London Academy of Maths

Proof by Induction - Divisibility

Question Paper

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1. Prove by induction that for all positive integers n,

$$f(n) = 3^{4n-2} + 2^{6n-3} \text{ is divisible by } 17$$

(6)

2. Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 7^{n-1} + 8^{2n+1}$$

is divisible by 57

(6)

3. (ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 12^n + 2 \times 5^{n-1}$$

is divisible by 7

(6)

4. (ii) Prove by induction that, for all positive integers n,

$$f(n) = 3^{3n-2} + 2^{4n-1}$$

is divisible by 11

(5)

5. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 7^n - 2^n \text{ is divisible by } 5$$

(6)

6. Prove by induction that, for $n \in \mathbb{Z}^+$

$$\mathrm{f}(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6)

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7. Prove by induction, that for $n \in \mathbb{Z}^+$,

(b) $\mathrm{f}(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

8 Prove by induction that $4 \times 8^n + 66$ is divisible by 14 for all integers $n \geqslant 0$. **[6]**

9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 8^n - 2^n$$

is divisible by 6

(6)

10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by } 5.$$

(6)

11 Prove that $2^{3n} - 3^n$ is divisible by 5 for all integers $n \geqslant 1$. **[5]**

12 Prove by induction that $5^n + 2 \times 11^n$ is divisible by 3 for all positive integers n. **[7]**

13 Prove by mathematical induction that $8^n - 3^n$ is divisible by 5 for all positive integers n. **[5]**

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14. Prove by induction that for all positive integers n,

$$f(n) = 2^{3n+1} + 3(5^{2n+1})$$

is divisible by 17

(6)

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15. (ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6)

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16. (ii) Prove by induction that, for all positive integers n,

$$f(n) = 3^{2n+4} - 2^{2n}$$

is divisible by 5

(5)

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17. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7

(6)

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18. Prove by induction that $2^{n+1}+5\times 9^{n}$ is divisible by 7 for all integers $n\geqslant 1$. **[6]**

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19. (ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 8^{2n+1} + 6^{2n-1}$$

is divisible by 7

(5)

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20 Prove by induction that $11 \times 7^{n} - 13^{n} - 1$ is divisible by 3, for all integers $n \geqslant 0$. **[5]**

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21. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

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22. $$\mathrm{f}(n) = 2^n + 6^n$$

(a) Show that $\mathrm{f}(k+1) = 6\mathrm{f}(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $\mathrm{f}(n)$ is divisible by 8.

(4)

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23. (ii) Prove by induction that, for $n \in \mathbb{N}$

$$f(n) = 5^{n+2} - 4n - 9$$

is divisible by 16

(5)

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24. Prove, by induction, that for $n \in \mathbb{Z}, n \geqslant 2$

$$4^n + 6n - 10$$

is divisible by 18

(5)

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25. (ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 5^{2n} + 3n - 1$$

is divisible by 9

(6)

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26. (ii) Prove by induction that, for all positive integers n,

$$f(n) = 3^{2n+3} + 40n - 27 \text{ is divisible by } 64$$

(6)

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27. (ii) Prove by induction that,

$$4^n + 6n + 8 \text{ is divisible by } 18$$

for all positive integers n.

(6)

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28 It is given that $u_n = 13^n + 6^{n-1}$, where n is a positive integer.

(i) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$. **[3]**

(ii) Prove by induction that u_n is a multiple of 7. **[4]**

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29 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

(i) Find u_1, u_2 and u_3. **[2]**

(ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. **[1]**

(iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part **(ii)** is correct. **[5]**

30 Prove by induction that the sum of the cubes of three consecutive positive integers is divisible by 9. **[5]**

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31. (i) $$\mathrm{f}(n) = 7^n(3n + 1) - 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $\mathrm{f}(n)$ is a multiple of 9

(6)