London Academy of Maths

Complex Number Basics

Question Paper

1 Show that $\dfrac{5}{2-4\mathrm{i}} = \frac{1}{2} + \mathrm{i}$. **[2]**

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2 The complex number $x+\mathrm{i}y$ is denoted by z. Express $3zz^{*}-|z|^{2}$ in terms of x and y. **[3]**

3 The complex numbers z and w are given by $z = 5 - 2\mathrm{i}$ and $w = 3 + 7\mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $4z - 3w$, **[2]**

(ii) z^*w. **[2]**

4 The complex numbers a and b are given by $a = 7 + 6\text{i}$ and $b = 1 - 3\text{i}$. Showing clearly how you obtain your answers, find

(i) $|a - 2b|$ and $\arg(a - 2b)$, **[4]**

(ii) $\dfrac{b}{a}$, giving your answer in the form $x + \text{i}y$. **[3]**

5 The complex numbers z and w are given by $z = 6 - \text{i}$ and $w = 5 + 4\text{i}$. Giving your answers in the form $x + \text{i}y$ and showing clearly how you obtain them, find

(i) $z + 3w$, **[2]**

(ii) $\dfrac{z}{w}$. **[3]**

6 Express $\frac{2+3\text{i}}{5-\text{i}}$ in the form $x + \text{i}y$, showing clearly how you obtain your answer. **[4]**

7 The complex number z_1 is $1+\mathrm{i}$ and the complex number z_2 has modulus 4 and argument $\frac{\pi}{3}$.

(i) Express z_2 in the form $a+b\mathrm{i}$, giving a and b in exact form. **[2]**

(ii) Express $\frac{z_2}{z_1}$ in the form $c+d\mathrm{i}$, giving c and d in exact form. **[2]**

8 Find real numbers a and b such that $(a-3\mathrm{i})(5-\mathrm{i}) = b-17\mathrm{i}$. **[5]**

9 **In this question you must show detailed reasoning.**

Given that $z_1 = 3 + 2\mathrm{i}$ and $z_2 = -1 - \mathrm{i}$, find the following, giving each in the form $a + b\mathrm{i}$.

(i) $z_1^* z_2$ **[2]**

(ii) $\dfrac{z_1 + 2z_2}{z_2}$ **[2]**

10 The complex numbers z and w are given by $z = 4 + 3\text{i}$ and $w = 6 - \text{i}$. Giving your answers in the form $x + \text{i}y$ and showing clearly how you obtain them, find

(i) $3z - 4w$, **[2]**

(ii) $\dfrac{z^*}{w}$. **[4]**

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11 The complex number w has modulus 6 and argument $\frac{2\pi}{3}$. Find $\frac{\sqrt{3}+2\mathrm{i}}{w}$, giving your answer in the form $x+\mathrm{i}y$, where x and y are exact real numbers. **[5]**

12 The complex number $2 - \text{i}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. **[2]**

(ii) Given that $az + bz^* = 4 - 8\text{i}$, find the values of the real constants a and b. **[5]**

13 The complex number z satisfies the equation $z + 2\mathrm{i}z^* = 12 + 9\mathrm{i}$. Find z, giving your answer in the form $x + \mathrm{i}y$. **[5]**

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14 The complex number $a + 5\text{i}$, where a is positive, is denoted by z. Given that $|z| = 13$, find the value of a and hence find $\arg z$. **[4]**

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15 The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x+\mathrm{i}y$, where x and y are exact real numbers, and showing clearly how you obtain them, find

(i) z, **[2]**

(ii) $\dfrac{1}{(z^*-5\mathrm{i})^2}$. **[5]**

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16 The complex number $a + \mathrm{i}b$ is denoted by z and the complex number $c + \mathrm{i}d$ is denoted by w.

It is given that $z^2 = z^*w$.

(i) Show that $2ab = ad - bc$. **[4]**

(ii) Given that the real part of $w = 0$, find the values of b in terms of a. **[6]**

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17 **In this question you must show detailed reasoning.**

The complex numbers z_1 and z_2 are given by $z_1 = 3 - 7\mathrm{i}$ and $z_2 = 2 + 4\mathrm{i}$.

(a) Express each of the following as exact numbers in the form $a + b\mathrm{i}$.

