London Academy of Maths

Series - Standard Formulae

Question Paper

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1 Find $\sum_{r=1}^{n}(2r-1)^2$, expressing your answer in a fully factorised form. **[6]**

2 Find $\sum_{r=1}^{n}(3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. **[7]**

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3 Find $\sum_{r=1}^{n}(4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form. **[6]**

4 Find $\sum_{r=1}^{n}(3r+1)(r-1)$, giving your answer in a fully factorised form. **[5]**

5 Find $\sum_{r=1}^{n}(r^2 - r - 8)$, giving your answer in a fully factorised form. **[5]**

6 Find $\sum_{r=1}^{n}(r+1)(r-1)$, expressing your answer in a fully factorised form. [6]

7 Use standard series formulae to find $\sum_{r=1}^{n} r(r^2+1)$, factorising your answer as far as possible. [6]

8 Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n} [(r+1)(r-2)] = \frac{1}{3}n(n^2 - 7)$. [6]

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9 Use standard series formulae to show that $\sum_{r=1}^{n}(r+2)(r-3) = \frac{1}{3}n(n^2-19)$. **[6]**

10 Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \tfrac{1}{4}n(n+1)(n+3)(n-2).$$ **[6]**

11 **(i)** Use standard series formulae to show that

$$\sum_{r=1}^{n}[r(r-1)-1] = \frac{1}{3}n(n+2)(n-2).$$ **[5]**

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12 Use standard series formulae to find $\sum_{r=1}^{n} r(r-2)$, factorising your answer as far as possible. **[5]**

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13 Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{10} r(3r-2) = 1045$. **[3]**

14 In this question you must show detailed reasoning.

Determine the value of $\sum_{r=1}^{50} r^2(16-r)$. **[3]**

15 In this question you must show detailed reasoning.

Determine the value of $\sum_{r=1}^{100}(2r+3)^2$. **[3]**

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16 Find $\sum_{r=1}^{n}(r+1)(r+5)$. Give your answer in a fully factorised form. **[4]**

17 Using standard summation formulae, find $\sum_{r=1}^{n}(r^2 - 3r)$, giving your answer in fully factorised form. **[3]**

18 Find $\sum_{r=1}^{n}(2r^2-1)$, expressing your answer in fully factorised form. **[4]**

19 Find $\sum_{r=1}^{n}(4r^3 + 6r^2 + 2r)$, expressing your answer in a fully factorised form. **[6]**

20 Find $\sum_{r=1}^{n}(r-1)(r+1)$, giving your answer in a fully factorised form. **[6]**

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21 Find $\sum_{r=1}^{n}(3r^2-5)$, expressing your answer in a fully factorised form. **[4]**

22 **(a)** Using standard summation formulae, write down an expression in terms of n for $\sum_{r=1}^{2n} r^3$. **[1]**

(b) Hence show that $\sum_{r=n+1}^{2n} r^3 = \frac{1}{4}n^2(an+b)(cn+d)$, where a, b, c and d are integers to be determined. **[5]**

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23 **(i)** Show that, when $n = 5$, $\sum_{r=n+1}^{2n} r^2 = 330$. **[1]**

(ii) Find, in terms of n, a fully factorised expression for $\sum_{r=n+1}^{2n} r^2$. **[4]**

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24 Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. **[6]**

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25 Find $\sum_{r=1}^{n} r(r^2 - 3)$, expressing your answer in a fully factorised form. **[6]**

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26 Find $\sum_{r=1}^{n} r^2(r-1)$, expressing your answer in a fully factorised form. [6]

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27 Find $\sum_{r=1}^{2n}\left(3r^2 - \frac{1}{2}\right)$, expressing your answer in a fully factorised form. **[6]**

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28 **(i)** Find $\sum_{r=1}^{n} r(r^2+r-7)$, giving your answer in a fully factorised form. **[5]**

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29 **(i)** Show that $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$. **[4]**

(ii) Hence find $\sum_{r=n}^{2n} r(r^2-2)$, giving your answer in a fully factorised form. **[5]**

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30 Use standard series formulae to show that $\sum_{r=1}^{n} r^2(3-4r) = \frac{1}{2}n(n+1)(1-2n^2)$. **[5]**

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31 **(i)** Using the standard summation formulae, find an expression for $\sum_{r=1}^{n}(1-2r)^2$ in terms of n. Give your answer in a fully factorised form. **[6]**

(ii) Hence evaluate $\sum_{r=25}^{75}(1-2r)^2$. **[2]**

32 **(i)** Show that $\sum_{r=1}^{n}(2r-1)=n^2$. **[3]**

(ii) Show that $\dfrac{\sum_{r=1}^{n}(2r-1)}{\sum_{r=n+1}^{2n}(2r-1)}=k$, where k is a constant to be determined. **[4]**

33 **(a)** Using standard summation formulae, show that $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$. **[4]**

(b) Use induction to prove the result in part **(a)**. **[6]**

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34 **(i)** Use standard series to show that

$$\sum_{r=1}^{n} r^2(2r-p) = \frac{1}{6}n(n+1)(3n^2+(3-2p)n-p),$$

where p is a constant. **[4]**

(ii) Given that the coefficients of n^3 and n^4 in the expression for $\sum_{r=1}^{n} r^2(2r-p)$ are equal, find the value of p. **[2]**

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35 Given that $\sum_{r=1}^{n}(ar^3 + br) \equiv n(n-1)(n+1)(n+2)$, find the values of the constants a and b. **[6]**

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36 **(i)** Using the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, prove that

$$\sum_{r=1}^{n} r^2(r+1) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

(ii) Prove the same result by mathematical induction. [8]

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37 In this question you must show detailed reasoning.

Determine the smallest value of n for which $\dfrac{1^2+2^2+\ldots+n^2}{1+2+\ldots+n} > 341$. **[4]**

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38 You are given that $\sum_{r=1}^{n}(ar+b) = n^2$ for all n, where a and b are constants.

By finding $\sum_{r=1}^{n}(ar+b)$ in terms of a, b and n, determine the values of a and b. **[6]**

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39 Find an expression for $1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \ldots + n(n+1)^2$ in terms of n. Give your answer in fully factorised form. **[3]**

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40 Using standard summation of series formulae, determine the sum of the first n terms of the series

$(1 \times 2 \times 4) + (2 \times 3 \times 5) + (3 \times 4 \times 6) + \ldots,$

where n is a positive integer. Give your answer in fully factorised form. **[6]**

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41 In this question you must show detailed reasoning.

The series S is defined as being the sum of the squares of all positive **odd** integers from 1^2 to 779^2.

Determine the value of S. **[5]**