London Academy of Maths

Vectors

Question Paper

1 **(i)** Find a vector which is perpendicular to both $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix}$. **[2]**

(ii) The cartesian equation of a line is $\frac{x}{2} = y - 3 = 2z + 4$.

Express the equation of this line in vector form. **[3]**

2 Find, to the nearest degree, the angle between the vectors $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$. **[3]**

3 **(a)** Determine whether the point (19, −12, 17) lies on the line $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. **[3]**

Vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$.

(b) **(i)** Find, in degrees, the angle between **a** and **b**. **[3]**

(ii) Find a vector which is perpendicular to both **a** and **b**. **[2]**

4 The vector **p**, all of whose components are positive, is given by $\mathbf{p} = \begin{pmatrix} a^2 \\ a-5 \\ 26 \end{pmatrix}$ where a is a constant.

You are given that **p** is perpendicular to the vector $\begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$.

Determine the value of a. **[4]**

5 The lines l_1 and l_2 have the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ -11 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -6 \\ 11 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

(a) Show that l_1 and l_2 intersect. **[4]**

(b) Write down the point of intersection of l_1 and l_2. **[1]**

6 **(a)** **(i)** Find $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. **[1]**

(ii) State a geometrical relationship between the answer to part **(a)(i)** and the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. **[1]**

(iii) Verify the relationship stated in part **(a)(ii)**. **[2]**

(b) Find the angle between the vectors $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$. **[3]**

7 **(a)** Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. **[1]**

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

(b) Show that the lines intersect, stating the point of intersection. **[5]**

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8 The lines L_1 and L_2 have the following equations.

$$L_1 : \mathbf{r} = \begin{pmatrix} -5 \\ 6 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix}$$

$$L_2 : \mathbf{r} = \begin{pmatrix} 24 \\ 1 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$$

(a) Show that L_1 and L_2 intersect, giving the position vector of the point of intersection. **[5]**

(b) Find the equation of the line which intersects L_1 and L_2 and is perpendicular to both. Give your answer in cartesian form. **[3]**

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9 Find the acute angle between the lines with vector equations $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. **[3]**

10 **(a)** Given that the lines $\mathbf{r}=\begin{pmatrix}0\\2\\2\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\3\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}-1\\2\\k\end{pmatrix}+\mu\begin{pmatrix}2\\3\\4\end{pmatrix}$ meet, determine k. **[5]**

(b) **In this question you must show detailed reasoning.**

Find the acute angle between the two lines. **[4]**

11 Vectors, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, are given by $\mathbf{a} = \mathbf{i} + (1-p)\mathbf{j} + (p+2)\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \mathbf{i} + 14\mathbf{j} + (p-3)\mathbf{k}$ where p is a constant.

You are given that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$.

Determine the possible values of p. **[6]**

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12 Determine whether the lines

$$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2} \qquad \text{and} \qquad \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$$

intersect or are skew. **[5]**

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13 The position vector of point A is $\mathbf{a} = -9\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$.
The line l passes through A and is perpendicular to $\mathbf{a}$.

(a) Determine the shortest distance between the origin, O, and l. **[2]**

l is also perpendicular to the vector $\mathbf{b}$ where $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$.

(b) Find a vector which is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. **[1]**

(c) Write down an equation of l in vector form. **[1]**

P is a point on l such that $PA = 2OA$.

(d) Find angle POA giving your answer to 3 significant figures. **[3]**

C is a point whose position vector, $\mathbf{c}$, is given by $\mathbf{c} = p\mathbf{a}$ for some constant p. The line m passes through C and has equation $\mathbf{r} = \mathbf{c} + \mu\mathbf{b}$. The point with position vector $9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$ lies on m.

(e) Find the value of p. **[3]**

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14 The line through points $A(8, -7, -2)$ and $B(11, -9, 0)$ is denoted by L_1.