(i) $3z_1 + 4z_2$ **[1]**

(ii) $z_1 z_2$ **[2]**

(iii) $\dfrac{z_1}{z_2}$ **[2]**

(b) Write z_1 in modulus-argument form giving the modulus in exact form and the argument correct to **3** significant figures. **[3]**

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18 You are given that $z = 3 - 4\mathrm{i}$.

(a) Find

- $|z|$,
- $\arg(z)$,
- z^*. **[3]**

On an Argand diagram the complex number w is represented by the point A and w^* is represented by the point B.

(b) Describe the geometrical relationship between the points A and B. **[2]**

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19 In this question you must show detailed reasoning.

In this question the principal argument of a complex number lies in the interval $[0, 2\pi)$.

Complex numbers z_1 and z_2 are defined by $z_1 = 3 + 4\mathrm{i}$ and $z_2 = -5 + 12\mathrm{i}$.

(a) Determine $z_1 z_2$, giving your answer in the form $a + b\mathrm{i}$. **[2]**

(b) Express z_2 in modulus-argument form. **[3]**

(c) Verify, by direct calculation, that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. **[3]**

20 **In this question you must show detailed reasoning.**

(a) Express $\dfrac{8+\mathrm{i}}{2-\mathrm{i}}$ in the form $a+b\mathrm{i}$ where a and b are real. **[2]**

(b) Solve the equation $4x^2-8x+5=0$. Give your answer(s) in the form $c+d\mathrm{i}$ where c and d are real. **[2]**

21 **In this question you must show detailed reasoning.**

(a) Solve the equation $2z^2 - 10z + 25 = 0$ giving your answers in the form $a + b\mathrm{i}$. **[2]**

(b) Solve the equation $3\omega - 2 = \mathrm{i}(5 + 2\omega)$ giving your answer in the form $a + b\mathrm{i}$. **[4]**

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22 **In this question you must show detailed reasoning.**

The complex numbers z_1 and z_2 are given by $z_1 = 2-3\mathrm{i}$ and $z_2 = a+4\mathrm{i}$ where a is a real number.

(i) Express z_1 in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. **[3]**

(ii) Find $z_1 z_2$ in terms of a, writing your answer in the form $c + \mathrm{i}d$. **[2]**

(iii) The real and imaginary parts of a complex number on an Argand diagram are x and y respectively. Given that the point representing $z_1 z_2$ lies on the line $y = x$, find the value of a. **[2]**

(iv) Given instead that $z_1 z_2 = (z_1 z_2)^*$ find the value of a. **[2]**

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23 The complex number z satisfies the equation $5(z-\mathrm{i}) = (-1+2\mathrm{i})z^*$.

Determine z, giving your answer in the form $a+b\mathrm{i}$, where a and b are real. **[5]**

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24 **(a)** The complex number $a+\mathrm{i}b$ is denoted by z.

(i) Write down z^*. **[1]**

(ii) Find $\mathrm{Re}(\mathrm{i}z)$. **[2]**

(b) The complex number w is given by $w=\dfrac{5+\mathrm{i}\sqrt{3}}{2-\mathrm{i}\sqrt{3}}$.

(i) **In this question you must show detailed reasoning.**

Express w in the form $x+\mathrm{i}y$. **[2]**

(ii) Convert w to modulus-argument form. **[2]**

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25 Two complex numbers are given by $u = -1 + \text{i}$ and $v = -2 - \text{i}$.

(a) **(i)** Find $u - v$ in the form $a + b\text{i}$, where a and b are real. **[1]**

(ii) **In this question you must show detailed reasoning.**

Find $\frac{u}{v}$ in the form $a + b\text{i}$, where a and b are real. **[3]**

(b) Express u in exact modulus-argument form. **[3]**

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26 **In this question you must show detailed reasoning.**

The complex number $\sqrt{3}+\mathrm{i}$ is denoted by z.

(a) By expanding $(\sqrt{3}+\mathrm{i})^5$, express z^5 in the form $a+b\mathrm{i}$ where a and b are real and exact. **[3]**

(b) **(i)** Express z in modulus-argument form. **[3]**

(ii) Hence find z^5 in modulus-argument form. **[2]**

(iii) Use this result to verify your answers to part **(a)**. **[2]**

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27 The complex number z satisfies the equation $z^2 - 4\mathrm{i}z^* + 11 = 0$.

