[8]**

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18 **In this question you must show detailed reasoning.**

You are given that $f(z) = 4z^4 - 12z^3 + 41z^2 - 128z + 185$ and that $2+i$ is a root of the equation $f(z) = 0$.

(a) Express $f(z)$ as the product of two quadratic factors with integer coefficients. **[5]**

(b) Solve $f(z) = 0$. **[3]**

Two loci on an Argand diagram are defined by $C_1 = \{z : |z| = r_1\}$ and $C_2 = \{z : |z| = r_2\}$ where $r_1 > r_2$. You are given that two of the points representing the roots of $f(z) = 0$ are on C_1 and two are on C_2. R is the region on the Argand diagram between C_1 and C_2.

(c) Find the exact area of R. **[4]**

(d) ω is the sum of all the roots of $f(z) = 0$.

Determine whether or not the point on the Argand diagram which represents ω lies in R. **[2]**

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19 **In this question you must show detailed reasoning.**

Two loci, C_1 and C_2, are defined as follows.

$C_1 = \left\{ z : \arg(z + 2 - \mathrm{i}) = \frac{1}{4}\pi \right\}$ and $C_2 = \left\{ z : \arg(z - 2 - \sqrt{3} - 2\mathrm{i}) = \frac{2}{3}\pi \right\}$

By considering the representations of C_1 and C_2 on an Argand diagram, determine the locus $C_1 \cap C_2$. **[7]**

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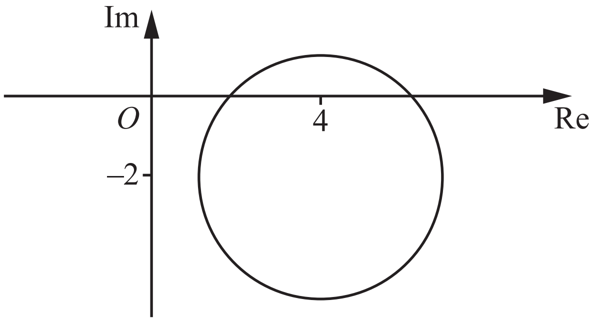
20 The Argand diagram shows a circle of radius 3. The centre of the circle is the point which represents the complex number $4-2\text{i}$.



(a) Use set notation to define the locus of complex numbers, z, represented by points which lie on the circle. **[2]**

The locus L is defined by $L=\{z : z\in\mathbb{C}, |z-\text{i}|=|z+2|\}$.

(b) On the Argand diagram in the Printed Answer Booklet, sketch and label the locus L. **[2]**

You are given that the locus $\left\{z : z\in\mathbb{C}, \arg(z-1)=\frac{1}{4}\pi, \operatorname{Re}(z)=3\right\}$ contains only one number.

(c) Find this number. **[2]**

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21 In this question you must show detailed reasoning.

(a) Find the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

The loci C_1 and C_2 are given by $|z| = |z - 2i|$ and $|z - 2| = \sqrt{5}$ respectively.

(b) (i) Sketch on a single Argand diagram the loci C_1 and C_2, showing any intercepts with the imaginary axis. **[3]**

(ii) Indicate, by shading on your Argand diagram, the region

$$\{z : |z| \leqslant |z - 2i|\} \cap \{z : |z - 2| \leqslant \sqrt{5}\}.$$ **[1]**

(c) (i) Show that both of the roots of the equation $2z^2 - 2z + 5 = 0$ satisfy $|z - 2| < \sqrt{5}$. **[2]**

(ii) State, with a reason, which root of the equation $2z^2 - 2z + 5 = 0$ satisfies $|z| < |z - 2i|$. **[1]**

(d) On the same Argand diagram as part **(b)**, indicate the positions of the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

22 In an Argand diagram the points representing the numbers $2 + 3\mathrm{i}$ and $1 - \mathrm{i}$ are two adjacent vertices of a square, S.

(a) Find the area of S. **[3]**

(b) Find all the possible pairs of numbers represented by the other two vertices of S. **[4]**

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23 Two loci, C_1 and C_2, are defined by

$$C_1 = \{z : |z| = |z - 4d^2 - 36|\}$$

$$C_2 = \left\{z : \arg(z - 12d - 3\text{i}) = \frac{1}{4}\pi\right\}$$

where d is a real number.

(a) Find, in terms of d, the complex number which is represented on an Argand diagram by the point of intersection of C_1 and C_2.

[You may assume that $C_1 \cap C_2 \neq \varnothing$.] **[6]**

(b) Explain why the solution found in part **(a)** is not valid when $d = 3$. **[2]**