(a) Find a vector equation for L_1. **[2]**

(b) Determine whether the point $(26, -19, -14)$ lies on L_1. **[2]**

The line L_2 passes through the origin, O, and intersects L_1 at the point C. The lines L_1 and L_2 are perpendicular.

(c) By using the fact that C lies on L_1, find a vector equation for L_2. **[4]**

(d) Hence find the shortest distance from O to L_1. **[2]**

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15 **(i)** Find the value of k such that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ k \end{pmatrix}$ are perpendicular. **[2]**

Two lines have equations $l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(ii) Find the point of intersection of l_1 and l_2. **[4]**

(iii) The vector $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$ is perpendicular to the lines l_1 and l_2.

Find the values of a and b. **[5]**

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16 Determine the acute angle between the line $\mathbf{r}=\begin{pmatrix}-\sqrt{3}\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\2\sqrt{3}\\-\sqrt{3}\end{pmatrix}$ and the *y*-axis. **[4]**

17 The line l has equations $\dfrac{x-1}{2} = \dfrac{y-1}{3} = \dfrac{z+1}{2}$ and the point A is (7, 3, 7). M is the point where the perpendicular from A meets l.

(i) Find, in either order, the coordinates of M and the perpendicular distance from A to l. **[7]**

(ii) Find the coordinates of the point B on AM such that $\overrightarrow{AB} = 3\overrightarrow{BM}$. **[3]**

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18 Points A, B and C have coordinates (4, 2, 0), (1, 5, 3) and (1, 4, −2) respectively.
The line l passes through A and B.

(a) Find a cartesian equation for l. **[3]**

M is the point on l that is closest to C.

(b) Find the coordinates of M. **[4]**

(c) Find the exact area of the triangle ABC. **[4]**

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19 Three intersecting lines L_1, L_2 and L_3 have equations

$$L_1\text{: } \frac{x}{2}=\frac{y}{3}=\frac{z}{1},\quad L_2\text{: } \frac{x}{1}=\frac{y}{2}=\frac{z}{-4}\quad \text{and}\quad L_3\text{: } \frac{x-1}{1}=\frac{y-2}{1}=\frac{z+4}{5}.$$

Find the area of the triangle enclosed by these lines. **[9]**

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20 The line segment AB is a diameter of a sphere, S. The point C is **any** point on the surface of S.

(a) Explain why $\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$ for **all** possible positions of C. **[3]**

You are now given that A is the point $(11, 12, -14)$ and B is the point $(9, 13, 6)$.

(b) Given that the coordinates of C have the form $(2p, p, 1)$, where p is a constant, determine the coordinates of the possible positions of C. **[6]**

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21 The equations of two **intersecting** lines are

$$\mathbf{r} = \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

where a is a constant.

(a) Find a vector, **b**, which is perpendicular to both lines. **[2]**

(b) Show that $\mathbf{b}.\begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} = \mathbf{b}.\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$. **[2]**

(c) Hence, or otherwise, find the value of a. **[2]**

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22 The points $P(3, 5, -21)$ and $Q(-1, 3, -16)$ are on the ceiling of a long straight underground tunnel. A ventilation shaft must be dug from the point M on the ceiling of the tunnel midway between P and Q to horizontal ground level (where the z-coordinate is 0). The ventilation shaft must be perpendicular to the tunnel.

The path of the ventilation shaft is modelled by the vector equation $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ is the position vector of M.

You are given that $\mathbf{b} = \begin{pmatrix} 1 \\ s \\ t \end{pmatrix}$ where s and t are real numbers.

(a) Show that $s = 2.5t - 2$. **[3]**

(b) Show that at the point where the ventilation shaft reaches the ground $\lambda = \frac{c}{t}$, where c is a constant to be determined. **[3]**

(c) Using the results in parts **(a)** and **(b)**, determine the shortest possible length of the ventilation shaft. **[6]**

(d) Explain what the fact that $\mathbf{b} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \neq \mathbf{0}$ means about the direction of the ventilation shaft. **[1]**