Given that $\mathrm{Re}(z) > 0$, find z in the form $a + b\mathrm{i}$, where a and b are real numbers. **[4]**

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28 In this question you must show detailed reasoning.

The complex numbers z_1 and z_2 are given by $z_1 = -2 + 2\mathrm{i}$ and $z_2 = 2\left(\cos\frac{1}{6}\pi + \mathrm{i}\sin\frac{1}{6}\pi\right)$.

(a) Find the modulus and argument of z_1. **[2]**

(b) Hence express $\dfrac{z_1}{z_2}$ in exact modulus-argument form. **[4]**

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29 **In this question you must show detailed reasoning.**

(a) Write the complex number $-24+7\mathrm{i}$ in modulus-argument form. **[3]**

(b) Solve the simultaneous equations given below, giving your answers in cartesian form.

$$\begin{aligned} \mathrm{i}z + 3w &= -7\mathrm{i} \\ -6z + 5\mathrm{i}w &= 3+13\mathrm{i} \end{aligned}$$

[4]

30 **In this question you must show detailed reasoning.**

(a) Solve the equation $x^2 - 6x + 58 = 0$. Give your solutions in the form $a + b\mathrm{i}$ where a and b are real numbers. **[3]**

(b) Determine, in exact form, $\arg(-10 + (5\sqrt{12})\mathrm{i})^5$. **[3]**

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31 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

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32 Use an algebraic method to find the square roots of the complex number $43-(6\sqrt{10})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[5]**

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33 **In this question you must show detailed reasoning.**

Solve the equation $4z^2 - 20z + 169 = 0$. Give your answers in modulus-argument form. **[5]**

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34 In this question you must show detailed reasoning.

(a) Use an algebraic method to find the square roots of $-16 + 30\text{i}$. **[5]**

(b) By finding the cube of one of your answers to part **(a)** determine a cube root of $\dfrac{-99+5\text{i}}{4}$.

Give your answer in the form $a + b\text{i}$. **[2]**

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35 In this question you must show detailed reasoning.

Use an algebraic method to find the square roots of $-77 - 36\mathrm{i}$. **[6]**

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36 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

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37 **(a)** **(i)** Find the modulus and argument of z_1, where $z_1 = 1 + \mathrm{i}$. **[2]**

(ii) Given that $|z_2| = 2$ and $\arg(z_2) = \frac{1}{6}\pi$, express z_2 in $a + b\mathrm{i}$ form, where a and b are exact real numbers. **[2]**

(b) Using these results, find the exact value of $\sin\frac{5}{12}\pi$, giving the answer in the form $\dfrac{\sqrt{m}+\sqrt{n}}{p}$, where m, n and p are integers. **[5]**

38 **(i)** Use an algebraic method to find the square roots of the complex number 5 + 12i. [5]

(ii) Find $(3 - 2\mathrm{i})^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

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39 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30\text{i}$.

(i) Write down the other root of the quadratic equation. **[1]**

(ii) Find the values of a and b. **[4]**

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. **[7]**

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40 **(i)** Use an algebraic method to find the square roots of the complex number $9+40\mathrm{i}$. **[6]**

(ii) Show that $9+40\mathrm{i}$ is a root of the quadratic equation $z^2-18z+1681=0$. **[1]**

(iii) By using the substitution $z=\dfrac{1}{u^2}$, find the roots of the equation $1681u^4-18u^2+1=0$. Give your answers in the form $x+\mathrm{i}y$, where x and y are real. **[4]**

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41 **(i)** Use an algebraic method to find the square roots of the complex number $5+12\mathrm{i}$. You must show sufficient working to justify your answers. **[5]**

(ii) Hence solve the quadratic equation $x^2-4x-1-12\mathrm{i}=0$

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42 On an Argand diagram, the point A represents the complex number z with modulus 2 and argument $\frac{1}{3}\pi$. The point B represents $\frac{1}{z}$.

(a) Sketch an Argand diagram showing the origin O and the points A and B. **[2]**

(b) The point C is such that OACB is a parallelogram. C represents the complex number w.

Determine each of the following.

- The modulus of w, giving your answer in exact form.
- The argument of w, giving your answer correct to **3** significant figures. **[7]**

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43 The complex number z, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4\mathrm{i}$.

(i) Use an algebraic method to find z. **[5]**

(ii) Show that $z^3 = 2 + 11\mathrm{i}$. **[1]**

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w. **[5]**